Contents

Acknowledgements 11

Notation 13

Main Acronyms 19

List of Algorithms 21

Introduction 27

Chapter 1. Constraint Networks 39

1.1. Variables and constraints 39
1.2. Networks of variables and constraints 51
1.2.1. Basic definitions 51
1.2.2. Associated (hyper)graphs 56
1.2.3. Instantiations and solutions 59
1.3. Examples of constraint networks 65
1.3.1. Queens problem 65
1.3.2. Crossword problem 68
1.3.3. Sudoku problem 70
1.3.4. Edge-matching puzzles 72
1.4. Partial orders, decisions, nogoods and properties 74
1.4.1. Partial orders 74
1.4.2. Decisions and nogoods 78
1.4.3. Properties on values and variables 81
1.5. Data structures to represent constraint networks 86
1.5.1. Representation of finite domains 86
1.5.2. Representation of constraints 90

Chapter 2. Random and Structured Networks 93

2.1. Random constraint networks 94
2.1.1. Classical models 94
2.1.2. Models RB and RD 100
2.1.3. Random constraint networks in intension 103
2.1.4. Benchmarks 107
2.1.4.1. Random series 107
2.1.4.2. Random series containing a small structure 108
2.2. Structured constraint networks 109
2.2.1. Backbones and backdoors 109
2.2.2. Cores and cliques 112
2.2.3. Acyclicity 115
2.2.4. Small world structure and morphing technique 121
2.2.5. Benchmarks 122
2.2.5.1. Main series 123
2.2.5.2. Other series 127

PART ONE. INFERENCE 133

Chapter 3. Consistencies 137

3.1. Basic consistencies 138
3.2. Stability of consistencies 143
3.3. Domain-filtering consistencies 150
3.4. Higher-order consistencies 162
3.4.1. Taking the right path 165
3.4.2. Relation-based consistencies 170
3.5. Global consistency 173
3.5.1. Identifying global consistency 173
3.5.2. Toward tractability 178
3.5.2.1. Relational CSP classes 179
3.5.2.2. Structural CSP classes 181
3.5.2.3. Hybrid CSP classes 183
3.6. Caveats about node, arc and path consistencies 184

Chapter 4. Generic GAC Algorithms 185

4.1. Coarse-grained propagation schemes 186
4.1.1. Arc-oriented propagation scheme 187
4.1.2. Variable-oriented propagation scheme 190
4.1.3. Applying forward checking 195
4.2. Iterating over valid tuples 197
4.3. GAC3 and GAC2001 200
4.4. More about general-purpose GAC algorithms 205

4.4.1. Important properties 205
4.4.1.1. Incrementality 205
4.4.1.2. Multi-directionality 207
4.4.1.3. Substitutability 207
4.4.2. Overview 209
4.5. Improving the efficiency of generic GAC algorithms 214
4.5.1. Exploiting cardinality of conflict sets 215
4.5.2. Exploiting residues 219
4.5.2.1. Algorithm GAC3$^{\mathrm{rm}}$ 219
4.5.2.2. Complexity issues 221
4.5.2.3. Residues within MAC 223
4.5.3. Exploiting bitwise operations 224
4.5.3.1. Binary representation 227
4.5.3.2. Algorithms AC3$^{\mathrm{bit}}$ and AC3$^{\mathrm{bit+rm}}$ 231
4.6. Experimental results 233
4.7. Discussion 236

Chapter 5. Generalized Arc Consistency for Table Constraints 239

5.1. Classical schemes 240
5.1.1. Positive table constraints 240
5.1.2. GAC-valid scheme 241
5.1.3. GAC-allowed scheme 242
5.1.4. Illustration 243
5.2. Indexing-based approaches 244
5.2.1. NextIn indexing 244
5.2.2. NextDiff indexing 249
5.3. Compression-based approaches 253
5.3.1. Tries 253
5.3.2. Multi-valued decision diagrams 257
5.3.3. Compressed tables 261
5.3.4. Deterministic finite automata 263
5.4. GAC-valid+allowed scheme 264
5.4.1. Using binary search 265
5.4.2. Using tries 267
5.5. Simple tabular reduction 269
5.5.1. Original algorithm 269
5.5.2. Optimizing STR 274
5.5.3. Relationship with GAC4 278
5.6. GAC for negative table constraints 279
5.6.1. Negative table constraints 279
5.6.2. GAC-valid scheme 279
5.6.3. GAC-valid+forbidden scheme 280
5.6.4. Compressed tuples and MDDs 281

5.7. Experimental results 283
5.8. Conclusion 286

Chapter 6. Singleton Arc Consistency 287

6.1. SAC1 and SAC2 289
6.2. SAC-Opt and SAC-SDS 290
6.3. SAC3 292
6.4. SAC3+ 296
6.5. Illustration 299
6.6. Weaker and stronger forms of SAC 300
6.6.1. Existential SAC 300
6.6.2. Weak k-singleton arc consistency 311
6.7. Experimental results 313
6.8. Conclusion 316

Chapter 7. Path and Dual Consistency 319

7.1. Qualitative study 321
7.2. Enforcing (conservative) path consistency 331
7.2.1. Algorithms PC8 and PC2001 331
7.2.2. Algorithms CPC8 and CPC2001 334
7.3. Enforcing strong (conservative) dual consistency 336
7.3.1. Algorithm sCDC1 336
7.3.2. Algorithm sDC2 339
7.3.3. Illustration 343
7.3.4. Discussion 343
7.3.4.1. Comparison with CPC 343
7.3.4.2. Path consistency by dual consistency 348
7.4. Experimental results 348
7.4.1. With CDC algorithms 349
7.4.2. With DC algorithms 349
7.5. Conclusion 353

PART TWO. SEARCH 355

Chapter 8. Backtrack Search 359

8.1. General description 361
8.2. Maintaining (generalized) arc consistency 367
8.3. Classical look-ahead and look-back schemes 370
8.3.1. A general backtracking algorithm 371
8.3.2. The kernel of the algorithm 373
8.3.3. Dealing with constraint propagation 373
8.3.4. Closely related algorithms 377

8.4. Illustrations 378
8.5. The role of explanations 383

Chapter 9. Guiding Search toward Conflicts 391

9.1. Search-guiding heuristics 392
9.1.1. Classical variable ordering heuristics 393
9.1.2. Value ordering heuristics 397
9.2. Adaptive heuristics 398
9.2.1. Using constraint weighting 399
9.2.2. Using impacts 404
9.3. Strength of constraint weighting 405
9.3.1. Boosting systematic search 405
9.3.2. Identifying small unsatisfiable cores 408
9.3.3. Questioning heuristic policies 412
9.3.4. Statistical analysis and extensions 413
9.4. Guiding search to culprit decisions 415
9.4.1. Nogood identification through testing-sets 416
9.4.2. Reasoning from the last conflict 418
9.4.3. Generalized reasoning from the last conflict 424
9.4.4. Experimental results 427
9.5. Conclusion 427

Chapter 10. Restarts and Nogood Recording 431

10.1. Restarting search 432
10.1.1. Heavy-tailed behavior 433
10.1.2. Restart strategies 434
10.2. Nogood recording from restarts 436
10.2.1. Reduced nld-nogoods 436
10.2.2. Extracting nogoods 439
10.3. Managing standard nogoods 441
10.3.1. Nogood constraints 441
10.3.2. Watched decisions 443
10.3.3. Making inferences 447
10.3.4. Complexity analysis 449
10.4. Minimizing nogoods 450
10.4.1. Minimal ϕ-nogoods 450
10.4.2. Minimization techniques 451
10.4.3. Complexity analysis 452
10.5. Experimental results 454
10.6. Conclusion 457

Chapter 11. State-based Reasoning 459

11.1. Inconsistent partial states 460
11.1.1. Definitions 460
11.1.2. Pruning the search tree 464
11.2. Learning from explanations and failed values 470
11.2.1. Learning generalized nogoods 470
11.2.2. Reasoning from failed values 473
11.3. Reducing elementary inconsistent partial states 476
11.3.1. E-eliminable variables 477
11.3.2. Proof-based extraction 481
11.3.3. Justification-based extraction 483
11.4. Equivalence detection 487
11.5. Experimental results 492
11.6. Conclusion 494

Chapter 12. Symmetry Breaking 495
Christophe LECOUTRE, Sébastien TABARY

12.1. Group theory 496
12.2. Symmetries on constraint networks 499
12.3. Symmetry-breaking methods 503
12.3.1. Symmetry-breaking constraints 504
12.3.2. Dynamic symmetry breaking 506
12.4. Automatic symmetry detection 508
12.5. Lightweight detection of variable symmetries 511
12.5.1. Locally symmetric variables 512
12.5.2. Computing normal forms of predicate expressions 516
12.5.3. Constructing lsv-graphs 518
12.6. A GAC algorithm for lexicographic ordering constraints 520
12.7. Experimental results 527

Appendices 531

A. Mathematical Background 531
A.1. Sets, relations, graphs and trees 531
A.2. Complexity 536
B. XML Representation of Constraint Networks 541

Bibliography 547

Index 571