Preface

In recent years quantitative finance has become an extraordinary field of research and interest, from an academic point of view as well as for practical applications.

At the same time, the pension issue is clearly a major economic and financial topic for the coming decades, in the context of the well-known longevity risk. The emergence and development of pension schemes in our modern societies can essentially be explained by two factors:

– The individual approach to life: our modern world tends to substitute for large multi-generational families, for an individual model where each person is assumed to be self-supporting, before and after retirement age. As a consequence, personal pension planning, including social security pensions and other incomes, becomes a necessity for everyone.

– The longevity syndrome: longevity has increased extraordinarily in recent decades, and nothing indicates a stopping or a reverse in this phenomenon could occur in the next few years. In the past it was usual to retire at 65 and then have a life expectancy of only a few more years. Nowadays, and in the near future, in many countries, it will be quite common for all of us to hope to survive until after 85!

The future of our pension systems is clearly one of the main economic challenges of the world for the coming decades. In the huge majority of countries, collective pension schemes have been created by states as well as by private companies or professional organizations. These systems can use very different tools, techniques, funding approaches or legal forms.

As described in Gronchi [GRO 99], pensions can be classified as a function of the calculation method of the benefits as:

– defined benefit (DB): the pension is a function of salaries and the number of years of service of the retiring person;

– defined contribution (DC): the pension is a function of the contribution that the retiring person paid during his life.

It can also be classified as a function of the financing technique as:

– pay as you go (PAYG): the pensions are paid by the current workers to the previous generation;

– funding: the pensions are generated by the savings accumulated by the contribution of the workers.

Surprisingly few books are devoted to the application of modern stochastic tools to pension analysis.

Therefore, the aim of this book is to fill this gap and to show how recent stochastic methods can be useful for the risk management of pension funds and the computation of market values. Optimal control methods will be especially developed and applied to fundamental problems such as the optimal asset allocation of the fund or the cost spreading of a pension scheme. In these various problems, financial as well as demographic and economic risks will be addressed and modeled.

The layout of this book is as follows: Chapters 1 and 2 present the fundamental issues of the next decades and classical theory of pension funding.

In Chapter 3, we introduce the minimal basic results concerning control theory, both for deterministic and stochastic formulation in order to prepare the models presented in Chapters 7 and 8.

Chapter 4 is devoted to concepts of defined contribution and defined benefit pension plans, while Chapter 5 presents some basic definitions on fair and market values for the evaluation of financial flows and stocks in the future – not only to improve the classical concept of present value, but also to fill up the constraints of new IFRS rules as well as Basel II and III rules for banks and Solvency II for insurance companies.

Chapter 6 first develops stochastic models for DC and then for DB, in order to measure the various risks faced by the pension fund and to propose eventual solvency buffers in the philosophy of Basel II or Solvency II. In particular, it illustrates the importance of time in the risk assessment of a pension fund.

Chapters 7 and 8 give the main results of the pension theory as various models for the optimal control of the investment strategy and the contribution process for

both defined contribution pension schemes and defined benefit pension schemes are presented.

Chapter 7 presents – also for DB pension schemes in a stochastic environment – important asymptotic results of evolution of the fund and the contributions.

Chapter 9 presents the construction of algorithms for the management of pension fund liabilities necessary for simulation models. Fundamentally we give two approaches to these models, one that is a semi-deterministic approach that we will call the *direct method* and the other based on the Monte Carlo method.

In short, with these last three chapters, it is possible to carefully study the evolution of a pension fund in the future; they constitute the core of this book.

The presentation, in Chapter 10, of discrete time homogeneous semi-Markov processes (DTHSMP), discrete time non-homogeneous (DTNHSMP) semi-Markov processes and the discrete time semi-Markov reward processes (SMRWP), is followed by Chapter 11, the last chapter of this book, which is devoted to another type of stochastic model of pension funds and manpower management study, called generalized non-homogeneous semi-Markov models.

This model is a general, rigorous and tractable stochastic evolution time model for pension funds, called the discrete time non-homogeneous semi-Markov pension fund model, taking into account economic, financial and demographic evolution factors so that it becomes a real-life model using important factors such as *seniority*, *general age dependence*, *rate of inflation* and *salary lines*. It can be particularly useful for the study of private pension fund evolution.

In conclusion, this book presents realistic stochastic models to carefully study the evolution of pension funds in the future, not only from a theoretical point of view but also from a practical point of view, as we present algorithms for the construction of simulation models.

We also present basic concepts in such a way that this book is relatively self-contained. Therefore, the book can be considered as the first textbook in the field of stochastic methods for pension funds and thus, it will be useful for graduate students in economics and actuarial science as well as for managers of pension funds and especially people involved in Solvency II for insurance companies and in Basel II and III for banks.