

Contents

Symbols and Constants	ix
Chapter 1. General Relativity	1
Brahim LAMINE	
1.1. The fundamentals of general relativity	1
1.1.1. The equivalence principle	1
1.1.2. Fundamentals of non-Euclidean geometry	14
1.2. Tensor analysis and curvature of space–time	33
1.2.1. Tangent space	33
1.2.2. Differentiation and parallel transport	43
1.2.3. Curvature	53
1.3. General relativity equations	68
1.3.1. Covariance principle	68
1.3.2. Energy–momentum tensor	70
1.3.3. Einstein equations	79
1.3.4. The Schwarzschild solution	85
1.4. References	104
Chapter 2. Compact Objects	107
Natalie WEBB	
2.1. Introduction	107
2.1.1. Compactness	109
2.2. White dwarfs	109
2.2.1. Formation	110
2.2.2. Electron degeneracy	111
2.2.3. Characteristics	115
2.2.4. Observations	118
2.3. Neutron stars	120
2.3.1. Formation	120

2.3.2. The neutron drip and baryonic degeneracy	121
2.3.3. Neutron star structure	122
2.3.4. The equation of state	123
2.3.5. Characteristics	126
2.3.6. Pulsars	129
2.3.7. Post-Keplerian parameters	133
2.3.8. Observations	136
2.4. Black holes	140
2.4.1. Formation	140
2.4.2. The mass	142
2.4.3. The radius	144
2.4.4. Black hole spin	145
2.4.5. Temperature	146
2.4.6. Gravitational redshift	147
2.4.7. Observations	147
2.5. Binary systems	153
2.5.1. Roche lobes	154
2.5.2. Binary evolution	156
2.5.3. Binary mass function	158
2.5.4. Accretion	160
2.5.5. Ejection	165
2.5.6. Observations	168
2.6. References	170
Chapter 3. Gravitational Waves	179
Patrice HELLO	
3.1. Gravitational waves in the context of general relativity	179
3.1.1. Some historical background	179
3.1.2. Properties of gravitational waves	181
3.1.3. Generation	184
3.1.4. Astrophysical sources and expected signals	191
3.2. Detection of gravitational waves	201
3.2.1. The effect of a gravitational wave on matter	201
3.2.2. Interferometric detectors (LIGO and Virgo)	204
3.2.3. Other detectors (LISA, pulsar chronometry)	217
3.2.4. Data analysis techniques	218
3.3. Gravitational wave astronomy	227
3.3.1. The fusion of black holes, GW150914 and others	228
3.3.2. GW170817 and the birth of multi-messenger astronomy	231
3.3.3. Constraints on the theories of gravitation	233
3.3.4. Astrophysical implications	240

3.3.5. Cosmology	246
3.4. The future	249
3.5. References	249
Chapter 4. The Friedmann-Lemaître Universe and Cosmological Expansion	253
Richard TAILLET	
4.1. Geometry	254
4.1.1. Comoving coordinates	255
4.1.2. Distances and coordinates	255
4.1.3. Metric over a sphere	256
4.1.4. FLRW metric	258
4.1.5. Area of a sphere of radius r	259
4.1.6. Volume of a sphere of radius r	260
4.2. Cosmological expansion	261
4.2.1. Redshift	261
4.2.2. Hubble–Lemaître law	263
4.2.3. Value of the Hubble constant	264
4.2.4. Remark on the interpretation of the Hubble–Lemaître law	265
4.2.5. Relationship to the Doppler effect	266
4.2.6. Domain of validity	266
4.2.7. Fundamental clarification	267
4.2.8. Decrease in physical velocities	267
4.3. The Friedmann–Lemaître equations	268
4.3.1. Writing the equations	268
4.3.2. Content of the Universe	269
4.3.3. Critical density and density parameters	271
4.3.4. Current values	273
4.3.5. Dominant component	273
4.3.6. The flatness problem	274
4.4. Temporal evolution	275
4.4.1. The evolution of the scale factor	275
4.4.2. Warning	275
4.4.3. The case of a flat universe with a single component	276
4.4.4. Initial singularity and age of the Universe	277
4.4.5. Big Bang	279
4.4.6. The case of an empty and flat universe with a cosmological constant	279
4.4.7. Flat universe containing matter and radiation ($\Lambda = 0$)	280
4.4.8. Case of a non-flat universe with matter	281
4.4.9. Matter and cosmological constant: our current Universe	286

4.4.10. Age of the Universe	287
4.4.11. The $\Omega_m^0 - \Omega_\Lambda^0$ plane	290
4.5. Horizons	294
4.5.1. Particle horizon	294
4.5.2. The horizon problem	296
4.5.3. Horizon and redshift	297
4.5.4. Event horizon	298
4.5.5. Visible universe	303
4.6. Distances	303
4.6.1. Scale of cosmological distances	303
4.6.2. Ambiguity in the concept of distance	304
4.6.3. Comoving distance – relation between r and z	305
4.6.4. Physical distance and angular size	306
4.6.5. A note on recessional velocity	310
4.7. The redshift–luminosity relation	311
4.7.1. Luminous flux	312
4.7.2. Example of a flat universe made up of matter and radiation	313
4.7.3. Example of a flat universe composed of a cosmological constant	316
4.7.4. Calculating and plotting the curve	316
4.8. Angular size distance	317
4.9. Cosmological supernovas	321
4.9.1. Supernovas	322
4.9.2. Images	326
4.9.3. Magnitudes	328
4.9.4. Spectral bands	329
4.9.5. K-correction	331
4.9.6. Standardizable candles	331
4.9.7. Constraints on cosmological parameters	331
4.10. Conclusion	332
4.10.1. Cosmological probes	332
4.10.2. Quintessence	334
4.10.3. What next?	335
4.11. References	335
List of Authors	337
Index	339