
Contents

Preface	xi
Notations	xiii
Introduction	xxv
Chapter 1. Elastic Domains: Yield Conditions	1
1.1. Introductory remarks	1
1.2. An overview of the model	4
1.2.1. The infinitesimal transformation framework	4
1.2.2. Time variable	4
1.3. One-dimensional approach.	5
1.3.1. Uniaxial tension test	5
1.3.2. Uniaxial tension-compression test	8
1.3.3. The Bauschinger effect	9
1.3.4. Other one-dimensional experiments	10
1.4. Multidimensional approach	11
1.4.1. A multidimensional experiment	11
1.4.2. Initial elastic domain	12
1.4.3. Work-hardening.	13
1.4.4. Perfectly plastic material	14
1.4.5. Bui's experimental results.	14
1.5. Yield conditions	16
1.5.1. Initial yield condition and yield function	16
1.5.2. Loading function and work-hardening	18
1.5.3. Simple work-hardening models	19
1.6. Yield criteria and loading functions	22

1.6.1. Convexity	22
1.6.2. Isotropy.	23
1.6.3. The Tresca yield criterion.	24
1.6.4. The von Mises yield criterion.	27
1.6.5. Other yield criteria for metals.	30
1.6.6. Yield criteria for anisotropic materials	31
1.6.7. Yield criteria for granular materials	35
1.7. Final comments	41
Chapter 2. The Plastic Flow Rule	43
2.1. One-dimensional approach.	43
2.1.1. Work-hardening material	43
2.1.2. Perfectly plastic material	45
2.2. Multidimensional approach for a work-hardening material.	46
2.2.1. Loading and unloading processes	46
2.2.2. General properties of the plastic flow rule	49
2.2.3. Plastic potential: associated plasticity	51
2.2.4. Principle of maximum plastic work	54
2.2.5. Validation of the principle of maximum plastic work	55
2.2.6. Piecewise continuously differentiable loading functions.	57
2.3. Multidimensional approach for a perfectly plastic material	59
2.3.1. Loading and unloading processes	59
2.3.2. Application of the principle of maximum plastic work.	61
2.3.3. Drucker's postulate.	62
2.4. Plastic dissipation	64
2.4.1. Plastic dissipation per unit volume.	64
2.4.2. Plastic dissipation and support function of the elastic domain	64
2.4.3. Plastic velocity jumps in the case of perfectly plastic materials.	65
2.5. Generalized standard materials	66
2.6. Mises', Tresca's and Coulomb's perfectly plastic standard materials.	69
2.6.1. Mises' perfectly plastic standard material	69
2.6.2. Tresca's perfectly plastic standard material	71
2.6.3. Coulomb's perfectly plastic standard material.	72
2.6.4. About edge and vertex regimes.	74
Chapter 3. Elastoplastic Modeling in Generalized Variables	77
3.1. About generalized variables	77
3.2. Elastic domains	78
3.2.1. Initial elastic domain	78
3.2.2. Work-hardening and perfect plasticity	79
3.3. The anelastic flow rule	80

3.3.1. Anelasticity or plasticity?	80
3.3.2. Principle of maximum work	81
3.3.3. The work-hardening anelastic flow rule	82
3.3.4. The “perfectly plastic” anelastic flow rule	83
3.3.5. Anelastic dissipation	84
3.4. Generalized continua	84
3.4.1. Curvilinear generalized continuum	84
3.4.2. Planar generalized continuum	90
Chapter 4. Quasi-static Elastoplastic Processes	101
4.1. Quasi-static loading processes.	101
4.1.1. Mechanical evolution within the SPH framework.	101
4.1.2. Quasi-static loading process within the SPH framework.	104
4.1.3. Statically admissible and kinematically admissible fields	104
4.1.4. Parametric problems	105
4.2. Quasi-static elastoplastic loading processes.	108
4.2.1. Problematics	108
4.2.2. Existence and uniqueness theorems	111
4.2.3. Uniqueness theorems for stress rates and strain rates	115
4.3. Response of a system made from an elastic and standard perfectly plastic material	116
4.3.1. Initial elastic domain of the system	116
4.3.2. Existence of the solution to the elastoplastic evolution problem	118
4.3.3. Solution to the elastoplastic evolution problem	119
4.3.4. Limit loads for the system.	120
4.3.5. Linear elastic response of the system	121
4.3.6. Anelastic response of the system.	122
4.3.7. Taking geometry changes into account	136
4.4. Response of a system made from a standard work-hardening elastoplastic material	139
4.4.1. Initial elastic domain of the system	139
4.4.2. Residual stress rates, residual strain rates.	140
4.4.3. Maximum work theorem	140
4.4.4. Summing up....	141
Chapter 5. Quasi-static Elastoplastic Processes: Minimum Principles	143
5.1. Elastic and standard perfectly plastic constituent material	143
5.1.1. Minimum principle for the stress rate field.	143
5.1.2. Minimum principle for the velocity field	146
5.1.3. Other expressions of the minimum principles	152
5.2. Elastic and standard positive work-hardening constituent material	153

5.2.1. Revisiting the constitutive equation	153
5.2.2. Minimum principle for the stress rate field	155
5.2.3. Minimum principle for the velocity field	156
5.2.4. Other expressions of the minimum principles	158
5.2.5. Historical comments	158
5.3. Minimum principles for the stress and strain fields	159
5.3.1. Colonnetti's theorem	159
5.3.2. Other expressions of Colonnetti's minimum principles	160
Chapter 6. Limit Loads: Limit Analysis	161
6.1. Limit loads and yield design (1).	161
6.2. Static approach, first plastic collapse theorem	163
6.2.1. Safe loads, interior approach	163
6.2.2. Lower bound theorem	164
6.3. Kinematic approach, second plastic collapse theorem.	165
6.3.1. Plastically admissible velocity fields	165
6.3.2. Kinematic necessary condition to be satisfied by safe loads.	167
6.3.3. Exterior approach, upper bound theorem	169
6.4. Combining static and kinematic approaches	170
6.4.1. Determination of a limit load	170
6.4.2. Association theorem	172
6.4.3. Duality	173
6.5. Limit analysis and the rigid, perfectly plastic material concept	173
6.5.1. Rigid and standard perfectly plastic model.	173
6.5.2. The connection with limit loads	174
6.6. Limit loads and yield design (2).	176
6.6.1. Fundamentals of the yield design theory	176
6.6.2. Resistance of the constituent material	177
6.6.3. Potentially safe loads, interior approach and lower bound theorem	178
6.6.4. Maximum resisting rate of work, exterior approach and upper bound theorem	178
6.6.5. Matching limit load and yield design theories	181
6.7. Two-dimensional limit analysis	182
6.7.1. Plane strain limit analysis problems	182
6.7.2. Partial static solutions to plane strain limit analysis problems	183
6.7.3. Complete static solutions to plane strain limit analysis problems.	183
6.7.4. Complete kinematic solutions to plane strain limit analysis problems	184
6.7.5. "Incomplete" solutions to plane strain limit analysis problems	188
6.7.6. Complete solutions to plane strain limit analysis problems	189
6.7.7. Comments	192

6.7.8. Plane stress limit analysis	194
6.7.9. Axially symmetric problems	196
6.8. Implementation	200
6.8.1. Analytical solutions	200
6.8.2. Analytical/numerical solutions	202
6.8.3. Numerical solutions	203
6.8.4. The example of a tantalizing problem	204
6.8.5. Final comments	208
References	211
Index	231