General Introduction

The mathematical results of the axiomatized theory of multicriterion decision-making as presented in this book are often surprising and counterintuitive. The French tradition will classify them as "pure" mathematics, because they are not without a strong relation to fundamental mathematics, while they nevertheless remain directly linked to phenomena of daily life. This proximity perhaps accounts for a part of the interest we hold for them.

The reader will therefore not be too surprised to see the first few chapters dedicated in part to philosophical and psychological considerations. These considerations justify the mathematical expositions that follow. They do not rely too heavily on formalism, but they are nevertheless very consequential.

In the 1960s, industrial decision makers acutely felt a need for multicriterion decision-making models. This need turned out to be difficult to meet. May's theorems, G. Debreu's thesis, Allais's famous paper on the "Critique of the American School's postulates", and above all Arrow's impossibility theorem – all these efforts showed that there was still indeed a long way to go and that "traditional" solutions were actually treacherous.

After the publication of Arrow's theorem [ARR 63], researchers' efforts have improved the analysis and rationalization of committee decision-making. Questions of strategy and cunning were a dominant theme. On these subjects, one may consult, for example, Sen [SEN 70], Fine & Fine [FIN 74], Gibbard [GIB 73], Satterswhaite [SAT 75], Moulin [MOU 80, 82, 83], and Tverski [TVE 81]. Numerous experimental and logical difficulties proved that the problem of multicriterion decision-making was far from solved.

Nevertheless, this need for a solution was particularly urgent, and commercial consulting organizations were aplenty. This is how one saw a plethora of intuitive

recipes crop up, all in full denial of the issue's true difficulty. They were more or less scientific, more or less falsifiable, and no meta-method permitted comparing their performances.

This is why models currently used in Operations Research for multicriterion ordering of a finite set of alternatives are still lacking in solid foundations. Even the methods most used are still subject to heated controversies.

Moreover, papers such as "Douze methods d'analyse multicritère", by Bernard and Besson [BER 71], had quickly shown that inventing a new technique was not difficult. Much harder, instead, was seriously evaluating that technique's legitimacy. Even today, specialized conferences present countless "new" heuristics that refer to some sort of numerical intuition, which makes impossible a proper evaluation of their comparative effectiveness (which is sometimes real with regard to the precise problems they solve).

A celebrated example in Europe is the case of ELECTRE (see Appendix 3.1), whose commercial success turned out to be remarkable. In its first developments (see Susman *et al.* [SUS 67]), with which I was very closely associated, the recipe was simple, a quasi-hoax. Rather than satisfying epistemological demands, it sought to satisfy the members of the research department who first conceived it: they just wanted to laugh. The authors were, moreover, not very proud of the joke. Even today, they are somewhat embarrassed to be associated with it.

Despite violent critiques, many excellent managers entertained the idea, over the course of 20 years (at least in Europe), that ELECTRE was the only legitimate technique to help difficult decision-making with multiple criteria. However, none of ELECTRE "methods" seems (up to the present) to satisfy a coherent and promising axiomatic system (see, e.g. [BOU 93]).

The same was true across the Atlantic for a "system to help decision makers", commercially baptized the *analytic hierarchy process* (AHP). Sold with great talent, accompanied by brilliant software, and conceived with the stamp of approval of a scientist whose competence on other questions is exceptional, it was admired to such an extent that various conferences were completely dedicated to it. However, this technique has up to now only deserved an axiomatization befitting a postmodern frenzy. A very recent effort at partial axiomatization, completely to the credit of the Chilean "fanatics" who authored it, lays bare the serious limitations to its validity. The lack of epistemological foundations of AHP provoked such an outcry from its detractors that calling their response violent can only be a euphemism.

As long as it remains a matter of specific problems with limited and exclusively material consequences, a battery of good, dedicated recipes – which

show creativity, intuition, and adaptability to specific conditions – may perhaps address such problems better than a single and rigid mathematical model, however elegant it may be.

But these specific recipes are often used very inappropriately. In this connection, just a few years ago, a very big European company, comprising thousands of engineers, was making a difficult decision in terms of risk to human life. They almost used a model that was completely obsolete, inappropriate, and denounced for a good 60 years for its discrepancies. Fortunately, at the eleventh hour, a senior manager realized the absurdity before it was carried out.

To avoid the drama that could result in such errors, this textbook offers decision makers and their consultants an approach taken from the lessons of Social Choice Theory. It hopefully allows them to identify a reasonable decision-making method. We have sought to respond, at least partially, to the real and legitimate demands of decision makers and honest consultants on the question of multicriterion analysis.

The word "reasonable" obviously deserves to be specified. We propose considering a method reasonable if, of course, it offers a legitimate and operational synthesis of available information, but also if:

– it neither needs prohibitive calculations nor uses a level of conceptual sophistication that would make the whole procedure unintelligible for the engineers who would be most often the ones who implement it;

– its degree of sophistication is adapted to the quality of the data that it has to synthesize;

– it does not produce a pseudo-scientific disguise for the decision maker who seeks only to justify a personal choice.

The primary need of decision makers is perhaps realizing that they may be skewed by a series of the biases they want to avoid. Therefore, to make decisions drawn up reasonably well, they need a true methodology.

This has to be completely different from a miraculous and alleged panacea-recipe, which would be presented as suiting any possible multicriterion decision problem [TAN 87].

One could and still can observe a rather surprisingly high credulity on the part of users, with little ambition for mental effort and quality.

This is why our axiomatic approach first aimed to raise the moral standard of the market of methods. In this time of drought for reason, the size of this market has produced too many temptations.

Very quickly, our own attempt showed that it was also possible to start to respond to the real needs of decision makers in the terms of multicriterion analysis.

If they are guided by healthy principles and some courage – and not primarily by a basic and perverted obsession with power – the user of axiomatized approaches should be able to avoid:

– techniques that are flexible enough to justify any which arbitrary choice after the fact;

– unstable techniques, prone to fragile results, which could strongly magnify certain personal biases unconsciously introduced;

– techniques so rigid that their application turns out to be ridiculously torturous.

We have deliberately concentrated our study on a limited set of multicriterion decision-making problems. Neurosciences inspired this limitation. The human brain is more prone to logical errors when it must choose between a *large* number of objects according to a *large* number of criteria, where the word "large" simply means more than five.

Moreover, two reasons brought us to focus our attention on criteria that are non-numerical structures. These structures are most often total orders, and in certain cases preorders, on the compared objects.

First, the Social Choice Theory was almost the only approach that concerned itself with laying the foundations for multicriterion decision-making procedures (This is true for domains where the decision consists of ordering a finite set of alternatives arranged according to a set of ordinal criteria.).

Second, we have known, since Eckenrode [ECK 65] and Johnsen [JOH 68], that, when it comes to preferences, the stability of ordinal evaluations is much higher than that of numerical evaluations: they correspond much better to our neurological functioning. These two reasons explain why we limited ourselves to the purely ordinal case.

In addition, we have sought to learn three lessons from the success of ELECTRE, MAUT, and AHP:

– decision makers like the fact that these techniques claim to model a somehow perfect decision maker, one who could extend to "big" and "complex" problems a psychologically natural technique successful on "small" and "simple" problems;

– decision makers considerably appreciate being able to understand the principles of relatively simple calculations;

– with their arsenal of parameters, these techniques give decision makers the impression they will not take up all their power. More modestly, one can even read in certain commercial brochures that the method offered is capable of producing good solutions all while assuring decision makers that their "exclusive power as managers" is not under threat.

One should therefore expect a certain suspicion toward "serious" methods, since they obviously threaten to demonstrate the shortcomings of the decision maker's "intuition".

It is thus natural that these pages begin by translating the results of the Social Choice Theory into the current language of Operations Research. This exercise then naturally led us to prove theorems that are able to respond to managers' specific needs, which we just described. The layout of the book reflects this scientific adventure.

The first part, after the description of the phenomenological and psychophysiological backdrop of the problem, gives a summary of how axiomatic systems, or the methods possibly taken at first glance to be the most habitual or natural, may lead to untenable paradoxes.

The second part seeks in particular to identify the truly legitimate application domain of the majority method to our difficult decision-making problems. To do this, we first ought to complete a list of effectiveness conditions for the majority method in a managerial context.

This part ends by noting that the managerial problems to which this method is ultimately applicable only form (statistically speaking) an asymptotically negligible set. In less technical terms, the situations in which the majority method may be legitimately used have only a faint chance of occurring in reality. Chapters 2 through 5 arise from a close collaboration between Hervé Raynaud and Kenneth J. Arrow.

The third and fourth parts deal with the axiomatization of other classical methods. This allows choosing, rejecting, or even constructing other methods that better respect the rationality proper to diverse problems in managerial decision-making. The third part concerns the particular set of problems associated with choice functions, whereas the fourth part concerns those associated with ordering functions.

The reader will therefore discover suggestions for non-contradictory axiomatic systems that are able to formalize specific properties of the tools that help along certain difficult decision-making processes. Identifying methods characterized by

these axiomatic systems also allowed us to bring to light the properties that could discredit them.

In particular, we have discussed "prudence", which extends the majority method. On this exact question, Kenneth J. Arrow's contribution was essential to sections 8.1, and to Chapters 13 and 14. The rest of the book is the sole responsibility of Hervé Raynaud – particularly what the reader may consider likely to provoke controversy, and the passages written in the first person of the singular. Otherwise, the ideas evoked in the book are due to colleagues and collaborators explicitly mentioned in the relevant passages.

The writing of this book and the corresponding investigations began at the Center for Organizational Efficiency and at the Stanford Institute for Mathematical Studies in the Social Sciences (contract ONR-N00014-792-0685 of the United States Office of Naval Research). The large part of the collaboration with Kenneth Arrow for this book took place here. The rest of the work was completed at the Academy of Sciences of Israel, the Joseph Fourier University in Grenoble the "*Décision*" working group of the Laboratoire des Structures Discrètes and then of the Laboratoire Leibniz, the Autonomous University of Mexico (Institute IIMAS), the University of Sherbrooke (Department of Mathematics), the Institute of Mathematical Research of Rio de Janeiro, and the Sigmund Freud University in Vienna and Paris. We thank all these organizations for their reception and support.

My first thanks go to my exceptionally motivated translator, Adwait Parker, a doctoral student in philosophy at Stanford, who helped me all throughout the translation of this book, especially to avoid writing these chapters with the style of a college freshman. I should particularly mention his patience and his determination, which led him to push back his return home to finish this work, even under the effects of a terrible flu.

The list of students, PhD candidates, researchers, and friends that have truly contributed to this work is very long, and I hope that the warmest thanks will reach them, in particular those with whom I have lost contact. For their cited results and our numerous conversations on the subject, equally at Stanford and Berkeley as at the Academy of Sciences of Israel and during various encounters, I must thank five of my colleagues who are no longer among us in person but have left a lot behind by way of their work: Maurice Allais, Amos Tverski, Jonas Salk, Claude Berge, and Gérard Debreu. I should also explicitly mention, for the importance of their contribution and their diverse forms of support: Jean Cottraux, Georges Escribano and my other very congenial colleagues at the Sigmund Freud University, Amartya Sen, R. Aumann, David Romero Vargas, Jean-Claude Vansnick, Carlos Bana e Costa, Mayra Trejos, Servio Guillen, Jean-Pablo Antun, Gert Köhler, Tahar Dridi,

Sylvain Durand, Jean-Guy Dion, Jenö Lehel, the late Robert Fortet, Dominique and Marie-Ange Helbois, Fabienne Guerra, Diane Zacher, Rose-Mary Ciernick, Patrick Suppes, Charlette Rodriguez, and Francis de Véricourt, Emmanuel Grizaud, Fabien Lamaison, Ihmed Othmani, Laura Plazzola Ramora, Mark Plant and Juan Carlos Leyva.

I must finally mention Eric Torreborre, Laurent Jacques, Pierre Grenet, Stéphane Pautremat, Arnaud Bruzat, and Ihmed Othmani who conducted a partial rereading of a first draft of this work. Last, my final thanks must of course go to Kenneth J. Arrow for his participation in a large part of these chapters and for his faithful and effective support through difficult moments.