Table of Contents

Preface xi

Chapter 1. Design at Serviceability Limit State (SLS) 1

1.1. Nomenclature 1
1.1.1. Convention with the normal vector orientation 1
1.1.2. Vectorial notation 1
1.1.3. Part of the conserved reference section 2
1.1.4. Frame 2
1.1.5. Compression stress $\sigma_{c,sup}$ in the most compressed fiber 2
1.2. Bending behavior of reinforced concrete beams – qualitative analysis 3
1.2.1. Framework of the study 3
1.2.2. Classification of cross-sectional behavior 5
1.2.3. Parameterization of the response curves by the stress σ_{s1} of the most stressed tensile reinforcement 5
1.2.4. Comparison of σ_{s1} of the tensile reinforcement for a given stress in the most compressed concrete fiber $\sigma_{c,sup}$ 6
1.2.5. Comparison of the bending moments 8
1.3. Background on the concept of limit laws 10
1.3.1. Limit law for material behavior 10
1.3.2. Example of limit laws in physics, case of the transistor 11
1.3.3. Design of reinforced concrete beams in bending at the stress Serviceability Limit State 12
1.4. Limit laws for steel and concrete at Serviceability Limit State 13
1.4.1. Concrete at the cross-sectional SLS 13
1.4.2. Steel at the cross-sectional SLS 13
1.4.3. Equivalent material coefficient 14
1.5. Pivots notion and equivalent stress diagram. 14

1.5.1. Frame and neutral axis 14
1.5.2. Conservation of planeity of a cross-section. 15
1.5.3. Planeity conservation law in term of stress 17
1.5.4. Introduction to pivot concepts 18
1.5.5. Pivot rules 19
1.6. Dimensionless coefficients. 20
1.6.1. Goal 20
1.6.2. Total height of the cross-section 21
1.6.3. Relative position of the neutral axis 21
1.6.4. Shape filling coefficient 22
1.6.5. Dimensionless formulation for the position of the center of pressure 23
1.7. Equilibrium and resolution methodology 24
1.7.1. Equilibrium equations 24
1.7.2. Discussion on the resolution of equations with respect to the number of unknowns 26
1.7.3 Reduced moments 27
1.7.4. Case of a rectangular section 29
1.8. Case of pivot A for a rectangular section 30
1.8.1. Studied section 30
1.8.2. Shape filling coefficient 30
1.8.3. Dimensionless coefficient related to the center of pressure 31
1.8.4. Equations formulation 32
1.8.5. Resolution 33
1.9. Case of pivot B for a rectangular section 35
1.9.1. Studied section 35
1.9.2. Shape filling coefficient 35
1.9.3. Dimensionless coefficient related to the center of pressure 35
1.9.4. Equations formulation 36
1.9.5. Resolution 37
1.9.6. Synthesis 38
1.10. Examples – bending of reinforced concrete beams with rectangular cross-section. 39
1.10.1. A design problem at SLS – exercise 39
1.10.2. Resolution in Pivot A – M_{ser} = 225 kN.m 42
1.10.3. Resolution in Pivot B – M_{ser} = 405 kN.m 45
1.10.4. Resolution in pivot AB 47
1.10.5. Design of a reinforced concrete section, an optimization problem 50
1.10.6. General design at Serviceability Limit State with tensile and compression steel reinforcements 54

1.11. Reinforced concrete beams with T-cross-section 58
1.11.1. Introduction 58
1.11.2. Decomposition of the cross-section 60
1.11.3. Case of pivot A for a T-cross-section 61
1.11.4. Case of pivot B for a T-cross-section 63
1.11.5. Example – design of reinforced concrete beams composed of T-cross-section 65

Chapter 2. Verification at Serviceability Limit State (SLS) 69

2.1. Verification of a given cross-section – control design 69
2.1.1. Position of the neutral axis 69
2.1.2. Equation of static moments for the determination of the position of neutral axis 70
2.1.3. Stress calculation – general case 72
2.1.4. Rectangular cross-section – verification of a given cross-section 74
2.1.5. T-cross-section – verification of a given cross-section 76
2.1.6. Example – verification of a reinforced T-cross-section. 79
2.1.7. Determination of the maximum resisting moment 80
2.2. Cross-section with continuously varying depth. 81
2.2.1. Triangular or trapezoidal cross-section 81
2.2.2. Equilibrium equations – normal force resultant 82
2.2.3. Equilibrium equations – bending resultant moment. 84
2.2.4. Case of pivot A for a triangular cross-section 86
2.2.5. Case of pivot B for a triangular cross-section 87
2.2.6. Static moment equation for a triangular cross-section 87
2.2.7. Design example of a triangular cross-section 88
2.3. Composed bending with combined axial forces 90
2.3.1. Steel reinforcement design for a given reinforced concrete section. 90
2.3.2. Determination of the position of the neutral axis – simple bending. 91
2.3.3. Determination of the position of the neutral axis – composed bending with normal force solicitation 92
2.3.4. Exercises for composed bending with normal force solicitation . . 96
2.4. Deflection at Serviceability Limit State 107
2.4.1. Effect of crack on the bending curvature relationship 107
2.4.2. Simply supported reinforced concrete beam 112
2.4.3. Calculation of deflection – safe approach. 113
2.4.4. Calculation of deflection – a more refined approach; tension stiffening neglected. 114

2.4.5. Calculation of deflection – a more refined approach; tension stiffening included . . . 116
2.4.6. Approximated approach . . . 118
2.4.7. Calculation of deflection – a structural example . . . 119

Chapter 3. Concepts for the Design at Ultimate Limit State (ULS) . . . 123

3.1. Introduction to ultimate limit state . . . 123
3.1.1. Yield design . . . 123
3.1.2. Application of yield design to the cantilever beam . . . 125
3.1.3. Inelastic (plasticity or continuum damage mechanics) bending-curvature constitutive law . . . 129
3.2. Postfailure analysis . . . 133
3.2.1. Historical perspective . . . 133
3.2.2. Wood's paradox . . . 135
3.2.3. Non-local hardening/softening constitutive law, a variational principle . . . 137
3.2.4. Non-local softening constitutive law: application to the cantilever beam . . . 144
3.2.5. Some other structural cases – the simply supported beam . . . 149
3.2.6. Postfailure of reinforced concrete beams under distributed lateral load . . . 152
3.3. Constitutive laws for steel and concrete . . . 156
3.3.1. Steel behavior . . . 156
3.3.2. Concrete behavior . . . 160
3.3.3. Dimensionless parameters at ULS . . . 170
3.3.4. Calculation of the concrete resultant for the rectangular simplified diagram . . . 174
3.3.5. Calculation of the concrete resultant for the bilinear diagram . . . 174
3.3.6. Calculation of the concrete resultant for the parabola–rectangle diagram . . . 179
3.3.7. Calculation of the concrete resultant for the law of Desayi and Krishnan . . . 183
3.3.8. Calculation of the concrete resultant for Sargin's law of Eurocode 2 . . . 187
3.3.9. On the use of the reduced moment parameter . . . 191

Chapter 4. Bending-Curvature at Ultimate Limit State (ULS) . . . 193

4.1. On the bilinear approximation of the moment-curvature relationship of reinforced concrete beams . . . 193
4.1.1. Phenomenological approach . . . 193
4.1.2. Moment-curvature relationship for concrete – brief overview . . . 196
4.1.3. Analytical moment-curvature relationship for concrete . . . 198

4.1.4. A model based on the bilinear moment-curvature approximation . . . 222
4.2. Postfailure of reinforced concrete beams with the initial bilinear moment-curvature constitutive law . . . 226
4.2.1. Elastic-hardening constitutive law . . . 226
4.2.2. Plastic hinge approach . . . 230
4.2.3. Elastic-hardening constitutive law and local softening collapse: Wood's paradox . . . 235
4.2.4. Elastic-hardening constitutive law and non-local local softening collapse . . . 238
4.3. Bending moment-curvature relationship for buckling and postbuckling of reinforced concrete columns . . . 242
4.3.1. A continuum damage mechanics-based moment curvature relationship . . . 242
4.3.2. Governing equations of the problem and numerical resolution . . . 245
4.3.3. Second-order analysis – some analytical arguments . . . 251
4.3.4. Postfailure of the non-local continuum damage mechanics column . . . 258

Appendix 1. Cardano's Method . . . 267

A1.1. Introduction . . . 267
A1.2. Roots of a cubic function – method of resolution . . . 268
A1.2.1. Canonical form . . . 268
A1.2.2. Resolution – one real and two complex roots . . . 269
A1.2.3. Resolution – two real roots . . . 271
A1.2.4. Resolution – three real roots . . . 271
A1.3. Roots of a cubic function – synthesis . . . 273
A1.3.1. Summary of Cardano's method . . . 273
A1.3.2. Resolution of a cubic equation – example . . . 274
A1.4. Roots of a quartic function – principle of resolution . . . 275

Appendix 2. Steel Reinforcement Table . . . 277

Bibliography . . . 279

Index . . . 293