Table of Contents

Preface ix

Chapter 1. Discrete-Time Markov Chains 1

1.1. Definitions and properties 1
1.2. Strong Markov property 5
1.3. Recurrent and transient states 8
1.4. State classification 12
1.5. Visits to a state 14
1.6. State space decomposition 18
1.7. Irreducible and recurrent Markov chains 22
1.8. Aperiodic Markov chains 30
1.9. Convergence to equilibrium 34
1.10. Ergodic theorem 41
1.11. First passage times and number of visits 53
1.11.1. First passage time to a state 53
1.11.2. First passage time to a subset of states 58
1.11.3. Expected number of visits 64
1.12. Finite Markov chains 68
1.13. Absorbing Markov chains 70
1.14. Examples 76
1.14.1. Two-state chain 76
1.14.2. Gambler's ruin 78
1.14.3. Success runs 82
1.15. Bibliographical notes 87

Chapter 2. Continuous-Time Markov Chains 89

2.1. Definitions and properties 92
2.2. Transition functions and infinitesimal generator 93

2.3. Kolmogorov's backward equation 108
2.4. Kolmogorov's forward equation 114
2.5. Existence and uniqueness of the solutions 127
2.6. Recurrent and transient states 130
2.7. State classification 137
2.8. Explosion 141
2.9. Irreducible and recurrent Markov chains 148
2.10. Convergence to equilibrium 162
2.11. Ergodic theorem 166
2.12. First passage times 172
2.12.1. First passage time to a state 172
2.12.2. First passage time to a subset of states 177
2.13. Absorbing Markov chains 184
2.14. Bibliographical notes 190

Chapter 3. Birth-and-Death Processes 191

3.1. Discrete-time birth-and-death processes 191
3.2. Absorbing discrete-time birth-and-death processes 200
3.2.1. Passage times and convergence to equilibrium 201
3.2.2. Expected number of visits 204
3.3. Periodic discrete-time birth-and-death processes 208
3.4. Continuous-time pure birth processes 209
3.5. Continuous-time birth-and-death processes 213
3.5.1. Explosion 215
3.5.2. Positive recurrence 217
3.5.3. First passage time 220
3.5.4. Explosive chain having an invariant probability 225
3.5.5. Explosive chain without invariant probability 226
3.5.6. Positive or null recurrent embedded chain 227
3.6. Absorbing continuous-time birth-and-death processes 228
3.6.1. Passage times and convergence to equilibrium 229
3.6.2. Explosion 231
3.7. Bibliographical notes 233

Chapter 4. Uniformization 235

4.1. Introduction 235
4.2. Banach spaces and algebra 237
4.3. Infinite matrices and vectors 243
4.4. Poisson process 249
4.4.1. Order statistics 252
4.4.2. Weighted Poisson distribution computation 255
4.4.3. Truncation threshold computation 258
4.5. Uniformizable Markov chains 263

4.6. First passage time to a subset of states 273
4.7. Finite Markov chains 275
4.8. Transient regime 276
4.8.1. State probabilities computation 276
4.8.2. First passage time distribution computation 280
4.8.3. Application to birth-and-death processes 282
4.9. Bibliographical notes 286

Chapter 5. Queues 287

5.1. The M/M/1 queue 288
5.1.1. State probabilities 290
5.1.2. Busy period distribution 311
5.2. The M/M/c queue 315
5.3. The M/M/∞ queue 318
5.4. Phase-type distributions 323
5.5. Markovian arrival processes 326
5.5.1. Definition and transient regime 326
5.5.2. Joint distribution of the interarrival times 336
5.5.3. Phase-type renewal processes 341
5.5.4. Markov modulated Poisson processes 342
5.6. Batch Markovian arrival process 342
5.6.1. Definition and transient regime 342
5.6.2. Joint distribution of the interarrival times 349
5.7. Block-structured Markov chains 352
5.7.1. Transient regime of SFL chains 354
5.7.2. Transient regime of SFR chains 363
5.8. Applications 370
5.8.1. The M/PH/1 queue 370
5.8.2. The PH/M/1 queue 372
5.8.3. The PH/PH/1 queue 372
5.8.4. The PH/PH/c queue 373
5.8.5. The $\mathrm{BMAP}/\mathrm{PH}/1$ queue 376
5.8.6. The $\mathrm{BMAP}/\mathrm{PH}/c$ queue 377
5.9. Bibliographical notes 380

Appendix 1. Basic Results 381

Bibliography 387

Index 395