Introduction

> "To the scientific mind, all knowledge is an answer to a question. Without questions there cannot be any scientific knowledge. Nothing is obvious. Nothing is given. Everything is constructed."
>
> Gaston Bachelard, 1938

Geographers deal with three types of issues.

First, they investigate the relative locations of populations, their activities such as industries or services, or their cultural properties. In order to respond to these topics, they make use of a range of theories on localization, which have been principally elaborated by spatial economists. For example, the theory developed by J.H. Von Thünen opened the way for cultural organization to be expressed as rings around a market before restructuring urban economies. A. Weber and W. Alonso concentrated on industrial localization, while W. Christaller and A. Lösch focused on localization of services in key areas. All of these theories are presented and applied in the combined works of M. Fujita *et al.* [FUJ 99] and P.-Ph. Combes *et al.* [COM 06]. Nevertheless, they remain incomplete, as they do not take account of all physical and cultural factors.

Second, geographers study the relationships between human societies and their physical surrounding, especially in classical geography. In the past, the emphasis was on the more or less determinant role of natural environments; now social and cultural dimensions have begun to take priority. In these studies, fewer theories are expounded and they are also less well developed. Geographers thus make use of their preferred empirical research in underlying theoretical contexts, for example Lamarck's or Darwin's theories of evolution. This is well proven by reading any of the major theses on climatic geomorphology or rural geography. The principle of gradualism has for a long time reigned unchallenged, with all reliefs and rural landscapes evolving slowly and uniformly.

Finally, geographers focus on terrestrial forms, whether they are physical, biological or socioeconomic. Seen from a particular point of view, geography is thus a morphology. It becomes morphogenetic when its followers rely on the emergence and evolution of terrestrial forms, such as landscape, town, region and continental transformations. In order to account for the emergence and succession of these forms, theories from physical science are used, such as Turing's theory, which we will make use of in Chapter 8. Life sciences take these theories and improve on them, as illustrated by the works of D'Arcy Thomson [DAR 94] and V. Fleury [FLE 09]. They are less common in social sciences, despite the recent success of the Schelling model [SCH 80], which renews our understanding of sociospatial segregation.

Whatever the question, and therefore whatever the geographical project used – i.e. whether classical or contemporary – spatial discontinuities, differentiations and disparities are always at the heart of the geographical project, as proclaimed by R. Brunet [BRU 68]. Geographers have always been united in considering fronts and frontiers, irregularities, diversities and disparities of territories as being central to their scientific programs. Certainly, every geographer follows his or her own methodology or favors one technique over another, but all concentrate their thoughts on the issues that they judge to be the most important.

Furthermore, these irregularities and spatial disparities are observed on all scales. P. Claval [CLA 68] strongly emphasized this in a reference work; but he was not alone. All geographers are sensitive to the multiscale characteristics of the behaviors being studied. The drafting of any job application for any university confirms this. In France, this theme is even followed in the "Aggregation" program [BAU 04]. The majority of geographical studies reflect on disparities and scales. Two recent contributions bring this to mind: the work of E. Sheppard [SHE 04] and the theme of the *Géopoint* 2010 symposium. Many others can also be quoted from all domains of geography.

This interest in multiscale systems is, moreover, not paricular to geography, but is found in a great many disciplines. Physicists have been plowing this field since Boltzman, who connected Clausius entropy, a macroscopic width, to the configuration of microstates. More generally, 20th Century statistical physics, which accords a priority to microstates, has been constructed in order to account for the laws of macrophysics discovered in the 19th Century. The contemporary work of

M. Laguës and A. Lesne [LAG 03], which is dedicated to this discipline, thus offers a host of recommendations to geographers and social science specialists.

It is the same for other disciplines. Some similar contributions come from the works of economists and sociologists faced with the double problem of combining inequality with scale. Long debates still divide the advocates of micro- versus macroeconomics or question the role of methodological individualism in sociology. Furthermore, in the area of life sciences, the principle of natural selection has changed. From the Darwinian level of species it is now identified as active at the cellular level in post-Darwinian theory.

Yet despite this, the fractal paradigm, which is the subject of this small collection of works, provides a new view of the disparities and at the same time of multiscale phenomena. It constitutes a sort of hinge, or bridge, which links these two main questions. It opens the way to a combined analysis of irregularity and level. From this fact all studies into morphology and territorial morphogenesis and, more generally, geographic studies are updated.

Initially, B. Mandelbrot [MAN 75] describes irregular mathematical objects as fractals, whose irregularities are the same on all scales. These abstract objects are said to be auto-similar and scale invariant. These two very similar concepts are not, however, synonyms, which is why they merit some preliminary discussion in this introduction.

The first concept, auto-similarity or internal similitude, is geometric in nature. It refers to an object comprised of sections that are copies of the object itself, which signifies that the whole is identical to its parts. Each section can also be broken down into subsections identical to itself. In reality, this iterative process, which is repeated infinitely in fractal mathematics, always has an upper and lower limit. In order to designate these double-limit fractals as "real" or "known", B. Mandelbrot preferred the term pre-fractal, but this concept was abandoned. These two limits however, should not be confused with resolution and range, which are two other limits that are dependent on observation and not the nature of the object.

Directly linked to this, the concept of scale invariance, or invariance by dilatation, is more statistical in nature. It indicates that a similar characteristic is observed at all scales. The fractal dimension is a measure of the rate of variation of data from one level to another, which is why B. Sapoval [SAP 97] considers fractals to be a geometry of probabilities. Thus, contrary to what some users believe, the fractal approach is not reduced to a geometric approach (the study of shapes).

Furthermore, scale invariance is a more general concept than auto-similarity. Scale invariance certainly encompasses auto-similarity, but also long-term or long-range dependence. Identification of this dependence is a result of attributing fractal characteristics to analyzed observations.

There is an abundance of literature on fractals that shows a number of authors prefer to think in terms of auto-similarity, geometry or the Koch or Peano shapes, while financial specialists, who are passionate about share prices, almost exclusively reflect on scale invariance, without recourse to geometry. In this work we try not to favor one viewpoint over the other, since we share B. Sapoval's idea, which in a way boils down to reuniting space and time.

This inherent link between geometry and probability also indicates that all scales are significant for the system under consideration. Better still, for a proper understanding of a fractal phenomenon, physicists have demonstrated that the interactions between scales have greater determinance than those between elements. This is doubtless true for other biological and social phenomena.

Before we consider moving away from mathematics for the first three chapters, remember that a fractal has a higher Hausdorff-Besicovitch dimension than topological dimension. A straight or broken line has a topological dimension equal to 1. However, if it is irregular, then its fractal dimension is greater than 1. Similarly, an irregular area will have a higher fractal than topological dimension of 2.

The Hausdorff-Besicovitch dimension, which measures the extent of this irregularity, is the logarithmic ratio between the number of internal homotheties of the object N and the inverse of this ratio, r [LOP 10]. This definition is sufficient to convey the fact that this dimension is difficult to determine for concrete physical objects. In order to overcome this problem, scientists from different disciplines use numerous algorithms. This explains the richness and complexity of Chapters 4, 5 and 6, which, without intending to be exhaustive, outline the vast number of fractal and multifractal dimensions.

The scope of application of fractals is widening beyond mathematics. First, they are described as physical or living shapes, such as a coastal outline, a river system or as a neural maze. At the same time, researchers are moving away from the study of shapes to that of processes in all branches of science, whether physical or economical, such as in linguistics. The fractal treatment of chronological sequences has been generalized, most notably in the areas of climatology and financial economics.

Several techniques have been developed in order to calculate the fractal dimension, depending on the circumstances encountered. Nevertheless, a fractal object always has a higher Hausdorff-Besicovitch dimension than its topological dimension. Too many studies seem to ignore this rule and instead consider the fractal dimension from the slope of a log-log or bilogarithmic adjustment; hence various authors produce examples where fractal dimensions are less than the topological dimensions of the objects being studied. Without exception, as we will see later in this book, the fractal dimension is calculated from the slope of the log-log graph, but it is not equal to this slope. It is derived from this by using a formula that is adapted to each fractal category.

Scientists are no longer content to qualify an object, to simply describe its irregularity in terms of an overall fractal dimension, equivalent to a statistical mean. They develop theories in order to explain fractality in global terms. Over recent decades, fractality has presented itself as a veritable paradigm, with its techniques, methods and theories.

In French geography, the pioneering works were those of A. Dauphiné [DAU 90-91, DAU 95] and P. Frankhauser [FRA 91, FRA 94]. Based on their work, this technique was expanded upon by the southern teams at the UMR ESPACE and the UMR ThéMA in Besançon. Other theoretical geography research teams have also richly contributed to this area, notably in Caen, Rouen, Grenoble, Pau, Strasbourg and Paris; not forgetting some brilliant "retirees" [BRI 04] and some French-speaking colleagues in Milan, Louvain, Switzerland and Québec. Today, over 100 French and French-speaking geographers understand and occasionally practice the fractal approach. This compilation work owes much to their research.

Quite clearly, the fractal approach is not exclusive to French geography. Some works were also expanded upon in geography in the English-speaking world following the publication edited by N. Lam and I. De Cola [LAM 93]. This has already uncovered explanations on fractal hydrological networks or on the localization of key places. The authors then called upon rigid explanations to account for irregularities measured using the fractal dimension. Additionally, in this landmark publication, two interventions demonstrate the benefit that geographers are able to draw from multifractal formalism. In particular, these authors provided geographers with computer programs, written in FORTRAN, which enabled fractal dimensions to be calculated and the first simulations of terrestrial reliefs to be created.

This publication was put together following seminal articles on ecology by P.A. Burrough [BUR 81], urban geography by M. Batty [BAT 85], geomorphology by M.F. Goodchild [GOO 87], and many others. It is difficult to name them all without forgetting some. Since these initial works, other schools have joined this vast movement, notably Chinese geographers from Peking University, who are very active in this field of research.

Well beyond geography, the fractal paradigm has flooded into all disciplines – the physical and chemical sciences [GOU 92], the life sciences, engineering

[ABR 02] and the economic and economic sciences [MAN 97, LEV 02]. Even philosophers are examining this paradigm, as attested by A. Boutot's book [BOU 93]. In the final chapters of his book, this philosopher advances a very informative classification for fractals.

Indeed, for the individual social sciences, the science and social science indices lists more than 20,000 works published in 2010, 10,574 in 2011 amongst a total of 340,201. We have not read all of these works, but they do provide valuable assistance to the geographer, since they introduce new methods and techniques. Furthermore, the most recent of these advance some of the formal and general theories that bypass rigid explanations and are too reductionist.

As with all compilations, this work is provisional. It is written for use by geographers and researchers from similar disciplines, such as ecologists, economists, historians and sociologists, which is why we are adopting a classical format, tackling descriptions of fractal phenomena before explaining them in later chapters. This description relies on a series of observations. However, there is nothing to prevent these exercises from being carried out on results from simulation, such as images produced by multiagent systems or macromodels that couple differential equations.

The first part of this book sets out a fractal panorama avoiding mathematical formalization, which is the unifying theme of the second part, as much as possible. Having observed the diversity and ubiquity of a fractal geographical world in Chapter 1, in Chapter 2 we will distinguish auto-similar fractals from those that are self-affine. Chapter 3 concludes this first part, with a rapid tour of the tools that have come about from the fractal dimension, from lacunarity to multifractal spectrums.

The next three chapters present a number of algorithms that are not the most widely used, but are best matched to the phenomena studied within social sciences and geography. After a study of auto-similar and self-affine fractals in Chapter 4, Chapter 5 deals with the fractal dimension of the rank-size rules. Generations of geographers, notably urban geography specialists, have relied on these rank-size rules. They are still being hotly debated and so merit special attention. Finally, multifractal formalism is tackled in Chapter 6. In all of these chapters, case studies enable us to judge the relevance of algorithms applied to "concrete" situations, with sets of data illustrating some diverse phenomena in one or two dimensions.

Above and beyond description, all geographers wish to bring about an explanation. The third part of this book comprises two chapters focused on comprehension, first disciplinary and second more general, on fractal shapes and processes. In Chapter 7 we show how it is possible to interpret fractal dimensions and power laws in geographical and, more generally, disciplinary terms. This chapter already demonstrates that the fractal is not simply a descriptive tool. It engages the geographer in new, illustrative ways. Pursuing our efforts at abstraction; in Chapter 8 we shed light on a number of general formal theories which bypass disciplinary frontiers. These controversial complexity theories will, despite their ambiguity, direct us towards a world where fractality rules.

Finally, the last chapter shows that the fractal approach raises questions about and provides answers regarding environmental management, or the tackling of problems related to town planning. Fractal reasoning is not a simple game played out on a computer screen or a highly theoretical exercise, but instead opens up some very real problems. Managing a fractal world imposes changes. This chapter is completed with a long appendix presenting computer programs written in the *Mathematica* language. Not yet optimized, these programs offer the potential to be used directly by young geographers who are interested in implementing these approaches.

The roots of this work are long-standing. It has thus benefited from constructive comments from our young southern colleagues, exchanges during thesis presentations and in meetings of the Dupont group, notably in the presence of the Avignon geographers, and from the *Théo Quant* symposiums. We would like to thank all of these contributors for their assistance, particularly Eric Bailly and Damienne Provitolo, who checked our occasionally too condensed reasoning and enabled us to improve the comprehension of this text, while patiently requesting clarifications. Whatever remains vague is quite clearly down to us.