Contents

Introduction xi

Chapter 1. Flaws in Materials 1

1.1. Introduction 1
1.2. The theoretical strength and the intrinsic strength of materials 2
1.3. The fracture strength of materials 5
1.3.1. Influence of a crack and a flaw 6
1.3.2. The resistance to crack extension 7
1.4. The flaws 11
1.4.1. Submicrostructure flaws 11
1.4.2. Processing flaws 12
1.4.3. Machining flaws 15
1.4.4. Flaws caused by in-service usage 16
1.5. Severity of individual flaws 17
1.5.1. Severity of cracks and voids 17
1.5.2. Severity of inclusions 19
1.6. Influence of flaw populations 21
1.6.1. Strength variability 22
1.6.2. Size effects 23
1.6.3. Influence of stress field 24
1.6.4. Influence of loading conditions 26
1.6.5. Influence of multimodal flaw populations 28
1.6.6. Effects of environment 29
1.7. Consequences of failure predictions 30

Chapter 2. Statistical-Probabilistic Approaches to Brittle Fracture: The Weibull Model 35

2.1. Introduction 35
2.2. Weibull statistical model 36
2.2.1. The weakest link concept 36
2.2.2. Probability of fracture under a uniaxial tensile stress 37
2.2.3. Statistical parameters 39
2.3. Probability of fracture for a uniaxial non-uniform tensile stress field 42
2.4. Probability of fracture from the surface of specimens 43
2.5. Weibull multiaxial analysis 43
2.6. Multiaxial approach based on the principle of independent action of stresses 46
2.7. Summary on the Weibull statistical model 47

Chapter 3. Statistical-Probabilistic Theories Based on Flaw Size Density 51

3.1. Introduction 51
3.2. Failure probability 52
3.3. Expressions for flaw size density and distribution 54
3.3.1. Description of complete flaw population 54
3.3.2. Statistical distribution functions for extreme values 57
3.4. Introduction of stress state 58
3.5. Models 59
3.5.1. Power-law flaw size density 59
3.5.2. The De Jayatilaka–Trustrum approach 60
3.6. Limits of the flaw size density-based approaches 61

Chapter 4. Statistical-Probabilistic Theories Based on Flaw Strength Density 63

4.1. Introduction 63
4.2. Basic equations of failure probability in the elemental strength approach 64
4.3. Elemental strength model for a uniform uniaxial stress state: Argon–McClintock development 66

4.4. The Batdorf model 68
4.4.1. The model 68
4.4.2. Examples of determination of failure probability using the Batdorf model: unixial, equibiaxial and equitriaxial tension 72
4.4.3. Discussion: comparison with the Weibull model 75
4.5. The multiaxial elemental strength model 76
4.5.1. The multiaxial elemental strength 76
4.5.2. The flaw density function (volume analysis) 78
4.5.3. Determination of local stress components 80
4.5.4. Probability of failure from surface flaws 81
4.5.5. Determination of functions $I_v(...)$ and $I_s(...)$ 82
4.5.6. Comparison with the Weibull model 87

Chapter 5. Effective Volume or Surface Area 91

5.1. Introduction 91
5.2. The Weibull model: the effective volume for a uniaxial stress state 91
5.3. The multiaxial elemental strength model: the effective volume for a multiaxial stress state 93
5.4. Analytic expressions for failure probability, effective volume or surface area (Weibull theory) 96
5.4.1. Compression 96
5.4.2. 3-point bending 97
5.4.3. 4-point bending 98
5.5. Some remarkable exact expressions for failure probability, effective volume or surface area (multiaxial elemental strength theory) 101
5.5.1. Uniaxial tension 101
5.5.2. Non-uniform uniaxial stress states 102
5.5.3. Multiaxial stress states: uniform stress state 103
5.5. Conclusion 107

Chapter 6. Size and Stress-State Effects on Fracture Strength 109

6.1. Introduction 109

6.2. Uniform uniaxial stress state 109
6.2.1. Effects of stressed volume or surface size on strengths 109
6.2.2. Respective effects of volume and surface on fracture. 112
6.3. Non-uniform uniaxial stress state 114
6.4. Multiaxial stress state: multiaxial elemental strength model 117
6.5. Applications 118
6.5.1. Influence of loading conditions. 118
6.5.2. Importance of stress effects on the fracture of fiber reinforced ceramic matrix composites 121
6.5.3. Influence of volume or surface size: disadvantages and benefits 125
6.5.4. Influence of shape and geometry: effects of surface-located and volume-located flaw populations. 126
6.6. Conclusion 131

Chapter 7. Determination of Statistical Parameters 133

7.1. Introduction. 133
7.2. Methods of determination of statistical parameters 134
7.2.1. Maximum likelihood technique. 135
7.2.2. Method of moments 136
7.2.3. Fitting theoretical distribution function to empirical one 137
7.2.4. Fitting failure probability computations to empirical values 138
7.2.5. Fitting the tensile behavior curve of multifilament bundles. 139
7.2.6. Examples. 140
7.3. Production of empirical data 142
7.4. Bias and variability 144
7.4.1. Bias of estimators and methods of estimation 145
7.4.2. Variability of statistical parameters 150
7.4.3. Goodness of fit 153

7.5. Effect of the presence of multimodal flaw populations 154
7.5.1. Exclusive populations 156
7.5.2. Concurrent populations 156
7.5.3. Partially concurrent populations 157
7.5.4. Concurrent populations of defects: separation of data 158
7.5.5. Concurrent populations of defects: maximum likelihood method 160
7.6. Fractographic analysis and flaw populations 161
7.7. Examples 161

Chapter 8. Computation of Failure Probability: Application to Component Design 169

8.1. Introduction 169
8.2. Computer programs for failure predictions 170
8.3. The CERAM computer program 172
8.4. Validation of the CERAM computer code 176
8.5. CERAM-based ceramic design 178
8.6. Relation test specimen/component: identification of allowable material properties 181
8.7. Determination of statistical parameters using CERAM 185
8.8. Application to multimaterials and composite materials 186
8.8.1. Prediction of damage by microcracking in ceramic composites 187
8.9. Conclusion 191

Chapter 9. Case Studies: Comparison of Failure Predictions Using the Weibull and Multiaxial Elemental Strength Models 193

9.1. Introduction 193
9.2. Predictions of failure under flexural load 194
9.2.1. Unimodal population of surface defects 194
9.2.2. Unimodal population of internal defects 202
9.2.3. Bimodal population of internal and surface-located flaws 208
9.3. Prediction of thermal shock failure 214
9.3.1. Quenching of alumina disks 215

9.3.2. Thermal fatigue 222
9.4. Conclusion 230

Chapter 10. Application of Statistical-Probabilistic Approaches to Damage and Fracture of Composite Materials and Structures 233

10.1. Introduction 233
10.2. Damage mode by successive cracking in continuous fiber reinforced composites 235
10.3. Flaw populations involved in damage and pertinent flaw strength density functions 237
10.4. Matrix fragmentation: series system model 240
10.4.1. Uniform tensile stress state: uniaxial elemental strength approach 241
10.4.2. Non-uniform stress state: multiaxial elemental strength approach 243
10.4.3. Influence of flaw strength density function 244
10.5. Approach based on Poisson process 246
10.6. The Monte Carlo simulation method 249
10.7. The fragment dichotomy-based model (parallel system) 250
10.7.1. Fragmentation of fibers (uniform stress state) 251
10.7.2. Fragmentation of the matrix in unidirectionally reinforced ceramic matrix composites 252
10.8. Evaluation of models: comparison to experimental data 256
10.9. Ultimate failure of unidirectionnally reinforced composite (Weibull model, uniform tension) in the presence of matrix damage 260
10.10. Application to composites: unified model 262
10.10.1. Uniform tension, unidirectional composites and the Weibull model 262
10.10.2. General approach, the multiaxial elemental strength model 264
10.11. Conclusion 266

Bibliography 269

Index 281