Table of Contents

Introduction xv

Chapter 1. Mathematical Prerequisites 1

1.1. Frequently used special functions 1
1.1.1. The “rectangle” function. 2
1.1.2. The “sinc” function 3
1.1.3. The “sign” function. 4
1.1.4. The “triangle” function 5
1.1.5. The “disk” function. 5
1.1.6. The Dirac δ function 6
1.1.6.1. Definition 6
1.1.6.2. Fundamental properties 8
1.1.7. The “comb” function 9
1.2. Two-dimensional Fourier transform 10
1.2.1. Definition and existence conditions 10
1.2.2. Theorems related to the Fourier transform 11
1.2.2.1. Linearity 13
1.2.2.2. Similarity. 13
1.2.2.3. Translation 13
1.2.2.4. Parseval’s theorem 13
1.2.2.5. The convolution theorem 14
1.2.2.6. The autocorrelation theorem 14
1.2.2.7. The duality theorem 14
1.2.3. Fourier transforms in polar coordinates 15
1.3. Linear systems 17
1.3.1. Definition 17
1.3.2. Impulse response and superposition integrals 18
1.3.3. Definition of a two-dimensional linear shift-invariant system. . . . 19

1.3.4. Transfer functions 20
1.4. The sampling theorem 21
1.4.1. Sampling a continuous function 21
1.4.2. Reconstruction of the original function 23
1.4.3. Space-bandwidth product 25

Chapter 2. The Scalar Theory of Diffraction 27

2.1. Representation of an optical wave by a complex function 28
2.1.1. Representation of a monochromatic wave 28
2.1.2. Complex amplitude of the optical field in space. 29
2.1.2.1. Plane waves 29
2.1.2.2. Spherical waves. 30
2.1.3. Complex amplitudes of plane and spherical waves in a front plane 31
2.1.3.1. Complex amplitude of a plane wave in a front plane 31
2.1.3.2. Complex amplitude of a spherical wave in a front plane. . . 32
2.2. Scalar theory of diffraction. 33
2.2.1. Wave equation 33
2.2.2. Harmonic plane wave solutions to the wave equation 34
2.2.3. Angular spectrum. 35
2.2.4. Kirchhoff and Rayleigh–Sommerfeld formulae 39
2.2.5. Fresnel approximation and Fresnel diffraction integral. 41
2.2.6. The Fraunhofer approximation 43
2.3. Examples of Fraunhofer diffraction patterns 45
2.3.1. Fraunhofer diffraction pattern from a rectangular aperture 45
2.3.2. Fraunhofer diffraction pattern from a circular aperture. 46
2.3.3. Fraunhofer diffraction pattern from a sinusoidal-amplitude grating 49
2.4. Some examples and uses of Fresnel diffraction. 51
2.4.1. Fresnel diffraction from a sinusoidal-amplitude grating 51
2.4.2. Fresnel diffraction from a rectangular aperture 54
2.5. Collins' formula 57
2.5.1. Description of an optical system by an ABCD transfer matrix 58
2.5.2. ABCD law and paraxial systems equivalent to a lens. 62
2.5.2.1. ABCD law of a spherical wave propagating across an optical system 62
2.5.2.2. System equivalent to a lens 64
2.5.2.3. Properties of the transfer matrix 65
2.5.3. Proof of Collins' formula 66
2.5.3.1. Transmission from a thin lens 67
2.5.3.2. Expression of the ideal image 68

2.5.3.3. Proof of Collins' formula 70
2.5.4. Comparison between Collins' formula and the Fresnel integral 72
2.6. Conclusion 74

Chapter 3. Calculating Diffraction by Fast Fourier Transform 77

3.1. Relation between the discrete and analytical Fourier transforms 78
3.1.1. Sampling and periodic expansion of a continuous two-dimensional function 78
3.1.2. The relation between the discrete and continuous Fourier transforms 79
3.2. Calculating the Fresnel diffraction integral by FFT 83
3.2.1. Calculating diffraction by the S-FFT method 84
3.2.2. Numerical calculation and experimental demonstration 86
3.2.3. The D-FFT method 88
3.2.4. Practical sampling conditions due to the energy conservation principle 89
3.2.5. Experimental demonstration of the D-FFT method 90
3.3. Calculation of the classical diffraction formulae using FFT 92
3.3.1. Kirchhoff and Rayleigh-Sommerfeld formulae in convolution form 92
3.3.2. Unitary representation of the classical diffraction formulae 94
3.3.3. Study of the sampling conditions of the classical formulae 95
3.3.4. Example of calculations of the classical diffraction formulae 97
3.3.5. Calculation of diffraction by convolution: summary 100
3.4. Numerical calculation of Collins' formula 101
3.4.1. Collin's direct and inverse formulae 101
3.4.2. Calculating Collins' formula by S-FFT 103
3.4.3. Calculating the inverse Collins' formula by S-FFT 105
3.4.4. Calculating Collins' formula by D-FFT 106
3.4.5. Calculating the inverse Collins' formula by D-FFT 108
3.4.6. Numerical calculation and experimental demonstration 109
3.4.6.1. Demonstration of the S-FFT and S-IFFT methods 110
3.4.6.2. Demonstration of the D-FFT method 111
3.5. Conclusion 113

Chapter 4. Fundamentals of Holography 115

4.1. Basics of holography 116
4.1.1. Leith–Upatnieks holograms 118
4.1.1.1. Illumination in the propagation direction of the original reference wave 120

4.1.1.2. Illumination with a wave propagating along the z-axis. . . . 121
4.1.2. Condition for the separation of the twin images and the zero order . . . 123
4.1.2.1. Case where the reference wave is planar . . . 123
4.1.2.2. The case where the reference wave is no longer planar . . . 125
4.2. Partially coherent light and its use in holography . . . 127
4.2.1. Analytic signal describing a non-monochromatic wave . . . 127
4.2.1.1. Analytic signal describing a monochromatic wave . . . 127
4.2.1.2. Analytic signal describing a non-monochromatic wave . . . 128
4.2.1.3. Analytic signal and spectrum of a laser wave . . . 129
4.2.2. Recording a hologram with non-monochromatic light . . . 131
4.2.3. Total coherence approximation conditions . . . 134
4.2.3.1. Identical wave train model . . . 134
4.2.3.2. Temporal coherence of a source emitting identical wave trains . . . 135
4.2.4. Recording a Fresnel hologram . . . 139
4.3. Study of the Fresnel hologram of point source . . . 141
4.3.1. Reconstructing the hologram of a point source . . . 142
4.3.1.1. Hologram illuminated by a spherical wave with the same wavelength . . . 142
4.3.1.2. Hologram illuminated by a spherical wave with a different wavelength . . . 145
4.3.1.3. Case where the reference and reconstruction waves are plane waves . . . 145
4.3.2. Magnifications . . . 146
4.3.2.1. Transverse magnification of the reconstructed image . . . 146
4.3.2.2. Longitudinal magnification . . . 147
4.3.3. Resolution of the reconstructed image . . . 147
4.3.3.1. Influence of the size of the illuminating source . . . 147
4.3.3.2. Influence of the spectral width of the light source . . . 148
4.4. Different types of hologram . . . 149
4.4.1. The Fraunhofer hologram . . . 149
4.4.2. The Fourier hologram . . . 150
4.4.2.1. Component $C\widetilde{u}_1(x_i, y_i)$ . . . 153
4.4.2.2. Component $C\widetilde{u}_2(x_i, y_i)$ . . . 153
4.4.2.3. Component $C\widetilde{u}_3(x_i, y_i)$ . . . 154
4.4.2.4. Component $C\widetilde{u}_4(x_i, y_i)$ . . . 154
4.4.3. The lensless Fourier hologram . . . 155
4.4.4. The image hologram . . . 158
4.4.5. The phase hologram . . . 160
4.5. Conclusion . . . 163

Chapter 5. Digital Off-Axis Fresnel Holography 165

5.1. Digital off-axis holography and wavefront reconstruction by S-FFT 166
5.1.1. Characteristics of the diffraction from a digital hologram impacted by a spherical wave 166
5.1.1.1. Virtual object 168
5.1.1.2. Conjugate object 169
5.1.1.3. Zero order 170
5.1.2. Optimization of the experimental parameters 172
5.1.3. Experimental reconstruction by S-FFT 173
5.1.4. Quality of the reconstructed image 176
5.2. Elimination of parasitic orders with the S-FFT method 183
5.2.1. Diffraction efficiency of a digital hologram 184
5.2.2. Methods of direct elimination 185
5.2.2.1. Method directly eliminating the object and reference waves 185
5.2.2.2. Method with an arbitrary phase shift of the reference wave 188
5.2.3. Method of extracting the complex amplitude of the object wave 189
5.3. Wavefront reconstruction with an adjustable magnification 191
5.3.1. Convolution with adjustable magnification 192
5.3.2. Experiment with adjustable magnification 195
5.3.3. Elimination of the perturbation due to the zero order 197
5.3.3.1. Spectral distribution and determination of the center of the object wave 197
5.3.3.2. Spectral position of the object wave 200
5.3.4. Method eliminating the perturbation due to the zero order 201
5.4. Filtering in the image and reconstruction planes by the FIMG4FFT method 202
5.4.1. Adjustable magnification reconstruction by the FIMG4FFT method 203
5.4.1.1. Filtering the image plane 203
5.4.1.2. Experimental results 205
5.4.1.3. Local reconstruction by the FIMG4FFT method 205
5.5. DBFT method and the use of filtering in the image plane 207
5.5.1. DBFT method 207
5.5.2. Sampling of the DBFT algorithm 208
5.5.3. Improvement of the DBFT method 210
5.5.4. Experimental demonstration of the DDBFT method 211
5.6. Digital color holography 212
5.6.1. Recording a digital color hologram 213

5.6.2. Standardization of the physical scale of reconstructed monochromatic images. 215
5.6.3. Fresnel transform with wavelength-dependant zero-padding 216
5.6.4. Experimental study of the different methods of reconstructing color images. 218
5.6.4.1. Two-color image calculated by the zero-padding method. 220
5.6.4.2. Reconstruction by the FIMG4FFT and the DDBFT with adjustable magnification 221
5.6.4.3. Three-color digital holography 222
5.7. Digital phase hologram 224
5.7.1. Formation of a digital phase hologram and reconstruction by S-FFT 225
5.7.2. Experimental demonstration 227
5.8. Depth of focus of the reconstructed image 229
5.8.1. Theoretical analysis 229
5.8.2. Comparison with a digital holographic simulation 231
5.8.3. Experiments 233
5.9. Conclusion 235

Chapter 6. Reconstructing Wavefronts Propagated through an Optical System 237

6.1. Theoretical basis 238
6.1.1. Case of a convergent lens 239
6.1.2. Impulse response of the process 240
6.1.2.1. Matrix description of the digital holographic system 240
6.1.2.2. Impulse response from the digital holographic system 241
6.1.2.3. Experimental reconstruction with an afocal system. 243
6.2. Digital holography with a zoom. 246
6.2.1. Principle of the zoom 247
6.2.2. Study of the zoom 248
6.2.3. Design of the zoom. 249
6.2.4. Experimental validation 251
6.3. Reconstructing an image by Collins' formula. 251
6.3.1. Reconstruction algorithm 251
6.3.1.1. Experimental setup 251
6.3.1.2. Reconstruction by calculating the inverse Collins formula 252
6.3.2. Adjustable-magnification reconstruction after propagation across an optical system 254
6.3.2.1. Reconstruction with an adjustable magnification in the detector space 255

6.3.2.2. Experimental demonstration for the reconstruction of color images . . . 257
6.4. Using the classical diffraction formulae to reconstruct the wavefront after propagation across an optical system. . . . 259
6.4.1. Use of the rigorous diffraction formulae . . . 259
6.4.1.1. Simulation of the overall process . . . 260
6.4.1.2. Experimental results . . . 264
6.4.2. Reconstruction of the object wave in the object and image spaces . . . 265
6.4.2.1. Parameters . . . 265
6.4.2.2. Reconstruction in the image space . . . 266
6.4.2.3. Reconstruction in the object space . . . 267
6.4.2.4. Experimental results . . . 268
6.5. Conclusion . . . 269

Chapter 7. Digital Holographic Interferometry and Its Applications . . . 271

7.1. Basics of holographic interferometry . . . 273
7.1.1. Reconstructing the phase of the object field . . . 273
7.1.2. Optical phase variations and the sensitivity vector . . . 274
7.1.3. Phase difference method . . . 275
7.1.4. Spatial filtering of the phase . . . 276
7.1.5. Phase unwrapping . . . 277
7.1.6. Out-of-plane sensitivity . . . 279
7.1.7. In-plane sensitivity . . . 280
7.1.8. 3D sensitivity . . . 280
7.1.9. Sensitivity variation across the field of view . . . 282
7.2. Digital holographic microscopy . . . 282
7.2.1. Principles and advantages . . . 282
7.2.2. Architectures . . . 283
7.2.3. Reconstruction of the object field . . . 285
7.2.4. Phase contrast . . . 287
7.3. Two-wavelength profilometry . . . 288
7.3.1. Principle . . . 288
7.3.2. Two-wavelength profilometry with spatio-chromatic multiplexing . . . 289
7.4. Digital holographic photomechanics . . . 291
7.4.1. Introduction . . . 291
7.4.2. Twin-sensitivity measurement with monochromatic multiplexing . . . 292
7.4.2.1. Principle . . . 292
7.4.2.2. Application: polymer concrete in a three-point bending test . . . 293

7.4.2.3. Experimental results 294
7.4.2.4. Comparison with finite element modeling 296
7.4.3. Twin-sensitivity measurement with spatio-chromatic multiplexing 297
7.4.3.1. Principle 297
7.4.3.2. Application to crack detection in electrical components 297
7.4.4. 3D measurement by three-color digital holography 299
7.4.4.1. Principle 299
7.4.4.2. Illustration 301
7.4.4.3. Application to the study of the mechanical behavior of composite materials 302
7.5. Time-averaged digital holography 303
7.5.1. Principle 303
7.5.2. Applications 304
7.5.2.1. Modal analysis 304
7.5.2.2. Analog/digital comparison 305
7.5.2.3. Time-averaged phase 307
7.6. Tracking high-amplitude vibrations 309
7.6.1. Introduction 309
7.6.2. Principle 310
7.7. Three-color digital holographic interferometry for fluid mechanics 314
7.7.1. Principle 314
7.7.2. Application to turbulent flows 317
7.8. Conclusion 318

Appendix. Examples of Digital Hologram Reconstruction Programs 319

A1.1. Diffraction calculation using the S-FFT algorithm 319
A1.1.1. Code for the program: LIM1.m 319
A1.1.2. Examples of diffraction calculations using LIM1.m 321
A1.2. Diffraction calculation by D-FFT 323
A1.2.1. Code for the program: LIM2.m 323
A1.2.2. Examples of diffraction calculations using LIM2.m 325
A1.3. Simulation of a digital hologram 326
A1.3.1. Code for the program: LIM3.m 326
A1.3.2. Example of the calculation of a hologram with LIM3.m 329
A1.4. Reconstruction of a hologram by S-FFT 330
A1.4.1. Code for the program: LIM4.m 330
A1.4.2. Example of reconstruction with LIM4.m 332

A1.5. Adjustable-magnification reconstruction by D-FFT 333
A1.5.1. Code for the program: LIM5.m 333
A1.5.2. Example of adjustable-magnification reconstruction with LIM5.m 336

Bibliography 339

Index 355