Preface

It is a common scheme in many sciences to study systems or signals by looking for *characteristic scales* in time or space. These are then used as references for expressing all measured quantities. Physicists may for instance employ the size of a structure, while signal processors are often interested in correlation lengths: (blocks of) samples whose distance is several times the correlation lengths are considered statistically independent. The concept of *scale invariance* may be considered to be the converse of this approach: it means that there is no characteristic scale in the system. In other words, all scales contribute to the observed phenomenon. This "non-property" is also loosely referred to as *scaling law* or *scaling behavior*. Note that we may reverse the perspective and consider scale invariance as the signature of a strong organization in the system. Indeed, it is well known in physics that invariance laws are associated with fundamental properties. It is remarkable that phenomena where scaling laws have been observed cover a wide range of fields, both in natural and artificial systems. In the first category, these include for instance hydrology, in relation to the variability of water levels, hydrodynamics and the study of turbulence, statistical physics with the study of long-range interactions, electronics with the so-called $1/f$ noise in semiconductors, geophysics with the distribution of faults, biology, physiology and the variability of human body rhythms such as the heart rate. In the second category, we may mention geography with the distribution of population in cities or in continents, Internet traffic and financial markets.

From a signal processing perspective, the aim is then to study *transfer mechanisms* between scales (also called "cascades") rather than to identify relevant scales. We are thus led to forget about scale-based models (such as Markov models), and to focus on models allowing us to study correspondences between many scales. The central notion behind scaling laws is that of *self-similarity*. Loosely speaking, this means that each part is (statistically) the same as the whole object. In particular, information gathered from observing the data should be independent of the scale of observation.

There is considerable variety in observed self-similar behaviors. They may for instance appear through scaling laws in the Fourier domain, either at all frequencies or in a finite but large range of frequencies, or even in the limit of high or low frequencies. In many cases, studying second-order quantities such as spectra will prove insufficient for describing scaling laws. Higher-order moments are then necessary. More generally, the fundamental model of self-similarity has to be adapted in many settings, and to be generalized in various directions, so that it becomes useful in real-world situations. These include self-similar stochastic processes, $1/f$ processes, long memory processes, multifractal and multifractional processes, locally self-similar processes and more. Multifractal analysis, in particular, has developed as a method allowing us to study complex objects which are not necessarily "fractal", by describing the variations of local regularity. The recent change of paradigm consisting of using *fractal methods* rather than studying *fractal objects* is one of the reasons for the success of the domain in applications.

We are delighted to invite our reader for a promenade in the realm of scaling laws, its mathematical models and its real-world manifestations. The 14 chapters have all been written by experts. The first four chapters deal with the general mathematical tools allowing us to measure fractional dimensions, local regularity and scaling in its various disguises. Wavelets play a particular role for this purpose, and their role is emphasized. Chapters 5 and 6 describe advanced stochastic models relevant in our area. Chapter 7 deals with fractional calculus, and Chapter 8 explains how to synthesize certain fractal models. Chapter 9 gives a general introduction to IFS, a powerful tool for building and describing fractals and other complex objects, while Chapter 10, of applied nature, considers the application of IFS to image compression. The four remaining chapters also deal with applications: various signal and image processing tasks are considered in Chapter 11. Chapter 12 deals with Internet traffic, and Chapter 13 with financial data analysis. Finally, Chapter 14 describes a fractal space-time in the frame of cosmology.

It is a great pleasure for us to thank all the authors of this volume for the quality of their contribution. We believe they have succeeded in exposing advanced concepts with great pedagogy.