General Introduction

In [DAU 03], André Dauphiné describes geography as the core of the complexity in human and social sciences. The information tools that permit us to enter the paradigm of geography's complexity were brought forth by Tobler and Hagerstand through the use of cellular automatons. Then, multi-agent systems appeared near the end of the 1980s thanks to the combined evolution of artificial intelligence, object-oriented programming and distributed intelligence, which were later developed into numerous fields such as physics, biology and computer science [WEI 89], [BRI 01]. Thus, numerous works have contributed to applying these computing and theoretical tools specifically to geography. These studies continue to appear today, in the works of different teams such as the geosimulation group, RIKS, CASA, Milan's politechnico and urban simulation (SIMBOGOTA) and city network studies from the Universities of Paris and Strasbourg in France. We will not explain these in detail.

Geography is essentially ingrained in space. The geographical map is its direct expression. If we are interested in complex processes, we must consider the interlocking organizational levels that are necessary to understand these phenomena. Modeling adds to the temporal and fundamental dimensions of the expression of dynamics. The multi-level representation in space forces us to address different temporality levels of processes in play.

This work will attempt to contribute to the challenge that is geographic complexity. Complexity is characterized as being a crossroads between physical and human sciences, by its intermediary position in overlapping levels of reality, which are spatial and temporal and finally, by its key position in the degrees of organization complexity, which is the position of human kind in both the living and mineral domains.

It took many years to accomplish this work in the area of geographic modeling. It is a product of reflection and fulfillment in the area of cartography, spatial analysis and geomatics. This study began at the beginning of the 1980s, and coincided with the arrival of micro-computers. This decade witnessed the construction of tools and concepts of solid and efficient representation of space. At the end of the 1990s and at the beginning of the 21st century, necessity passed to the next level: dynamic spatial simulation. It was imposed by the powerful level attained by computers, by the mature development of cartography software and by spatial analysis, through the development of complexity theories and associated simulation tools, since developed in other areas, such as physics.

Through this work, our goal is to share our knowledge in the area of modeling spatial dynamics, based on a systemic, individual-centered and distributed approach. This work is also the continuation of diverse contributions on this theme in works such as [GUE 08] and [AMB 06]. Here we will present a more personal analysis through our theoretic reflections and by means of a few of our realizations which were developed by our research team that are not isolated from the national and international abundance of such productions. We want this work to be an educational tool for students, geographic researchers, developers and computer scientists who wish to learn more about modeling in geography.

The mathematical aspect of certain developments should not alienate the literary reader as the formulae and mathematic notations are not necessary for its general comprehension and may be disregarded during a qualitative reading. These developments are generally associated with text explaining them in today's terms. The formal aspect must therefore enable the reader to learn about this area and to deal with these notions. They can seem repellent at times but we need to overcome that perception if we wish to numerically test or program these methods or models. Nevertheless, many of these formalisms deal with very simple concepts, and in this work, we have made a constant effort to accompany these formalisms with a simple explanation and to give meaning to the symbols and notations in the text.

Starting with the most general concepts of structure, organization and system, we will firstly approach the fundamental notion of space. The richness of this concept is shown through different formalizations that lead us to geometries, topologies and metrics defined through space. Then, we approach the concepts of matter and object to finally introduce time. This allows us to approach the notions of processes and interaction that are fundamental in dynamic geographic modeling. After this section, presenting the foundations of geographic space modeling we will work with the computing tools of dynamic modeling, which are the geographic cellular automatons (GCA) which enable us to have a general model of a GCA. Then, we generalize it to construct a general system of geographic agents (SGA) model, based on a formal ontology constructed on the Agent-Organization-Behavior triptych where the geographic object appears as a dual entity between the individual and the group. This formal ontology is mathematically formalized as it lets us elaborate a construction totally independent of all technological constraints and provides a rigorous theoretical framework. Thus, we can think of geographic objects according to a more realistic approach, even if it remains simplified. The mathematic formalization enables us to think of continuum or infinity without being preoccupied by the limitations of a computer in which everything must be explicit[1], enumerated and finished. We need a theoretic and suppler framework to formalize this construction.

Firstly, the set theory and the logic of predicates currently form an elementary basis which is recognized for all mathematical formalizations. We have come a long way from the beginning of the set theory of entities which was initiated by Cantor at the end of the 19th century, a time when fundamental paradoxes shook its axiomatic structure. The set theory has reached its maturity while being conscious of its limits, for example, knowing how to distinguish between what is a set and what is not (which we will call a "family" or a "collection"). There is no formal definition of the notion of "set". It is a primary definition of the theory. Nevertheless, the family of all sets is not a set in itself, as a set must be clearly defined, either by the thorough and non superfluous list of its elements, (it is therefore defined "in extension") or by a property characteristic of its elements, (it is then defined "in comprehension"). Another essential rule exists so that the theory does not contradict itself. This has to do with the relation of belonging: a set cannot belong to itself. However, the notion of sub-sets gives birth to the relation of inclusion, which is a relation of order defined on the set $P(E)$ of the parts of a set E.

The inclusion relation is reflexive, as opposed to belonging, which is anti-reflexive. Thus, in the set theory, a set contains itself but does not belong to itself. With such precautions, Russel's paradox no longer exists. In fact, this paradox rested upon a particular set, formed by all the sets which do not contain themselves. This paradox resulted from the fact that this set could either contain or not contain itself. These improvements are linked with others in Zermelo-Fraenkel's axiomatic. The latter confers great weight to this theory. Even if it is not the only one at the basis of a set theory, it is widely used today. It will eventually be accompanied by other complementary axioms, such as the choice axiom, and the continuum hypothesis.

---

1 Contrary to mathematics that are based on an implicit syntactic construction (a definition once stated is assumed to be known afterwards) and on the implicit contents. For example, we only know out of the real numbers those which can be formulated or made explicit, but there is an infinity of numbers that will never be made explicit. Many mathematical objects are implicitely definite by theorems of existence, but we either don't know or cannot always determine them effectively.

A few other set theories have been formed, such as the theory of types (Whitehead, Russell) and the theory of classes (von Neumann, Godel). In spite of their differences, these theories now appear to be converging translations of the same mathematical reality. Other tentatives of axiomatization were developed in different directions and some of those were formalized. Such is the case for mereology which is a more logical theory formalized by the logician Stanisław Leśniewski (1886-1939). This theory does not form a more fecund advance for our work than the "standard" set theory. For example, one of the main principles of the complexity paradigm is that the whole is more than the sum of its parts. In the set theory, like in mereology, this affirmation is false. The definition of a complex system rests upon a richer concept than a simple set formed of elements (and of parts).

We propose to formulate this enrichment, which is not contradictory by the use of the set theory. This ontological construction is not limited to the single use of the set theory. The whole structure of algebra, geometry, topology and analysis, whose coherence and language rest upon the set theory, will be useful for us at many levels. Nevertheless, we do not want to elaborate a mathematical theory formulated by a series of theorems and demonstrations. We also do not want to elaborate on new axioms. We will only use the mathematical language to define entities of our ontology and to show its coherence. Thus, the level of mathematical knowledge used remains elementary.

In order to define this ontological construction step in a geographic realm, we will start from nothingness with the localizations constituted by what is left of the world, keeping only localizations and coordinates. This nothingness is formed by the space[2] of geometry which is void of all matter and content. It allows for the construction of geometric forms and permits them to be put in relation, through topology, in order to construct more complex abstract objects. In the meantime, the profound essence of objects only appears with the introduction of the concept of matter and energy. How can matter be formalized in this geometric space? Does a point, a line or a surface still exist when space becomes material? We will finally examine how the acknowledgement of time permits us to construct facts and behaviors. For example, it permits the birth, the development and the death of either physical, living, social or imaginary beings. It also enables us to add depth of history and incertitude of the future to the diversity of spatial reality. Thus, it seems that the physical triptych of space-time-matter is the preliminary conceptual pedestal on which our ontological construction Agent-Organization-Behavior (AOB) is based. This confirms that the laws of physics do not only apply to life sciences of man and

2 This term is used in a voluntarily ambiguous manner to evoke geography's disciplinary area, but also to indicate that we are situated in a physical space, which is mathematically formalized.

society. If each level of reality possesses its own laws, they keep their vertical coherence, which is to say that each level cannot contradict laws acquired at lower levels.

The concepts of agent and organization are at the heart of geographic object construction. They define a geographic object dually, which consists of a more or less abstract membrane, the external side of which is turned towards the exterior world with which it acts. This realm is formed on the one hand by a diverse part of agent-objects of the same level, more or less evolved but nevertheless of the same general conception, and on the other hand by an englobing system into which all of these objects are integrated. It also consists of an internal side which presents the object as an organization turned towards the isolated depth of its interior for which its parts are still agent-objects forming a system. These two sides express the fundamental interaction which is the object's essence that is active and evolutive (some would say "inactive"). These qualities permit a co-construction (or even a co-evolution) from both the collective exterior and interior universes. If the AOB ontology was initially inspired by Jacques Ferber's AGR work (Agent-Group-Role), it defers from the duality of agent and organization which integrates this auto-reference and its internal and external environments which derive from it. Geography's main interest with respect to this structure derives from the fact that it expresses a systematic, multi-level vision. Furthermore, it permits us to identify the exterior and interior limits of the model. Thus, we can often identify three levels of modeling (but this number is not limited, i.e. macro, meso and micro).

The macro level is limited by the global system's envelope (which corresponds to the entire model) and contains the highest level of organization. The main level of the system's objects is constructed in this environment (the one that contains objects we study) which are at the meso-level. These objects can themselves be structured by terminal objects so they cannot be deteriorated by more elementary objects. This is what we call the "particular" or "micro" level. If the problem persists, we can always add more levels. This representation (see Figure I.1) is evident to an individual observer (or an individual observed by the modeler), who can see at its meso-level the grouping of the other individuals of this level, who can internally "feel" the grouping of these micro-level internal components and who are also in relation to this system in which it evolves (at the macro-level).

Moreover, these two formalization steps that we use, mathematically and informatically, are not antagonistic but complementary and can be mutually enriched. Our method will thus be presented more often as an object or as a concept in the form of a description, as is customary in geography. Then we will mathematically formalize it, and/or see how it can be translated in a conceptual, structural or algorithmic computer science formalization.

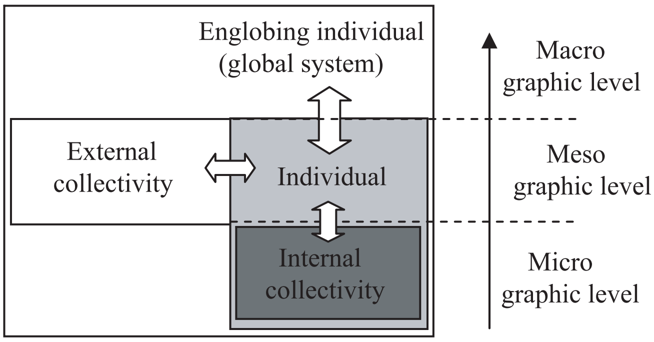

Figure I.1. *Relations between the individual, collective, internal and external*

Notations used

We constantly use two formalization methods: mathematic and algorithmic. These two methods conform with slightly different conventions so it is therefore a good idea to know the difference, depending on the context. The mathematic language generally uses one symbol (typically a letter), sometimes accompanied by an index to represent an entity as either a variable, an element, a set, a function, an unknown, etc. When we associate two letters which represent numbers, this often signifies that we multiply them. On the contrary, in computer science, as the number of symbols in a program or in an algorithm can be large, we represent an entity by a rather explicit name, by using many letters. The same formula or series of calculations can be written in two manners, and the same symbols can have different significations:

– In *mathematical language*, the symbols are written in italics to differentiate them from common language. The multiplication operation is implied (or more rarely indicated by a point). The expression of equality a=b indicates a mathematical equality, which means that a and b are two ways of signifying the same quantity or the same element of a set. We use specific symbols for operations (summation, integration, fraction line, square root, etc.).

So,

$$y = y_{\min} + jp$$

the number is y equal to the y minimum added to the product of j by p numbers

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

indicates that the number x bar (showing an average in statistics), is equal to the opposite of n multiplied by the sums of x_i for the index i varying from 1 to *n*, which makes us divide the sum of x_i by *n*:

$$\overline{x} = \frac{x_1 + x_2 + ... + x_i + ... + x_n}{n}$$

– In *algorithmic language*, we use the "typewriter" font where we often use a syntactic approach close to the Pascal language. The multiplication is then represented by a star. The symbol of equality "=" does not have the same sense as it does in mathematics. Here it is a logical operation that gives the "true" result if the left and right members represent the same quantity or quality, otherwise giving a "false" result. We must not confuse the symbol of equality with the symbol of allocation, noted in the Pascal language ":=" or sometimes in the algorithmic language, by an arrow "←". The expressions "*a* := *b*+*c*" or "*a* ← *b* +*c*" mean that we read the values contained in the memory files named b and c, that we use the *b*+*c* addition and that we write (or store) the result in the "a" memory file. In algorithmics, we do not use special symbols (Greek, etc.), we only use keyboard symbols. The entities are often named by chains of many characters. The point represents a separator between a complex entity (an object) and a component of this entity (an attribute, property or method).

For example, the two following formulae could be written as:

```
y := DTM.yMin + j*DTM.PasY;
```

where "yMin" and "PasY" are fields (attributes) of the "DTM" object. We can also have an algorithmic style of writing, like in the following example that calculates the average of values contained in the X chart.

```
AveX :=0 ;
for i :=1 to n do
AveX := AveX + X[i]
end ;
AveX := AveX/n
```