
Contents

FOREWORD	ix
INTRODUCTION	xiii
CHAPTER 1. WORDS AND SEQUENCES FROM SCRATCH	1
1.1. Mathematical background and notation	2
1.1.1. About asymptotics	4
1.1.2. Algebraic number theory	5
1.2. Structures, words and languages	11
1.2.1. Distance and topology	16
1.2.2. Formal series	24
1.2.3. Language, factor and frequency	28
1.2.4. Period and factor complexity	33
1.3. Examples of infinite words	36
1.3.1. About cellular automata	43
1.3.2. Links with symbolic dynamical systems	46
1.3.3. Shift and orbit closure	59
1.3.4. First encounter with β -expansions	62
1.3.5. Continued fractions	69
1.3.6. Direct product, block coding and exercises	70
1.4. Bibliographic notes and comments	77

CHAPTER 2. MORPHIC WORDS	85
2.1. Formal definitions	89
2.2. Parikh vectors and matrices associated with a morphism	96
2.2.1. The matrix associated with a morphism	98
2.2.2. The tribonacci word	99
2.3. Constant-length morphisms	107
2.3.1. Closure properties	117
2.3.2. Kernel of a sequence	119
2.3.3. Connections with cellular automata	120
2.4. Primitive morphisms	122
2.4.1. Asymptotic behavior	127
2.4.2. Frequencies and occurrences of factors	127
2.5. Arbitrary morphisms	133
2.5.1. Irreducible matrices	134
2.5.2. Cyclic structure of irreducible matrices	144
2.5.3. Proof of theorem 2.35	150
2.6. Factor complexity and Sturmian words	153
2.7. Exercises	159
2.8. Bibliographic notes and comments	163
CHAPTER 3. MORE MATERIAL ON INFINITE WORDS	173
3.1. Getting rid of erasing morphisms	174
3.2. Recurrence	185
3.3. More examples of infinite words	191
3.4. Factor Graphs and special factors	202
3.4.1. de Bruijn graphs	202
3.4.2. Rauzy graphs	206
3.5. From the Thue–Morse word to pattern avoidance	219
3.6. Other combinatorial complexity measures	228
3.6.1. Abelian complexity	228
3.6.2. k -Abelian complexity	237
3.6.3. k -Binomial complexity	245

3.6.4. Arithmetical complexity	249
3.6.5. Pattern complexity	251
3.7. Bibliographic notes and comments	252
BIBLIOGRAPHY	257
INDEX	295
SUMMARY OF VOLUME 2	303