
Contents

| | |
|--|------|
| Introduction | xi |
| Table of Notations | xiii |
| Chapter 1. Vector Calculus | 1 |
| 1.1. Vector space | 1 |
| 1.1.1. Definition | 1 |
| 1.1.2. Vector space – dimension – basis | 2 |
| 1.1.3. Affine space. | 3 |
| 1.2. Affine space of dimension 3 – free vector | 4 |
| 1.3. Scalar product $\bar{a} \cdot \bar{b}$ | 5 |
| 1.3.1. Properties of the scalar product | 6 |
| 1.3.2. Scalar square – unit vector | 6 |
| 1.3.3. Geometric interpretation of the scalar product | 7 |
| 1.3.4. Solving the equation $\bar{a} \cdot \bar{x} = 0$ | 9 |
| 1.4. Vector product $\bar{a} \wedge \bar{b}$ | 9 |
| 1.4.1. Definition | 9 |
| 1.4.2. Geometric interpretation of the vector product | 10 |
| 1.4.3. Properties of vector product | 11 |
| 1.4.4. Solving the equation $\bar{a} \wedge \bar{x} = \bar{b}$ | 11 |
| 1.5. Mixed product $(\bar{a}, \bar{b}, \bar{c})$ | 12 |
| 1.5.1. Definition | 12 |
| 1.5.2. Geometric interpretation of the mixed product | 12 |
| 1.5.3. Properties of the mixed product | 13 |
| 1.6. Vector calculus in the affine space of dimension 3 | 15 |
| 1.6.1. Orthonormal basis | 15 |

| | |
|--|-----------|
| 1.6.2. Analytical expression of the scalar product | 16 |
| 1.6.3. Analytical expression of the vector product | 16 |
| 1.6.4. Analytical expression of the mixed product | 17 |
| 1.7. Applications of vector calculus | 18 |
| 1.7.1. Double vector product | 18 |
| 1.7.2. Resolving the equation $\vec{a} \cdot \vec{x} = b$ | 22 |
| 1.7.3. Resolving the equation $\vec{a} \wedge \vec{x} = \vec{b}$ | 23 |
| 1.7.4. Equality of Lagrange | 25 |
| 1.7.5. Equations of planes | 25 |
| 1.7.6. Relations within the triangle | 27 |
| 1.8. Vectors and basis changes | 28 |
| 1.8.1. Einstein's convention | 28 |
| 1.8.2. Transition table from basis (e) to basis (E) | 30 |
| 1.8.3. Characterization of the transition table | 32 |
| Chapter 2. Torsors and Torsor Calculus | 35 |
| 2.1. Vector sets | 35 |
| 2.1.1. Discrete set of vectors | 35 |
| 2.1.2. Set of vectors defined on a continuum | 36 |
| 2.2. Introduction to torsors | 37 |
| 2.2.1. Definition | 37 |
| 2.2.2. Equivalence of vector families | 38 |
| 2.3. Algebra torsors | 38 |
| 2.3.1. Equality of two torsors | 38 |
| 2.3.2. Linear combination of torsors | 39 |
| 2.3.3. Null torsors | 39 |
| 2.3.4. Opposing torsor | 40 |
| 2.3.5. Product of two torsors | 40 |
| 2.3.6. Scalar moment of a torsor – equiprojectivity | 41 |
| 2.3.7. Invariant scalar of a torsor | 43 |
| 2.4. Characterization and classification of torsors | 43 |
| 2.4.1. Torsors with a null resultant | 43 |
| 2.4.2. Torsors with a no-null resultant | 45 |
| 2.5. Derivation torsors | 48 |
| 2.5.1. Torsor dependent on a single parameter q | 49 |
| 2.5.2. Torsor dependent of n parameters q_i functions of p | 51 |
| 2.5.3. Explicitly dependent torsor of $n + l$ parameters | 52 |

| | |
|--|-----|
| Chapter 3. Derivation of Vector Functions | 55 |
| 3.1. Derivative vector: definition and properties | 55 |
| 3.2. Derivative of a function in a basis | 56 |
| 3.3. Deriving a vector function of a variable. | 57 |
| 3.3.1. Relations between derivatives of a function in different bases | 57 |
| 3.3.2. Differential form associated with two bases. | 63 |
| 3.4. Deriving a vector function of two variables | 65 |
| 3.5. Deriving a vector function of n variables | 68 |
| 3.6. Explicit intervention of the variable p | 70 |
| 3.7. Relative rotation rate of a basis relative to another | 71 |
| Chapter 4. Vector Functions of One Variable Skew Curves | 73 |
| 4.1. Vector function of one variable. | 73 |
| 4.2. Tangent at a point M | 74 |
| 4.3. Unit tangent vector $\bar{\tau}(q)$ | 76 |
| 4.4. Main normal vector $\bar{N}(q)$ | 77 |
| 4.5. Unit binormal vector $\bar{\beta}(q)$ | 79 |
| 4.6. Frenet's basis | 80 |
| 4.7. Curvilinear abscissa | 81 |
| 4.8. Curvature, curvature center and curvature radius | 83 |
| 4.9. Torsion and torsion radius. | 84 |
| 4.10. Orientation in (λ) of the Frenet basis. | 87 |
| Chapter 5. Vector Functions of Two Variables Surfaces | 91 |
| 5.1. Representation of a vector function of two variables | 91 |
| 5.1.1. Coordinate curves | 91 |
| 5.1.2. Regular or singular point – tangent plane – unit normal vector | 93 |
| 5.1.3. Distinctive surfaces | 95 |
| 5.1.4. Ruled surfaces | 101 |
| 5.1.5. Area element | 110 |
| 5.2. General properties of surfaces | 111 |
| 5.2.1. First quadratic form | 111 |
| 5.2.2. Darboux–Ribaucour's trihedral | 114 |
| 5.2.3. Second quadratic form | 119 |
| 5.2.4. Meusnier's theorems | 121 |

| | |
|---|------------|
| 5.2.5. Geodesic torsion | 123 |
| 5.2.6. Prominent curves traced on a surface | 125 |
| 5.2.7. Directions and principal curvatures of a surface | 127 |
| Chapter 6. Vector Function of Three Variables: Volumes | 135 |
| 6.1. Vector functions of three variables | 135 |
| 6.1.1. Coordinate surfaces. | 135 |
| 6.1.2. Coordinate curves. | 136 |
| 6.1.3. Orthogonal curvilinear coordinates | 136 |
| 6.2. Volume element | 137 |
| 6.2.1. Definition. | 137 |
| 6.2.2. Applications to traditional coordinate systems. | 138 |
| 6.3. Rotation rate of the local basis. | 139 |
| 6.3.1. Calculation of the partial rotation rate $\overset{1}{\delta}(\lambda, e)$ | 140 |
| 6.3.2. Calculation of the rotation rate | 143 |
| Chapter 7. Linear Operators | 145 |
| 7.1. Definition | 145 |
| 7.2. Intrinsic properties. | 145 |
| 7.3. Algebra of linear operators. | 147 |
| 7.3.1. Unit operator. | 147 |
| 7.3.2. Equality of two linear operators | 147 |
| 7.3.3. Product of a linear operator by a scalar | 147 |
| 7.3.4. Sum of two linear operators. | 148 |
| 7.3.5. Multiplying two linear operators | 148 |
| 7.4. Bilinear form | 149 |
| 7.5. Quadratic form | 150 |
| 7.6. Linear operator and basis change | 150 |
| 7.7. Examples of linear operators. | 152 |
| 7.7.1. Operation $\underline{f} = \underline{a} \wedge \underline{F}$ | 152 |
| 7.7.2. Operation $\underline{f} = \underline{a} \wedge (\underline{a} \wedge \underline{F})$ | 152 |
| 7.7.3. Operation $\underline{f} = \underline{a}(\underline{b} \cdot \underline{F})$ | 153 |
| 7.7.4. Operation $\underline{f} = \underline{a} \wedge (\underline{F} \wedge \underline{a})$ | 155 |
| 7.8. Vector rotation $\mathcal{R}_{\underline{u}, \underline{a}}$ | 156 |
| 7.8.1. Expression of the vector rotation | 156 |
| 7.8.2. Quaternion associated with the vector rotation $\mathcal{R}_{\underline{u}, \underline{a}}$ | 159 |
| 7.8.3. Matrix representation of the vector rotation | 160 |
| 7.8.4. Basis change and rotation vector | 162 |

| | |
|--|-----|
| Chapter 8. Homogeneity and Dimension. | 165 |
| 8.1. Notion of homogeneity | 165 |
| 8.2. Dimension | 165 |
| 8.3. Standard mechanical dimensions. | 166 |
| 8.4. Using dimensional equations | 168 |
| Bibliography | 171 |
| Index. | 173 |