
Contents

Notations	xi
Preface	xxi
Part 1. Typical Processes in Queues	1
Chapter 1. The Poisson Process	3
1.1. Review of the exponential distribution.	3
1.1.1. Definitions.	3
1.1.2. The properties of an exponential distribution.	4
1.2. Poisson process	10
1.2.1. Definitions.	10
1.2.2. Properties of the Poisson process	12
1.3. Exercises	16
Chapter 2. Markov Chains	21
2.1. Markov chains in discrete time	21
2.1.1. Definitions.	21
2.1.2. Evolution of a stochastic vector over time	26
2.1.3. Asymptotic behavior	30
2.1.4. Holding time in a state	32
2.1.5. Time-reversible chain	33
2.1.6. Reversible Markov chains	34
2.1.7. Kolmogorov's criterion.	34
2.2. Markov chains in continuous time	35
2.2.1. Definitions.	35
2.2.2. Evolution over time.	38
2.2.3. Resolving the state equation	41

2.2.4. Asymptotic behavior	42
2.3. Birth and death process	43
2.3.1. Definition	43
2.3.2. Infinitesimal stochastic generator	43
2.3.3. Stationary distribution	44
2.4. Exercises	45
Part 2. Queues	51
Chapter 3. Common Queues	53
3.1. Arrival process of customers in a queue	53
3.1.1. The Poisson process	53
3.1.2. Using the Poisson distribution $\mathcal{P}(\lambda)$	54
3.1.3. Exponential distribution of delay times	55
3.2. Queueing systems	57
3.2.1. Notation for queueing systems.	58
3.2.2. Little distributions	59
3.2.3. Offered traffic	60
3.3. M/M/1 queue.	60
3.3.1. Stationary distribution	61
3.3.2. Characteristics of the M/M/1 queue	62
3.3.3. Introducing a factor of impatience.	64
3.4. M/M/ ∞ queue	65
3.5. M/M/n/n queue.	66
3.5.1. Stationary distribution	67
3.5.2. Erlang-B formula	67
3.5.3. Characteristics of the M/M/n/n queue.	68
3.6. M/M/n queue.	68
3.6.1. Stationary distribution	69
3.6.2. Erlang-C formula	70
3.6.3. Characteristics of the M/M/n queue	70
3.7. M/GI/1 queue	71
3.7.1. Stationary distribution	71
3.7.2. Characteristics of the M/GI/1 queue.	73
3.8. Exercises	74
Chapter 4. Product-Form Queueing Networks	79
4.1. Jackson networks	80
4.1.1. Definition of a Jackson network	80
4.1.2. Stationary distribution	81
4.1.3. The particular case of the Jackson theorem for open networks	84
4.1.4. Generalization of Jackson networks: BCMP networks	84

4.2. Whittle networks.	85
4.2.1. Definition of a Whittle network	85
4.2.2. Stationary distribution	88
4.2.3. Properties of a Whittle network	88
4.3. Exercise.	89
Part 3. Teletraffic.	91
Chapter 5. Notion of Teletraffic	93
5.1. Teletraffic and its objectives	93
5.2. Definitions	94
5.2.1. Measures in teletraffic	94
5.2.2. Sources and resources	95
5.2.3. Requests and holding time	96
5.2.4. Traffic	97
5.3. Measuring and foreseeing traffic	101
5.3.1. Traffic and service quality	101
5.3.2. Measuring traffic	102
5.3.3. Markovian model of traffic	102
5.3.4. Economy and traffic forecasting.	103
5.4. Exercises	103
Chapter 6. Resource Requests and Activity	107
6.1. Infinite number of sources	107
6.1.1. Distribution of requests in continuous time.	107
6.1.2. Distribution of requests in discrete time	110
6.1.3. Duration of activity distributions	113
6.1.4. Distribution of busy sources	115
6.2. Finite number of sources	115
6.2.1. Modeling with birth and death processes	116
6.2.2. Distribution of requests.	117
6.3. Traffic peaks and randomness	118
6.3.1. Traffic peaks.	118
6.3.2. Pure chance traffic	119
6.4. Recapitulation	119
6.5. Exercises	120
Chapter 7. The Teletraffic of Loss Systems	123
7.1. Loss systems	124
7.1.1. Definitions.	124
7.1.2. Blocking and loss	124

7.2. The Erlang model	126
7.2.1. Infinite number of resources	127
7.2.2. Finite number of resources	128
7.2.3. Erlang-B formula	131
7.2.4. Dimensioning principles	132
7.3. Engset model	133
7.3.1. Sufficient number of resources	133
7.3.2. Insufficient number of resources	135
7.3.3. On the Engset loss formula.	137
7.4. Imperfect loss systems	137
7.4.1. Loss probability in an imperfect system with limited and constant accessibility	137
7.4.2. Losses in a system with limited and variable accessibility	138
7.5. Exercises	138
Chapter 8. Teletraffic in Delay Systems	143
8.1. Delay system	143
8.1.1. Description	143
8.1.2. Characteristics of delay.	144
8.2. Erlang model	145
8.2.1. Infinitely long queue	145
8.2.2. Erlang-C formula	146
8.2.3. Distribution of delays.	147
8.3. Finite waiting capacity model	150
8.3.1. Queues of finite length	150
8.3.2. Limitations affecting the delay.	151
8.4. Palm model.	151
8.4.1. M/M/n/N/N queue	152
8.4.2. Characteristics of traffic	153
8.5. General distribution model for activity	153
8.5.1. The Pollaczek–Khinchine formula	153
8.5.2. Activity with a constant duration	154
8.6. Exercises	155

Part 4. Answers to Exercises	161
Chapter 9. Chapter 1 Exercises	163
Chapter 10. Chapter 2 Exercises	171
Chapter 11. Chapter 3 Exercises	185
Chapter 12. Chapter 4 Exercise	197
Chapter 13. Chapter 5 Exercises	201
Chapter 14. Chapter 6 Exercises	205
Chapter 15. Chapter 7 Exercises	207
Chapter 16. Chapter 8 Exercises	211
Part 5. Appendices	219
Appendix 1	221
Appendix 2	227
References	233
Index	235