

---

# Contents

---

<b>Foreword</b> . . . . .	ix
FéLix DARVE	
<b>Preface</b> . . . . .	xiii
<b>Chapter 1. Fabric, Stress, and Strain</b> . . . . .	1
1.1. Particle size and shape . . . . .	1
1.2. Granular fabric . . . . .	4
1.2.1. Spatial description . . . . .	5
1.2.2. Topological description . . . . .	7
1.2.3. Directional description . . . . .	8
1.2.4. Particle-body and particle-surface orientations . . . . .	11
1.2.5. Contact fabric . . . . .	13
1.2.6. Voids fabric . . . . .	16
1.3. Granular stress . . . . .	28
1.3.1. Average stress . . . . .	28
1.3.2. Continuum equivalent stress . . . . .	33
1.4. Granular deformation . . . . .	43
1.4.1. Strain in tessellated regions . . . . .	48
1.4.2. Strain without spatial tessellation . . . . .	55
1.4.3. Bulk strain estimates for up-scaling analyses . . . . .	59
<b>Chapter 2. Contact Interaction</b> . . . . .	65
2.1. Contact kinematics . . . . .	65
2.2. Contact mechanics: normal force . . . . .	73

2.2.1. Contact between smooth spheres . . . . .	75
2.2.2. Experiments and pressure dependence of bulk stiffness . . . . .	77
2.2.3. Contact between non-spherical contours . . . . .	80
2.2.4. Contact between rough contours . . . . .	82
2.3. Contact mechanics: tangential force . . . . .	84
2.3.1. The Cattaneo-Mindlin model . . . . .	85
2.3.2. Linear-frictional model . . . . .	91
2.4. Contact mechanics: rolling resistance . . . . .	93
2.4.1. Rolling moment . . . . .	94
2.4.2. Creep-friction . . . . .	97
<b>Chapter 3. Numerical Simulation . . . . .</b>	<b>101</b>
3.1. Numerical methods for discrete particle systems . . . . .	101
3.1.1. Particle shape . . . . .	105
3.1.2. Boundaries . . . . .	108
3.1.3. Assembly and particle sizes . . . . .	110
3.1.4. Assembly creation . . . . .	112
3.1.5. Contact detection . . . . .	113
3.1.6. Particle orientation and rotation kinematics . . . . .	114
3.2. Discrete element method . . . . .	120
3.2.1. Time step, mass and damping . . . . .	124
3.2.2. Strain rate and quasi-static performance . . . . .	127
3.2.3. Kinetics of non-symmetric particle shapes . . . . .	128
3.3. Stiffness matrix methods . . . . .	130
3.4. Contact dynamics method . . . . .	143
3.4.1. Contact velocities . . . . .	144
3.4.2. Complementarity relations . . . . .	148
3.4.3. Contact forces . . . . .	149
<b>Chapter 4. Loading, Movement, and Strength . . . . .</b>	<b>153</b>
4.1. Deformation and strength . . . . .	153
4.1.1. Two phases of deformation . . . . .	158
4.2. Trends in movements and rotations . . . . .	161
4.2.1. Affine motion . . . . .	162
4.2.2. Deviations from affine motion . . . . .	163
4.2.3. Local strains . . . . .	173
4.2.4. Particle rotations . . . . .	174

---

4.2.5. Contact rolling . . . . .	178
4.3. Coordination number, redundancy and contact longevity . . . . .	178
4.3.1. Redundancy . . . . .	180
4.3.2. Contact longevity and creation rates . . . . .	185
4.4. Anisotropy induced by loading . . . . .	187
4.4.1. Anisotropy of particle orientations . . . . .	188
4.4.2. Anisotropy of contacts . . . . .	189
4.4.3. Voids anisotropy . . . . .	192
4.4.4. Fabric anisotropy and stress . . . . .	194
4.5. Density distribution of contact force . . . . .	199
4.6. Effects of macro- and micro-scale characteristics . . . . .	203
4.6.1. Initial anisotropy . . . . .	203
4.6.2. Particle shape . . . . .	204
4.6.3. Inter-particle friction . . . . .	206
4.6.4. Other contact characteristics . . . . .	207
4.7. Localization and patterning . . . . .	208
4.7.1. Force chains . . . . .	209
4.7.2. Circulation cells . . . . .	213
4.7.3. Micro-bands . . . . .	214
4.7.4. Shear bands . . . . .	216
<b>Permissions</b> . . . . .	229
<b>Notation</b> . . . . .	231
<b>Bibliography</b> . . . . .	233
<b>Index</b> . . . . .	255