Series Editor
Henri Maître

Topographical Tools for Filtering and Segmentation 1

Watersheds on Node- or Edge-weighted Graphs

Fernand Meyer

Color section

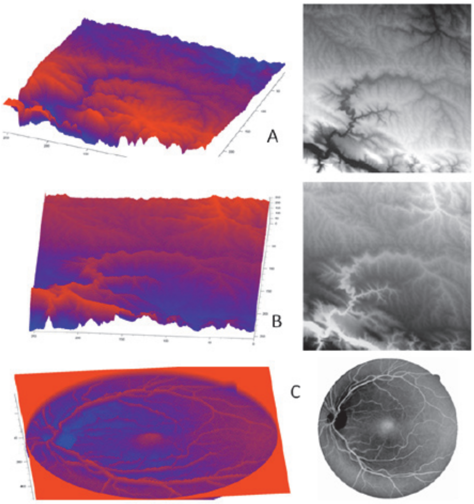

Figure 1.1. *a) A real topographic surface, represented as a relief on the left and as a digital elevation model on the right-hand side. b) A topographic surface which does not exist in reality. The preceding relief has been turned upside down; the digital elevation model being the negative of the preceding one. c) Angiography of a human eye on the right-hand side and its representation as a topographic surface on the left-hand side*

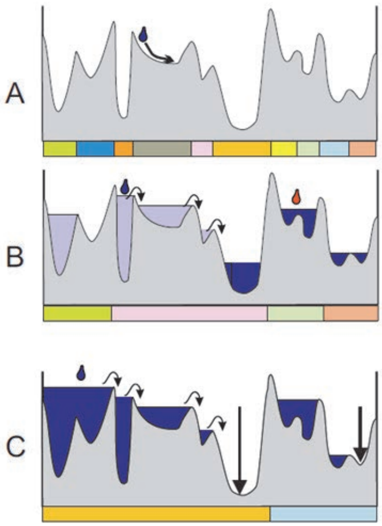

Figure 1.2. *The catchment zones associated with the various ways of flooding a topographic surface. The watershed partition is situated below each topographic surface, with different colors assigned to each tile. a) An unflooded topographic surface, in which all regional minima generate a catchment zone. The trajectory of a drop of water is highlighted. b) On this flooded topographic surface, there remain four regional minimum lakes. All others are full lakes and belong to the catchment zone of a regional minimum lake. Adding a drop of water to a full lake provokes a cascade of overflows, until reaching a regional minimum lake. c) A similar situation to that in B. This particular flooding is the highest flooding for which the regional minima marked by a vertical arrow remain dry. It contains two regional minima*

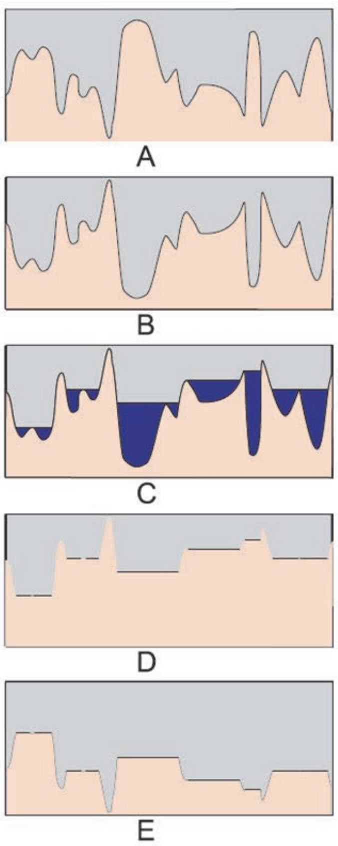

Figure 1.3. *a) Initial topographic surface f. b) Mirror function $0 - f$. c) A particular flooding $Fl\ (0 - f)$ of the mirror function where the lakes are highlighted. d) The same topographic surface as in C, without highlighting the lakes. e) The mirror function of the topographic function of D: $Rz\ (f) = 0 - Fl\ (0 - f)$*

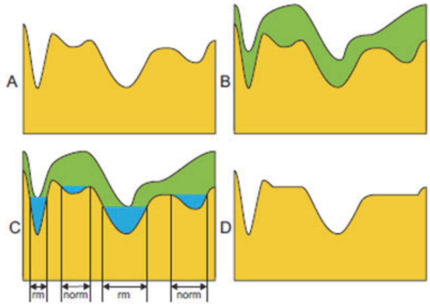

Figure 1.4. *Using dominated flooding as a filtering tool: a) A topographic relief f in yellow. b) A ceiling function $g \geq f$ in green. c) The highest flooding of f under g creates four lakes. The lakes $\langle$mr$\rangle$ are regional minimum lakes, whereas the lakes $\langle$nomr$\rangle$ are non-regional minimum lakes. d) Filling the non-regional minimum lakes and suppressing the regional minimum lakes produce a filtered version of A, in which only the most salient wells are preserved*

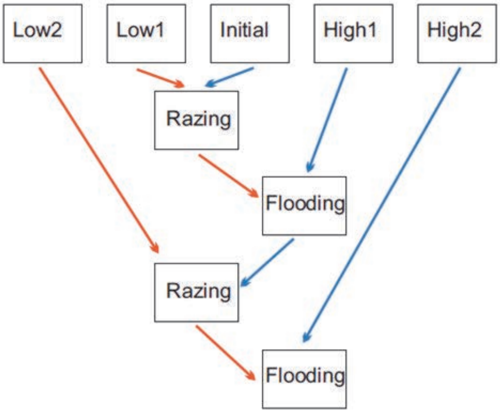

Figure 1.7. *Alternate sequential razings and floodings for filtering images*

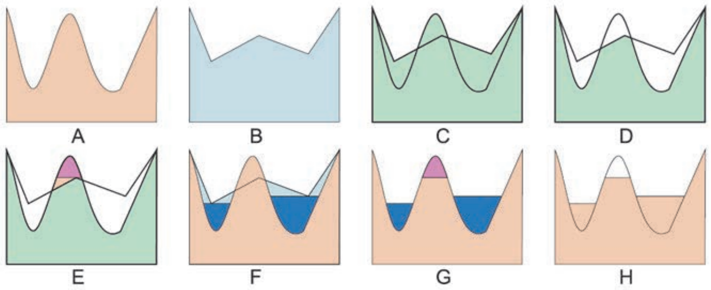

Figure 1.9. *A combination of flooding and razing in order to flatten a function f bounded by a second function g*

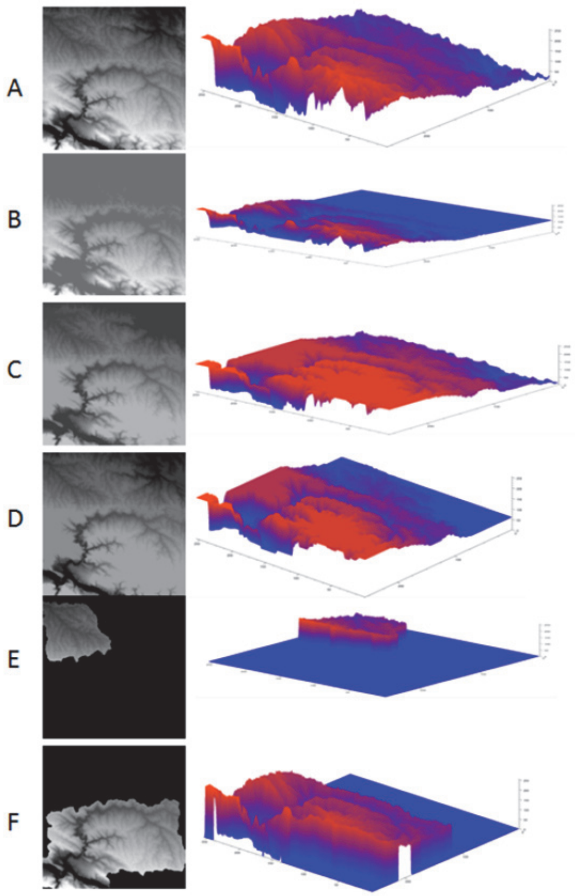

Figure 1.10. *a) R = initial relief; b) flooding of R; c) razing of R; d) flooding followed by razing of R; e) catchment zone of a regional minimum; f) catchment zone of another regional minimum*

Figure 1.11. *An image (the luminance of a painting by Gauguin), its gradient image and the watershed partition associated with the gradient. This largely oversegmented partition counts 4,888 tiles*

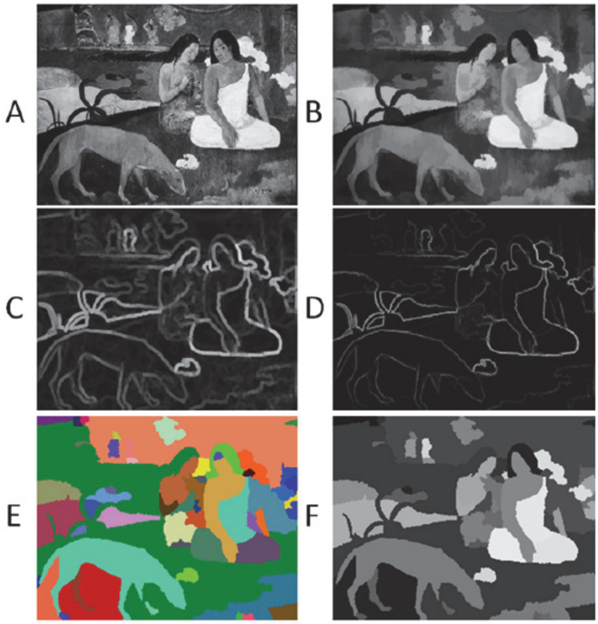

Figure 1.12. *Filtering the initial image and its gradient in order to obtain sparse segmentations with a low number of significative regions: a) Initial image. b) Filtering by alternative razing and flooding. c) A ceiling function for the gradient image, obtained by dilating it and adding a constant to it. d) The flooded gradient image. e) The watershed partition of the filtered gradient image of image D. f) A mosaic image obtained by replacing each tile of the watershed partition by its mean value*

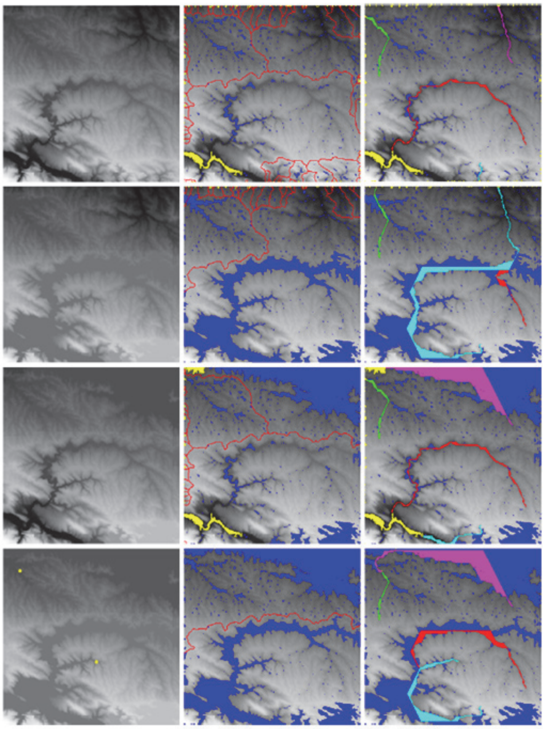

Figure 1.13. *Catchment zones and flowing paths of a topographic surface if we modify the outflow points of the relief. In each row, a different outflow pattern is chosen: the complete boundary (which is the real situation) in the top row, the upper border for the second row, the left-side border for the third row and finally two yellow nodes (seen in the leftmost image) in the last row. For each row, we show the flooded topographic surface in gray on the left-hand side, in the center, the flooded zones in blue, with the limits of the catchment zones in red and the outflow points in yellow. On the right-hand side the flowing paths of marked points in the image are indicated, at identical positions in all four rows*

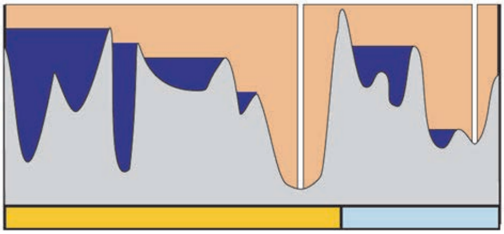

Figure 1.14. *A dominated flooding of a topographic relief in which two regions are marked. The ceiling function (in orange) is equal to the relief in the marked zones, and equal to the maximal altitude elsewhere. The resulting dominated flooding has the marked regions as only regional minima, all lakes which are created are full lakes*

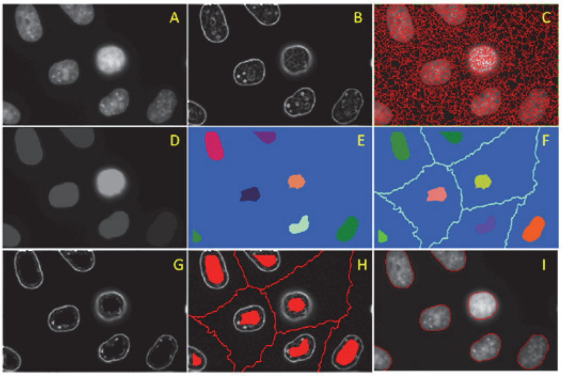

Figure 1.16. *Segmentation of cell nuclei. a) Initial image. b) Gradient image. c) Contours of the watershed partition associated with the gradient image showing a severe oversegmentation, even in the background, due to the presence of low contrasted regional minima in the gradient image. d) Filtering of the chromatine structure of the nuclei using razing followed using flooding. e) inside markers of the nuclei derived from image D. f) The watershed partition of the initial image associated with the markers in image E. g) A ceiling function is constructed, equal to 0 on the markers in F and to the maximum gray tone everywhere else. The highest flooding of the gradient image is now regularized. h) The regional minima in red of the regularized gradient of G. i) The contours in red of the watershed partition of H, yielding the final segmentation*

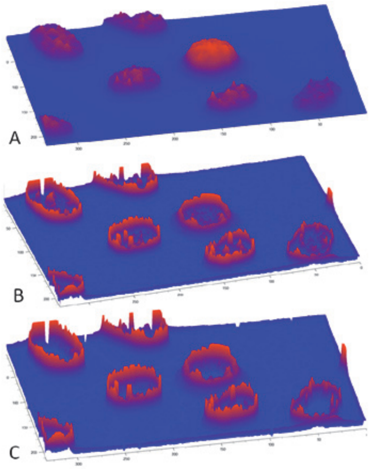

Figure 1.17. *a) A 3D representation of cell nuclei. b) The gradient image of the nuclei with many minima inside and outside the cells. c) The regularized gradient image in which only one regional minimum remains in each nucleus and in the background*

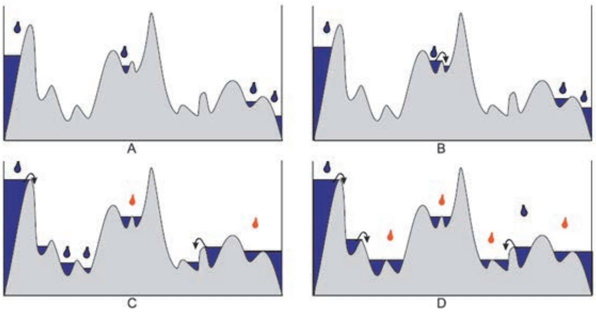

Figure 1.18. *Creating critical flooding of a topographic surface: a) Creation of the four first lakes. b) The second lake is full and flows over neighboring basins, creating a new lake. c) The red drops fall into regional minimum lakes, increasing their level. The blue drops increase the level of the other lakes until they are full and flow over into a neighboring basins. d) Critical flooding: all lakes are full up to their lowest pass point. The blue drops provoke overflows of the full lakes they are falling in. The red drops fall into regional minimum lakes, whose level is increased with any additional drop of water*

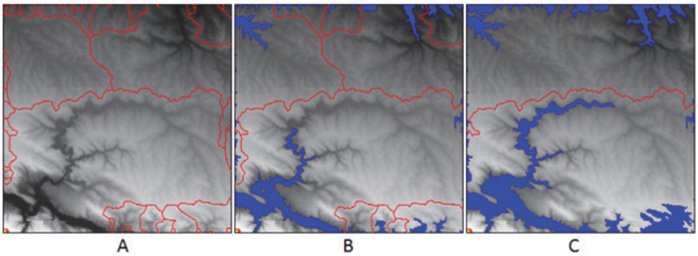

Figure 1.19. *Three levels of the waterfall hierarchy: a) The initial topographic surface with the limits of its catchment basins. b) The new catchment basins after the first waterfall flooding flooded areas are in blue. c) The second waterfall flooding, the floods larger areas and the associated watershed partition now has only two regions*

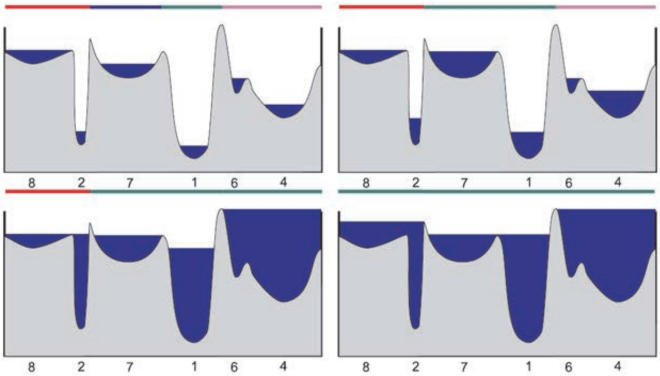

Figure 1.21. *Example of a height synchronous flooding. Four levels of flooding are illustrated; each of them is topped by a figuration of the corresponding catchment basins*

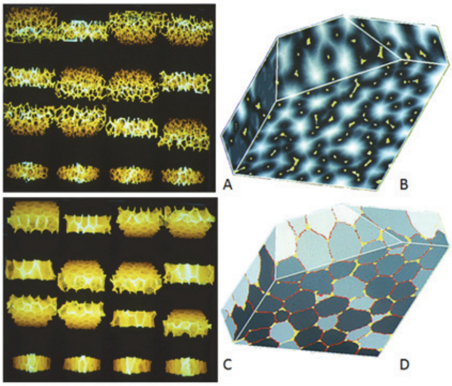

Figure 1.24. *Segmentation of the 3D structure of a polyurethane foam: a) The initial binary image representing a 3D tomography of the foam. b) The distance function to the foam skeleton of A. c) The limits of the watershed partition associated with the inversed distance function. d) The resulting segmentation in which each tile of the watershed partition has been assigned a distinct gray tone, the boundaries between them being highlighted in red*

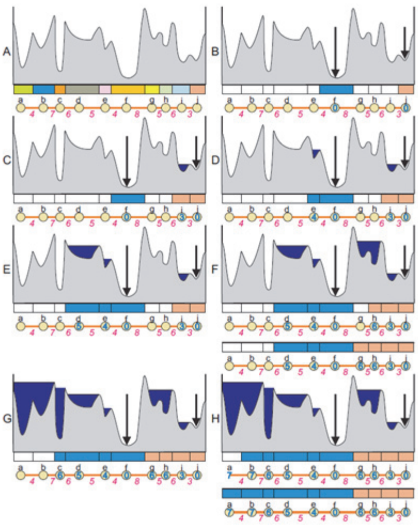

Figure 1.25. *Flooding the region neighborhood graph of a one-dimensional topographic relief (see the text for the description of each step)*

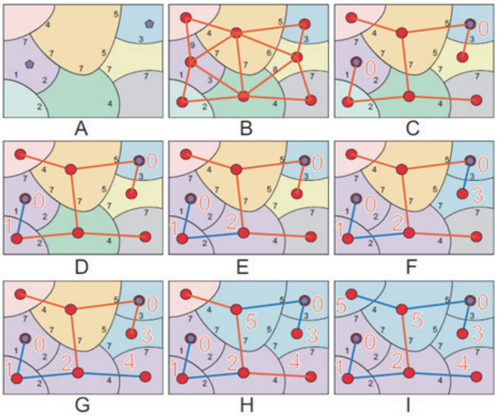

Figure 1.26. *Flooding a region neighborhood graph of a two-dimensional partition (see the text for the description of each step)*

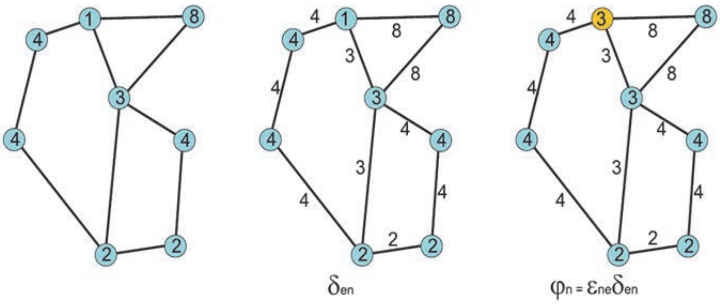

Figure 4.1. *From left to right, a node-weighted graph $G(\nu, nil)$, the edge-weighted graph $G(nil, \delta_{en}\nu)$ and the node-weighted graph $G(□_n \nu, nil)$. The orange node on the right has its weight changed between $G(\nu, nil)$ and $G(□_n \nu, nil)$*

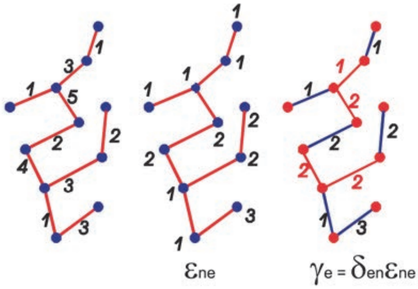

Figure 4.2. *From right to left, an edge-weighted graph $G(nil, \eta)$, the node-weighted graph $G(\varepsilon_{ne}\eta, nil)$ and on the right the graph $G(nil, \gamma_e \eta)$. The red edges in the graph $G(nil, \gamma_e \eta)$ have their weights lowered between the graphs $G(nil, \eta)$ and the graph $G(nil, \gamma_e \eta)$. These edges are not the lowest edges of one of their extremities*

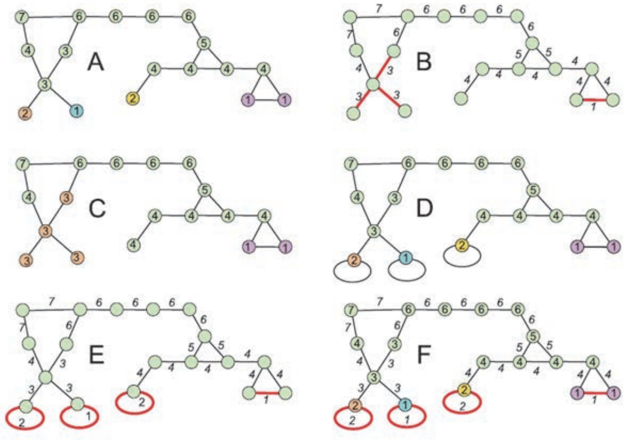

Figure 5.1.

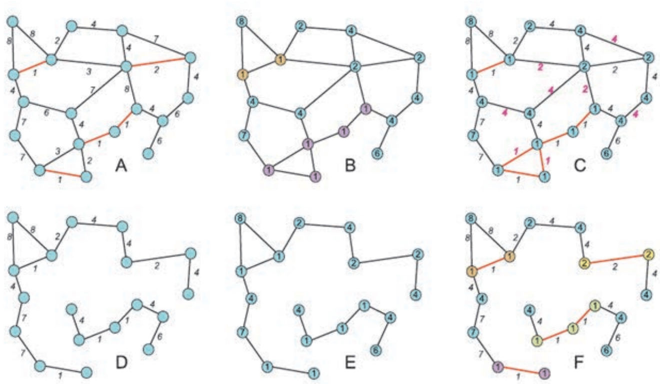

Figure 5.2.

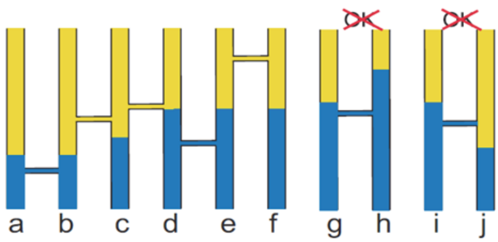

Figure 5.3.

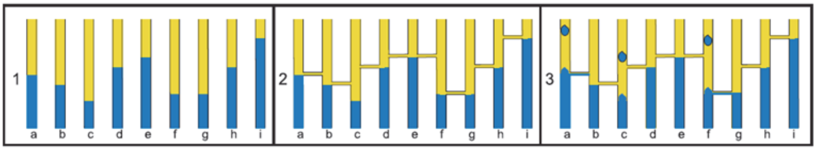

Figure 5.4. *The graph $G(\nu, \delta_{en}\nu)$ is edge unstable but not necessarily node unstable: 1) Each tank represents a node of the graph $G(\nu, nil)$, filled to the level ν. 2) Pipes link the tanks corresponding to neighboring nodes in the graph $G(\nu, \delta_{en}\nu)$; the pipe e_{pq} is placed at the level $(\delta_{en}\nu)_{pq}$. The resulting network is edge unstable, as lowering the edge level of a pipe produces an overflow through this pipe. 3) The tank network representing $G(\nu, \delta_{en}\nu)$ is not node unstable, as it is possible to add a drop of water into the tank c, representing an isolated regional minimum, without provoking an overflow through an adjacent pipe*

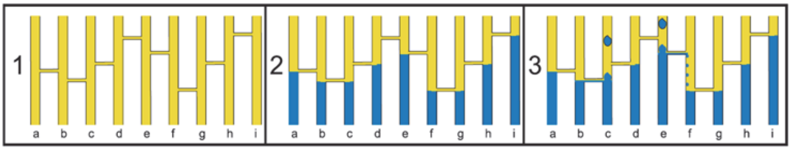

Figure 5.5. *The graph $G(\varepsilon_{ne}\eta, \eta)$ is node unstable but not necessarily edge unstable: 1) A network of tanks and pipes representing the graph $G(nil, \eta)$; the pipe linking neighboring nodes p and q is placed at the level η_{pq}. 2) The tanks are filled up to their lowest adjacent pipe. Thus, adding a drop of water into a tank provokes an overflow through this lowest adjacent pipe. The tank network is node unstable. 3) However, the tank network representing $G(\varepsilon_{ne}\eta, \eta)$ is not edge unstable, as there are pipes which are not traversed by an overflow if we add a drop of water in one of the adjacent tanks. This is the case for the pipe e_{de}*

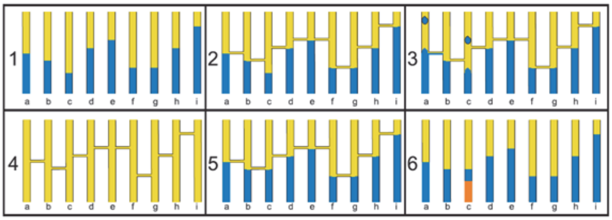

Figure 5.6. *We represent a node-weighted graph $G(\nu, nil)$ by the tanks in image 1, connected by pipes at level $\delta_{en}\nu$ in image 2. The resulting graph $G(\nu, \delta_{en}\nu)$ is not node unstable as shown in image 3: a drop of water is added into the tank c, representing an isolated regional minimum, without provoking an overflow. Image 4 represents the graph $G(nil, \delta_{en}\nu)$; each tank is filled up to the level $\varepsilon_{ne}\delta_{en}\nu = \square_n\nu$ in image 5. Comparing the fillings of the tanks in images 1 and 5 shows that the level of the isolated regional minimum c has been filled up to its lowest adjacent node. Image 6 shows the initial level of water in pipe c in red and the final level in blue*

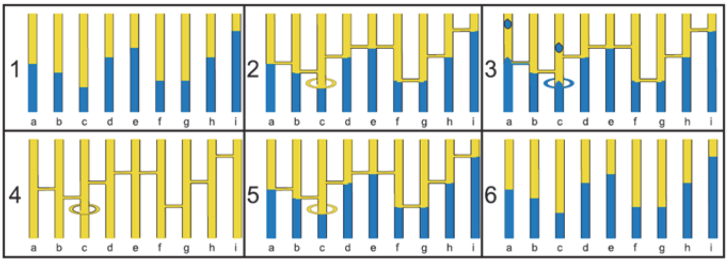

Figure 5.7.

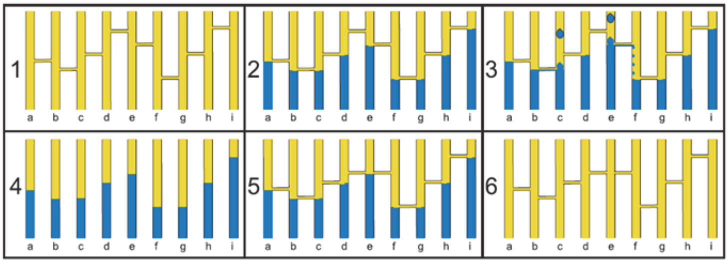

Figure 5.8.

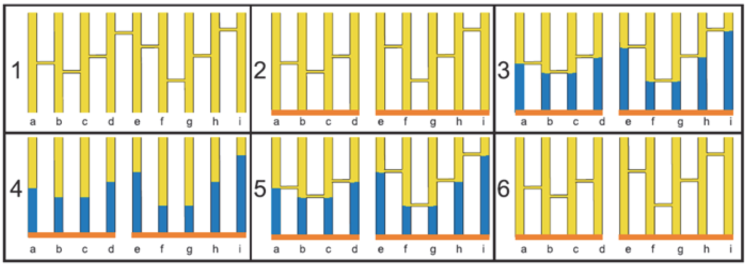

Figure 5.9.

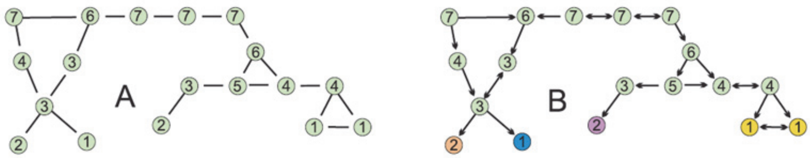

Figure 6.1.

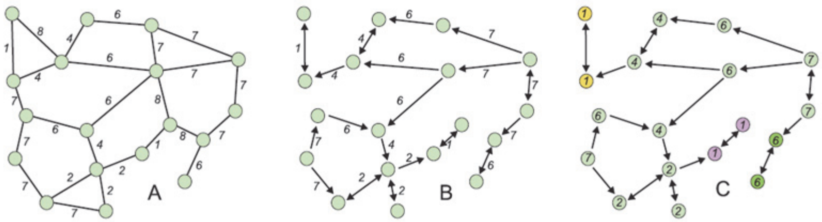

Figure 6.2.

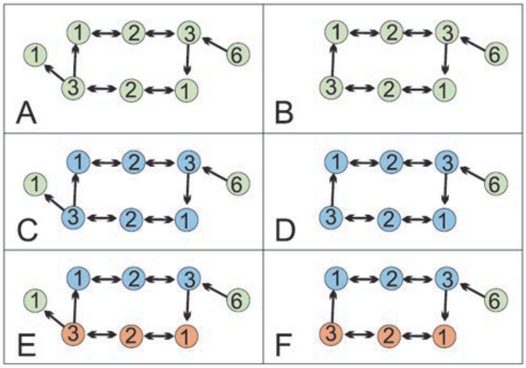

Figure 6.3. *a, b) Two digraphs. c) The blue nodes and each of the green nodes form a smooth zone. The green node with weight 1 forms a dead end. d) The green node forms a smooth zone. The blue nodes form a smooth zone and a dead end. e,f) The blue nodes, the orange nodes and each of the green nodes form a flat zone of the graphs E and F. The node with weight 1 is a black hole of the graph E. The graph F has no black hole*

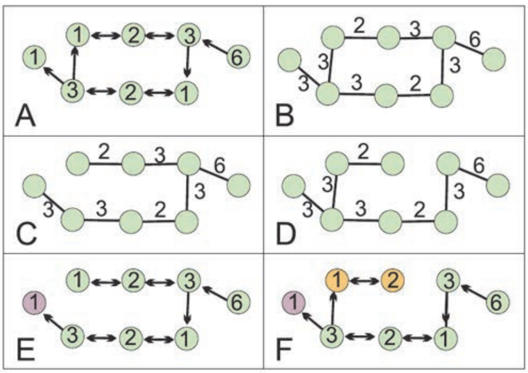

Figure 6.4.

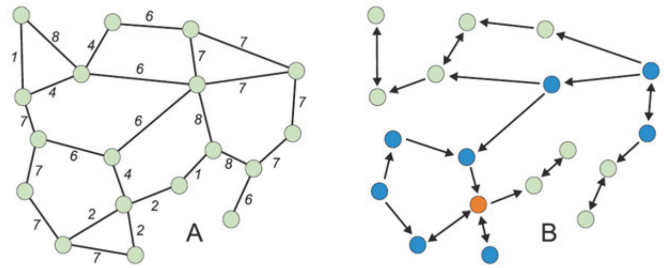

Figure 6.6. *An edge-weighted graph and its associated gravitational graph. A red node is highlighted and its upstream (in blue) is extracted*

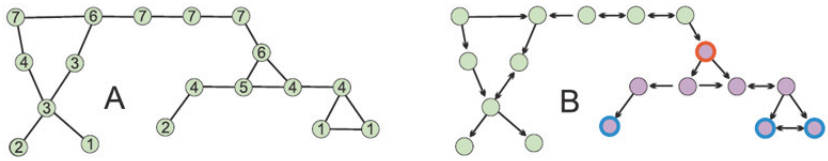

Figure 6.7. *A node-weighted graph and its associated gravitational graph. A node with a red contour is highlighted and its downstream (in violet) is extracted. The downstream contains two distinct black holes, highlighted with a thick blue contour*

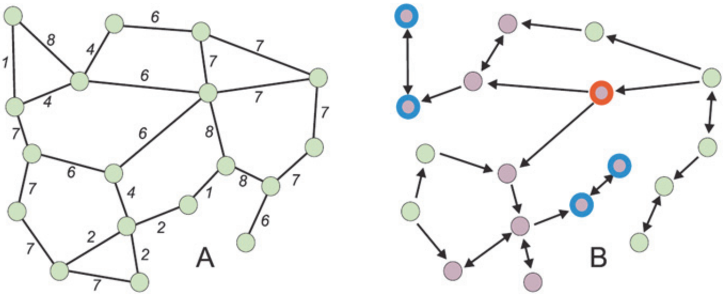

Figure 6.8. *An edge-weighted graph and its associated gravitational graph. A node with a red contour is highlighted and its downstream (in violet) extracted. The downstream contains two distinct black holes, highlighted with a thick blue contour*

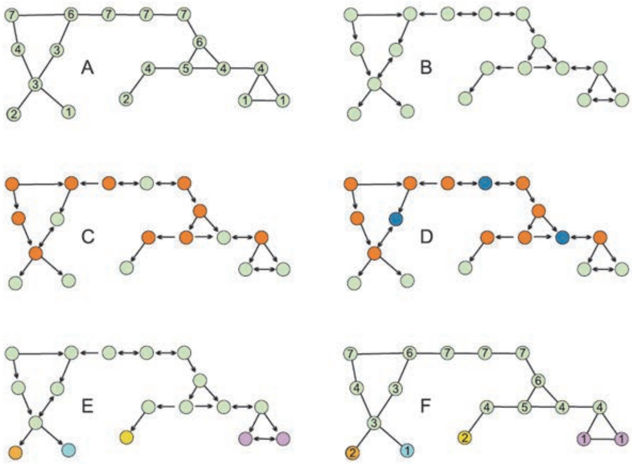

Figure 6.9. *A node-weighted graph is transformed into a gravitational graph on which the black holes are detected and labeled: a) A node-weighted graph. b) The associated weightless gravitational digraph. c) In red, the nodes s with a neighboring node u such that $s \rightarrow u$ and u _ s. d) In blue, the upstream of the red nodes; the blue and red nodes together are the complement of the black holes. e) The black holes, complement of the blue and red nodes; each black hole has been labeled. f) The black holes are the regional minima of the initial node-weighted graphs*

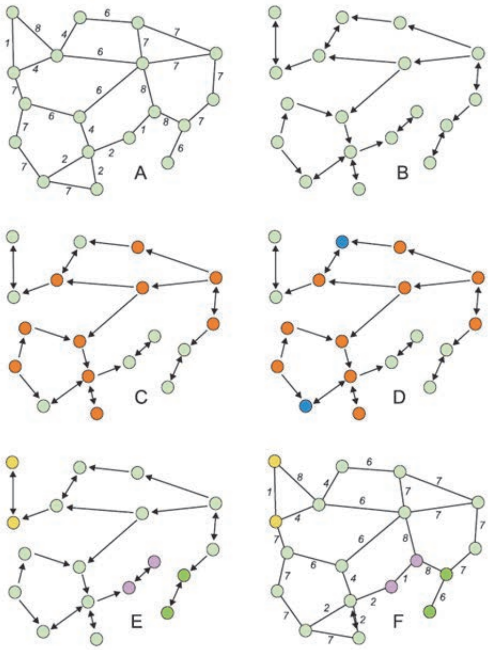

Figure 6.10. *An edge-weighted graph is transformed into a gravitational graph on which the black holes are detected and labeled: a) An edge-weighted graph. b) The associated weightless gravitational digraph. c) In red, the nodes s with a neighboring node u such that $s \rightarrow u$ and u _ s. d) In blue, the upstream of the red nodes; the blue and red nodes together are the complement of the black holes. e) The black holes, complement of the blue and red nodes; in addition, each black hole has been labeled. f) The black holes are the regional minima of the initial edge-weighted graph*

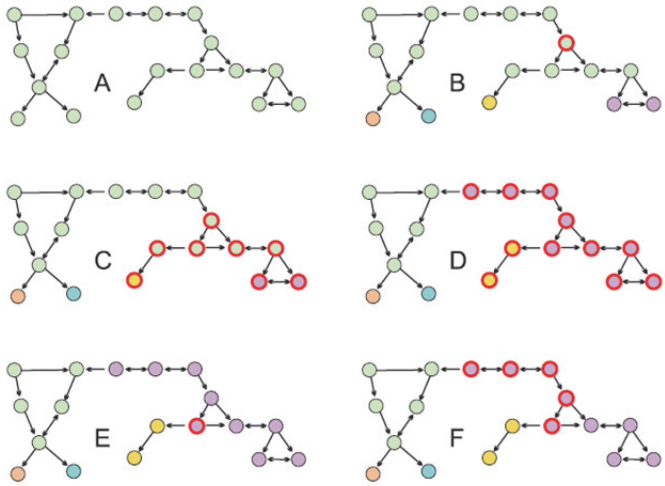

Figure 6.11. *a) A gravitational graph. b) A node has been selected, highlighted by a red contour. c) The downstream of the marked node contains a yellow and a violet black hole. d) The upstream of the downstream of the marked node. e) The highlighted node has a violet label and points towards a yellow label: it belongs to the overlapping of the violet and yellow catchment zones. f) The upstream of this marked node highlights the overlapping of the catchment zones of the yellow and the violet black holes*

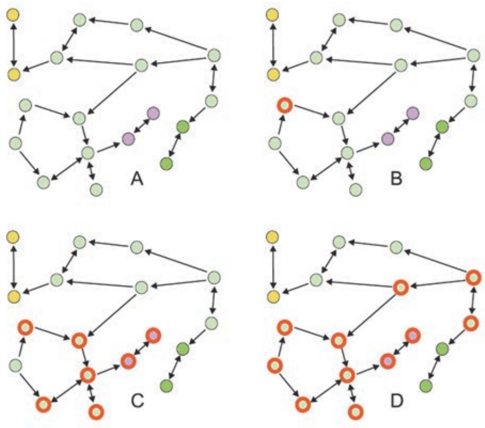

Figure 6.12. *a) A gravitational digraph. b) A node has been selected, highlighted by a red contour. c) The downstream of the marked node. d) The upstream of the downstream of the marked node, containing a violet black hole*

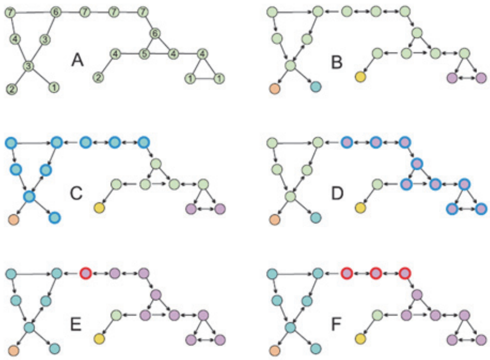

Figure 6.13. *a) A node-weighted graph. b) The associated gravitational digraph with labeled black holes. c) The upstream of the blue black hole has been constructed. d) The upstream of the violet black hole has been constructed. e) The upstream of the blue and the violet black holes has been constructed; the violet label, being higher, occupies the overlapping zone of both catchment zones. The nodes with a violet label pointing towards a node with a blue label are highlighted. Those nodes belong to the blue and violet catchment zones. f) The upstream of these nodes represent the nodes belonging to the violet and the blue catchment zones*

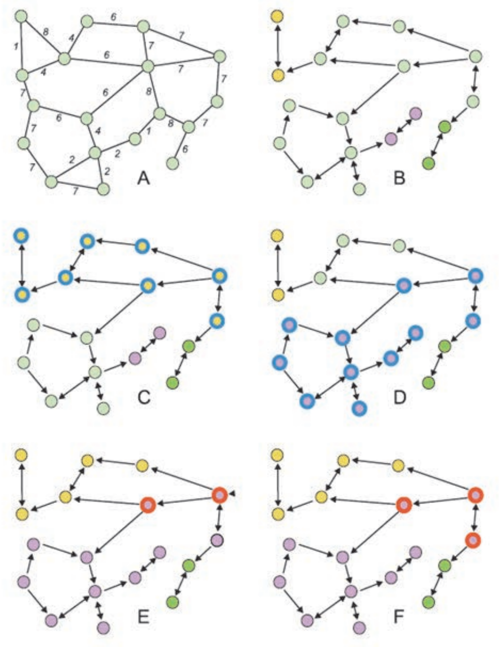

Figure 6.14. *a) An edge-weighted graph. b) The associated gravitational graph with labeled black holes. c) The upstream of the yellow black hole has been constructed. d) The upstream of the violet black hole has been constructed. e) The upstream of the yellow and the violet black holes has been constructed; the violet label, being higher, occupies the overlapping zone of both catchment zones. The nodes with a violet label pointing towards a node with a yellow label are highlighted. Those nodes belong to the yellow and violet catchment zones. f) The upstream of these nodes represent the nodes belonging to the yellow and the blue catchment zones.We check that it is indeed the intersection of the catchment zones detected in C and D*

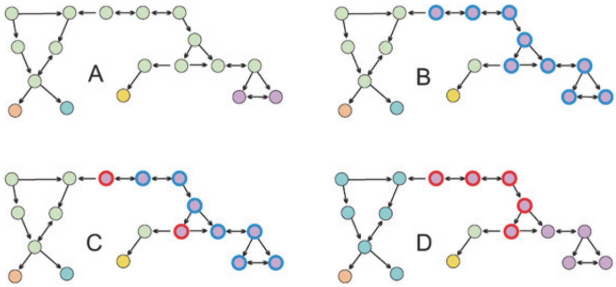

Figure 6.15. *a) A gravitational graph with labeled black holes. b) The catchment zone of the black hole labeled in violet has been constructed. c) The nodes of this catchment zone pointing towards a node with another label or without a label are highlighted. d) The upstream of these nodes represents the overlapping zone of the violet catchment zone*

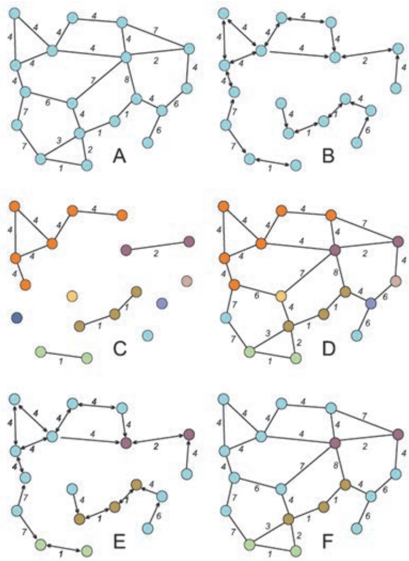

Figure 6.16.

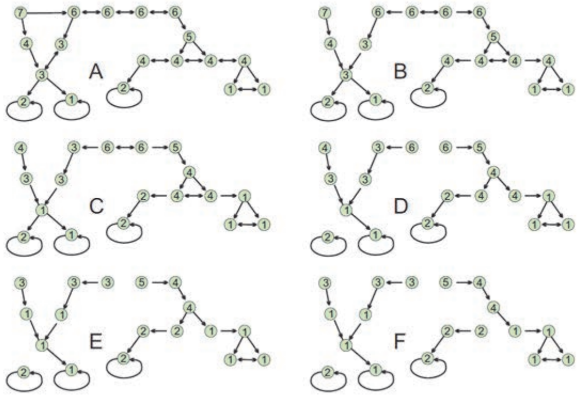

Figure 8.1.

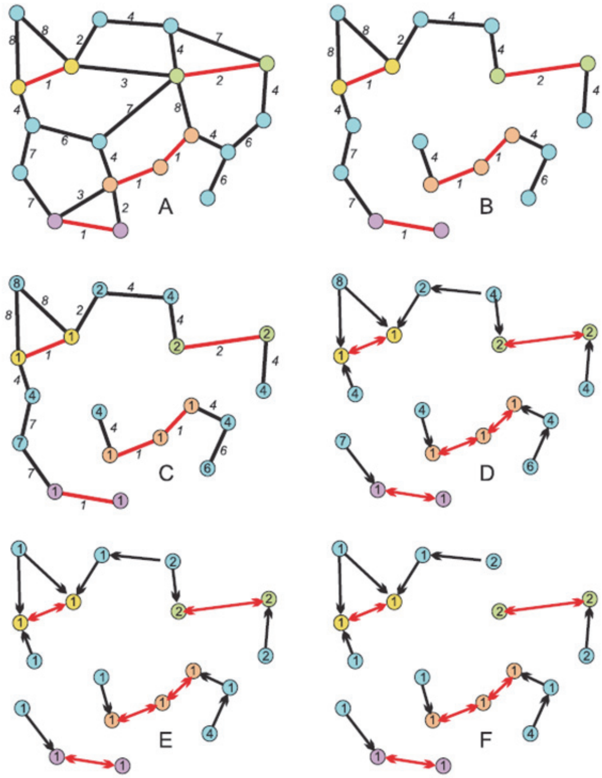

Figure 8.2.

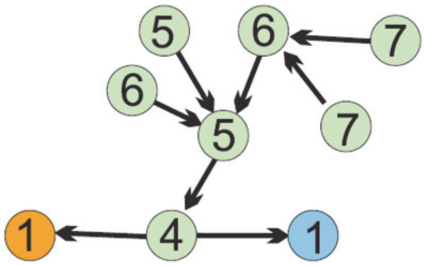

Figure 8.4. *A button hole: it is impossible, without some arbitrary choice, to attribute the green nodes to one of the regional minima with weight 1*

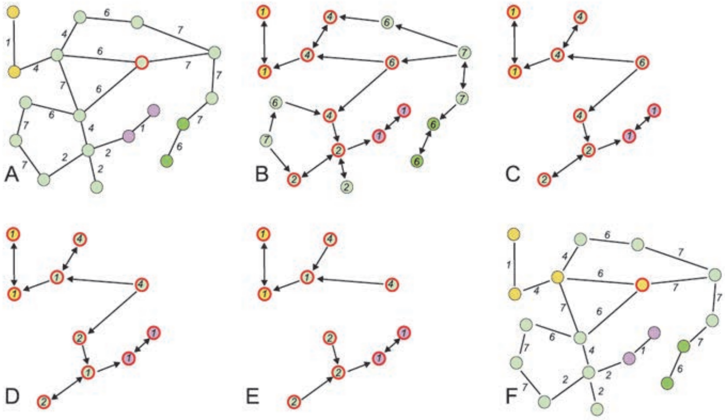

Figure 8.5.

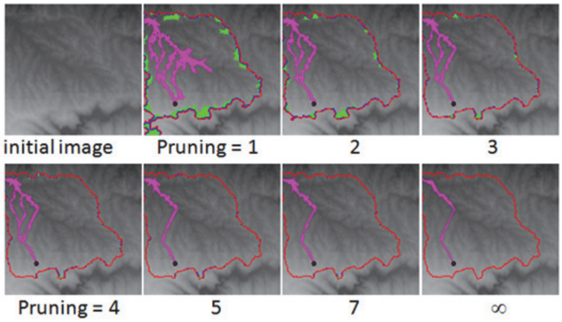

Figure 8.6. *The flowing paths (in violet) and the catchment zone of the dark dot are compared for increasing degrees of pruning* ($1, 2, 3, 4, 5, 7, \infty$)

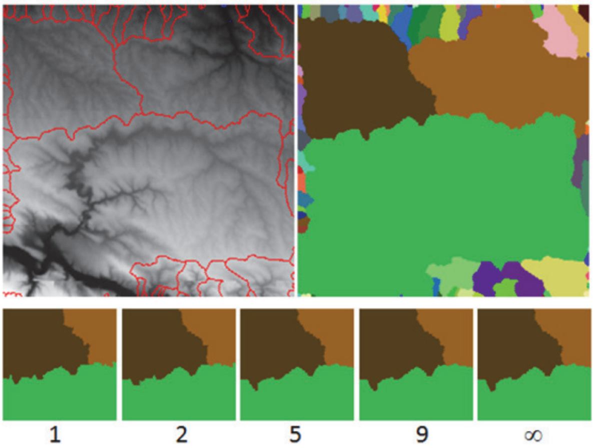

Figure 8.7. *Catchment zones obtained for increasing pruning intensities*

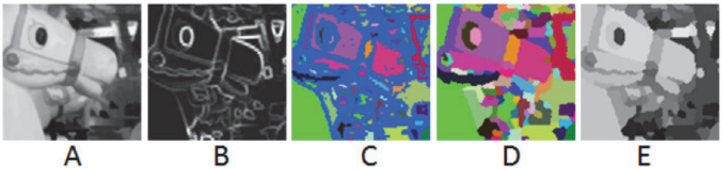

Figure 8.8. *Typical sequence of segmentation. The watershed partition is obtained by the core-expanding algorithm driven by a hierarchical queue*

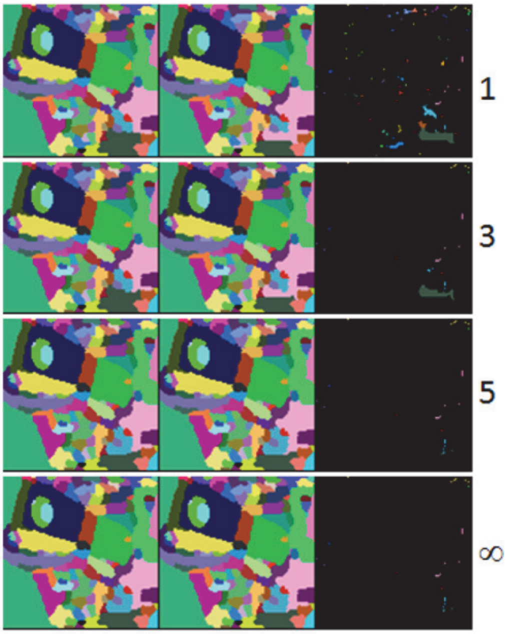

Figure 8.9.

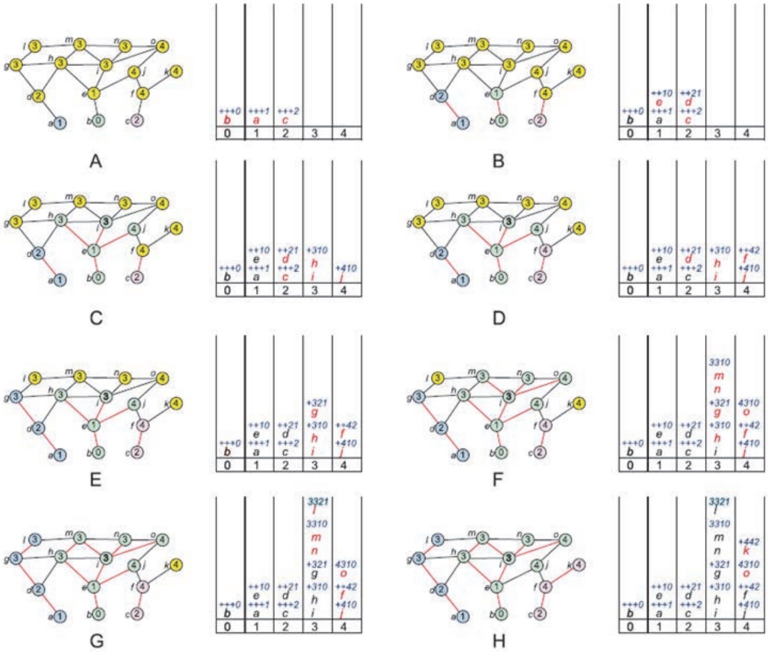

Figure 9.1. *The HQ flooding algorithm constructing all steepest flooding paths*

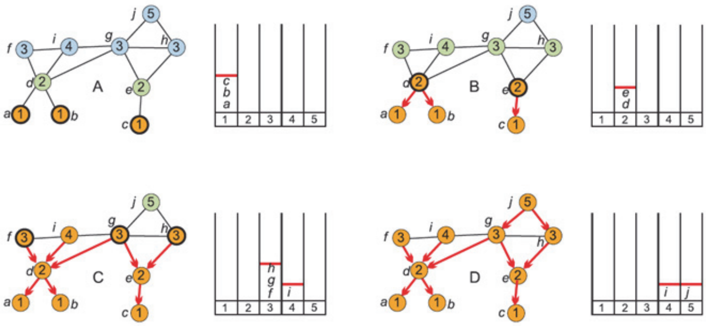

Figure 9.2. *A hierarchical queue algorithm for constructing the steepest paths (bold red edges) in a node-weighted graph*

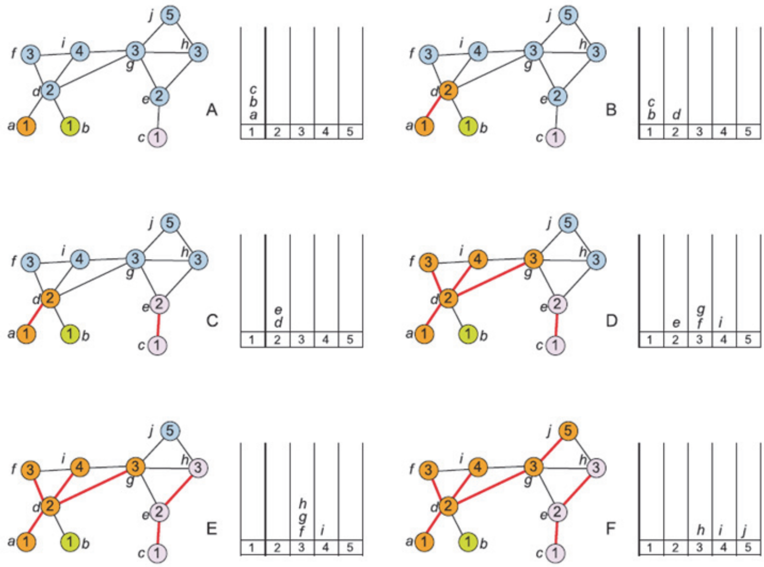

Figure 10.1. *The hierarchical queue algorithm constructing the watershed partition of a node-weighted graph*

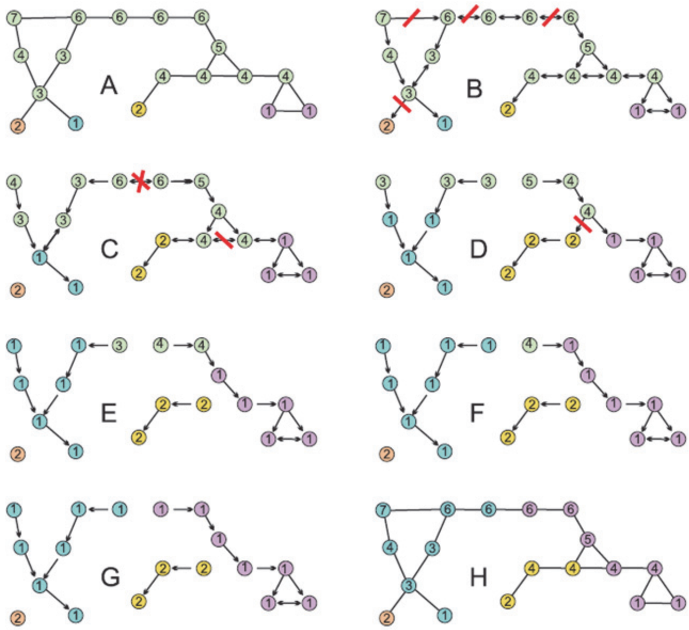

Figure 10.2.

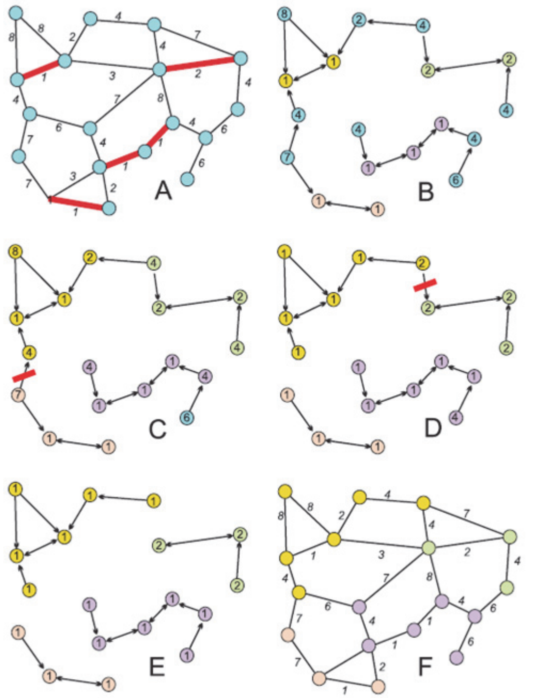

Figure 10.3.

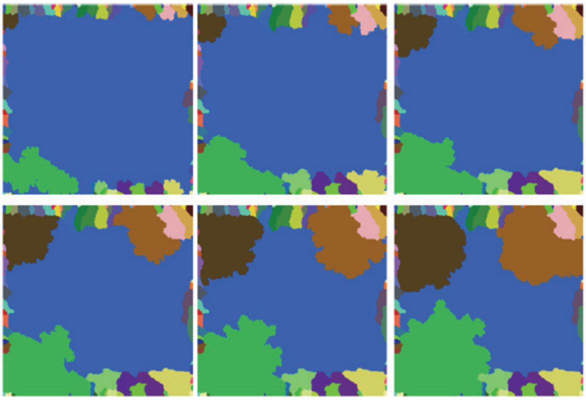

Figure 10.4. *Extension of the catchment basins after 20, 40, 60, 80, 100 and 120 iterations of the combined adaptative erosion of gray tones, pruning and dilation of labels*

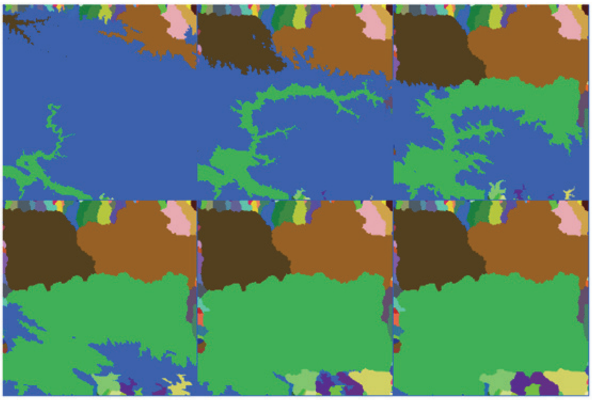

Figure 10.5. *Construction of the catchment basins with an algorithm based on uniform flooding*

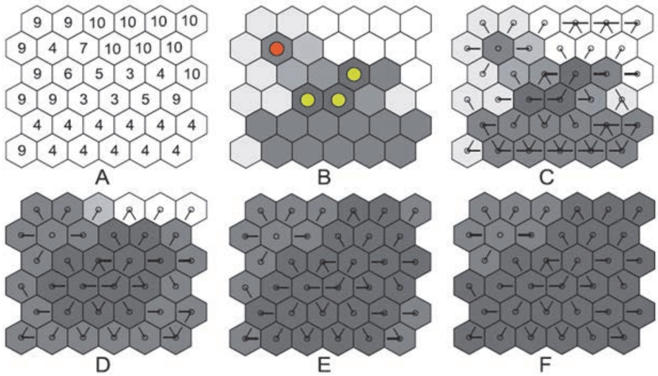

Figure 12.6.

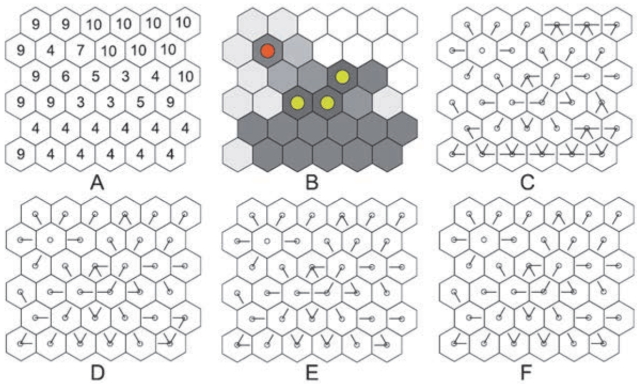

Figure 12.7.

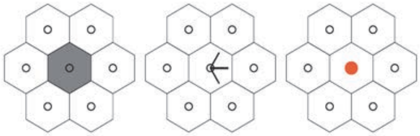

Figure 12.8. *In the case of images, we have three images, the first expressing the gray tones of the pixels, the second encoding the arcs that have their origin in the central point and the last encoding labels*

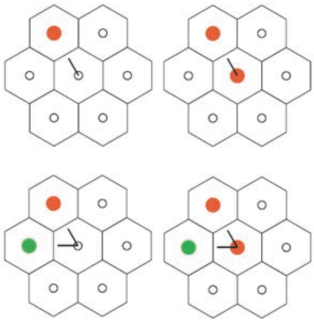

Figure 12.10. The upstream operator

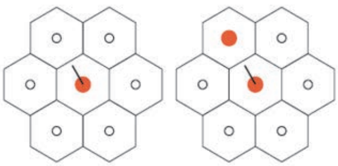

Figure 12.11. *The downstream operator*

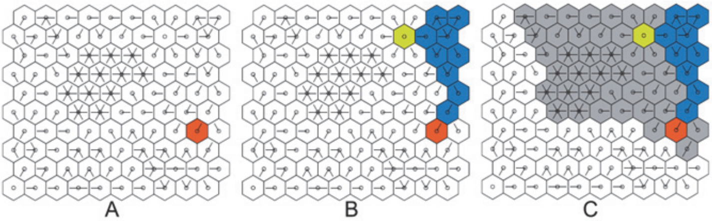

Figure 12.12.

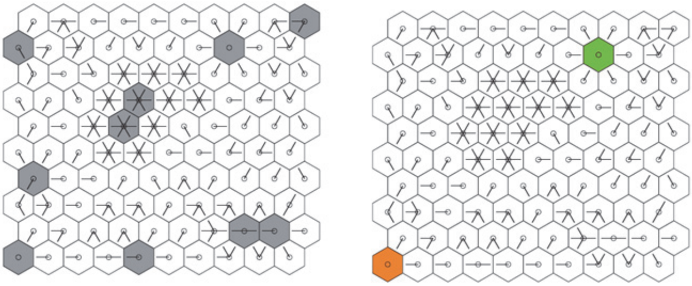

Figure 12.13.

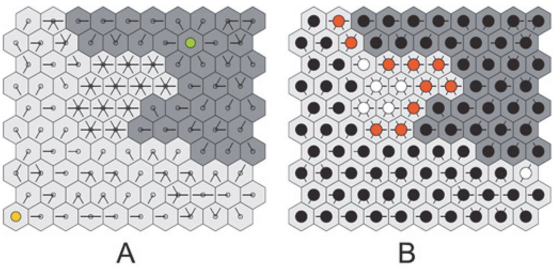

Figure 12.14.

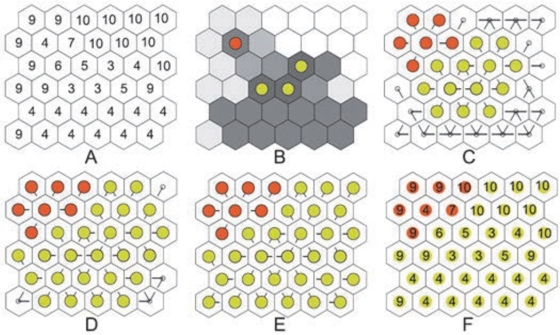

Figure 12.15.

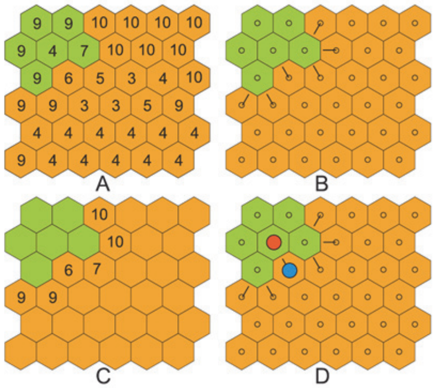

Figure 12.16.

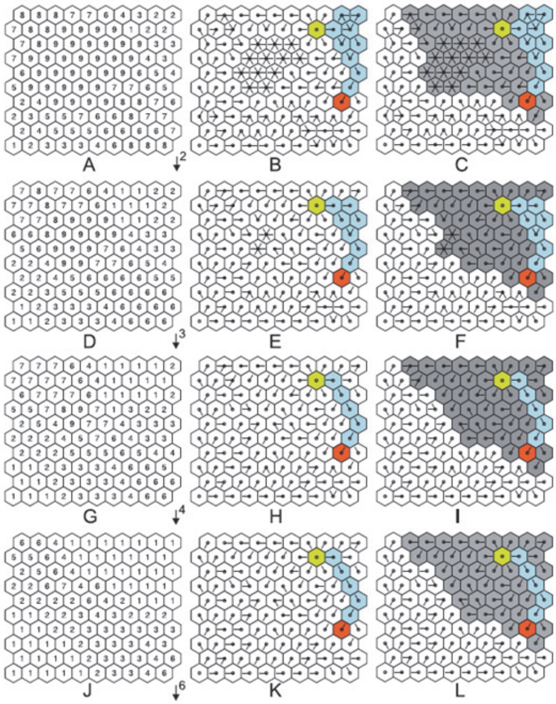

Figure 13.1. *The downstream and upstream of the red node in the same digraph after increasing pruning intensities to equal 2,3,4 and 6. The left image presents the node weigths, the central image the downstream propagation of the red node and the right image the catchment zone of the red node, obtained by upstream propagation of the downstream*

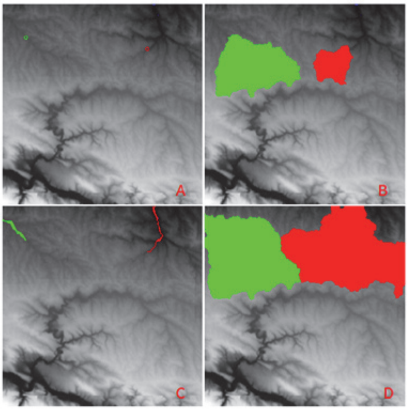

Figure 13.2. *a) A digital elevation model in which a green and a red dot have been hand marked. b) The upstream of the marked nodes, with the same color. c) The downstream of the marked nodes follows a river leading outside of the domain, towards the sea. d) The downstream of the upstream represents the catchment zone of both marked nodes. The catchment zone holding the label with the highest priority covers the catchment zone with a lower priority. Here, the red label has a higher priority than the green label*

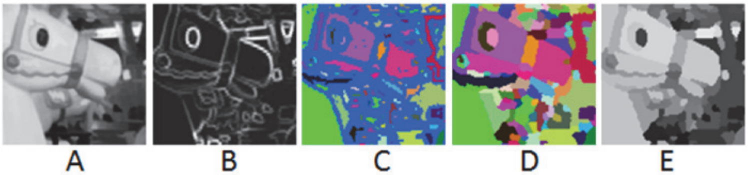

Figure 14.1. *Typical sequence of segmentation. The watershed partition is obtained by the core-expanding algorithm driven by a hierarchical queue*

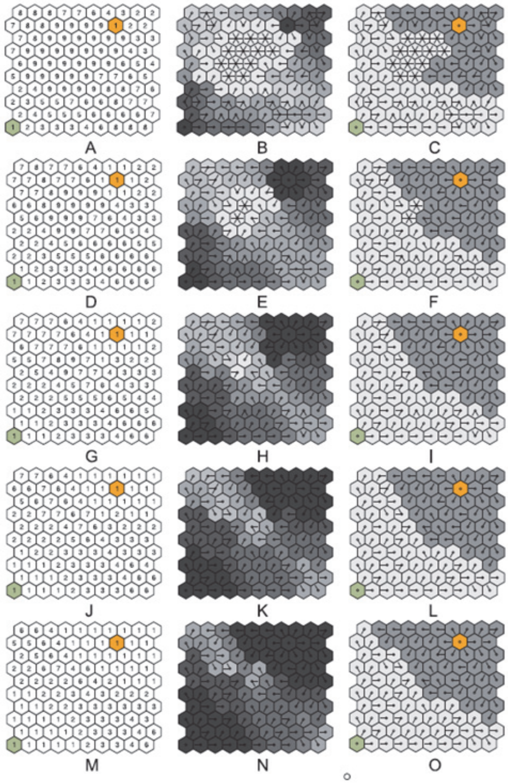

Figure 14.2. *Comparing the dead leaves tessellations for various pruning intensities, from 2 in the first row to 6 in the last row. The last column shows the dead leaves tessellation associated with each degree of pruning*

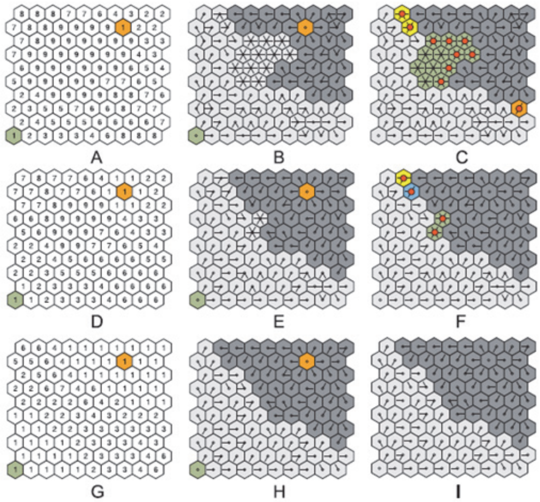

Figure 14.3.

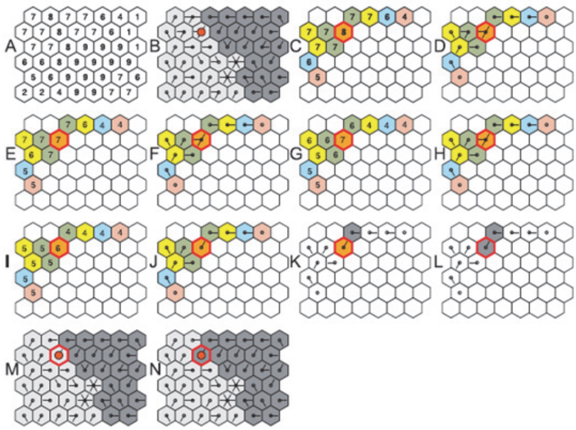

Figure 14.4. *Two-step segmentation: the node with a red dot belongs to an overlapping zone of a digraph. An additional pruning in the downstream of this node is applied to assign it to the correct catchment zone*

Figure 14.5.

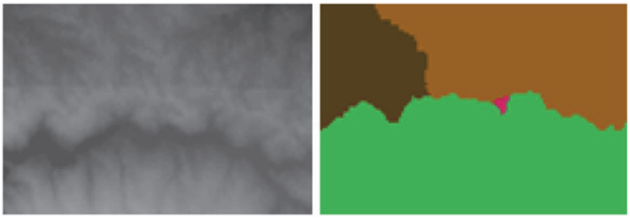

Figure 14.6.

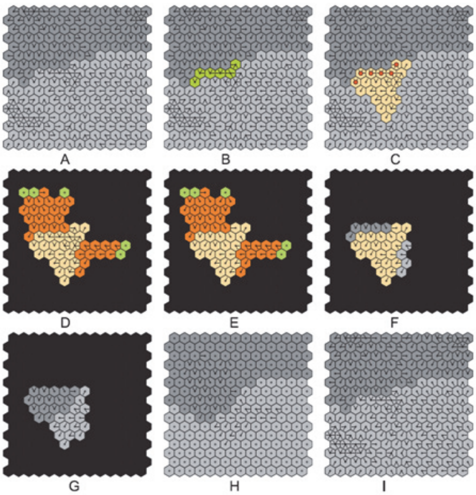

Figure 14.8.

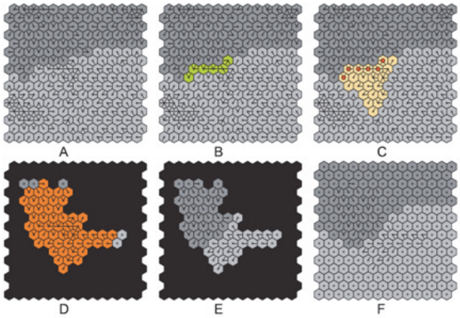

Figure 14.9.

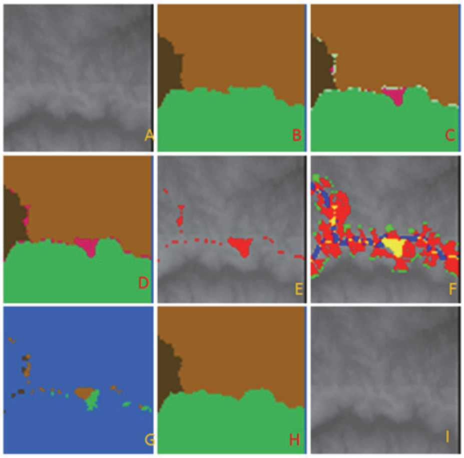

Figure 14.10.

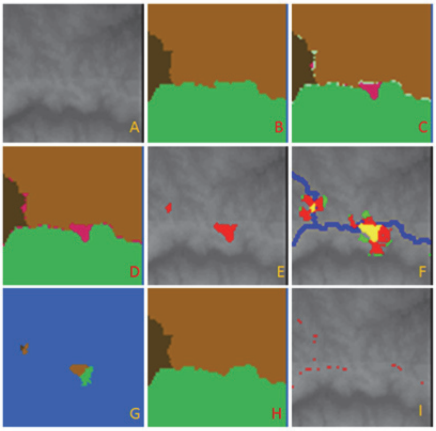

Figure 14.11.

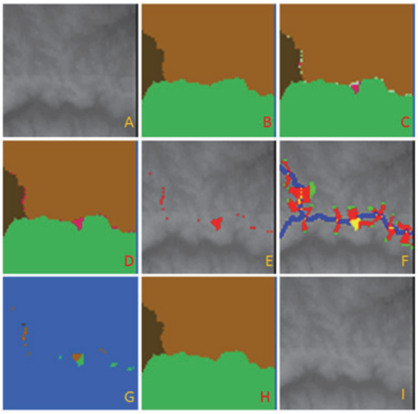

Figure 14.12.

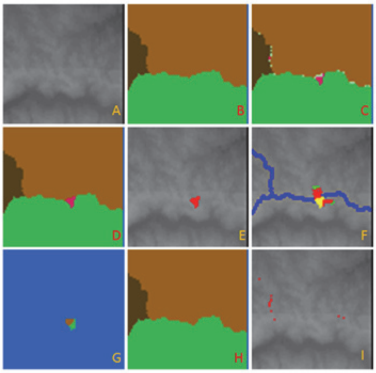

Figure 14.13.

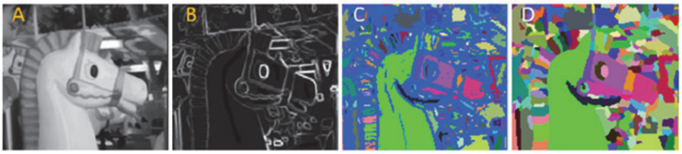

Figure 14.14. *The classical sequence for watershed segmentation. a) Initial image. b) The modulus of the gradient (Beucher gradient: dilation–erosion). c) Detection and labeling of the regional minima. d) Watershed partition obtained by the core-expanding algorithm applied to the gradient image*

Figure 14.15. *Segmenting marked regions. a) Initial image on which markers have been superimposed. b) The partition resulting from the segmentation. c) The markers have been labeled. d) All regions of the partition touched by a marker get the label of this marker*

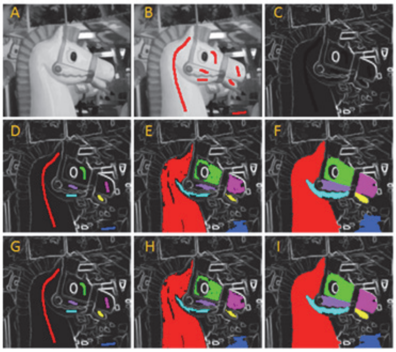

Figure 14.16.

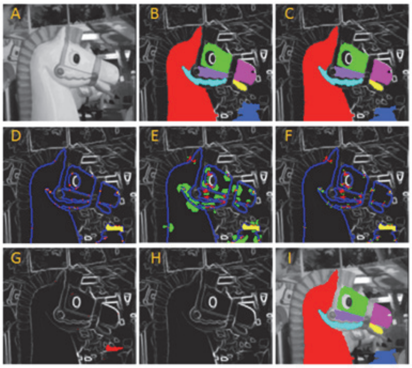

Figure 14.17.

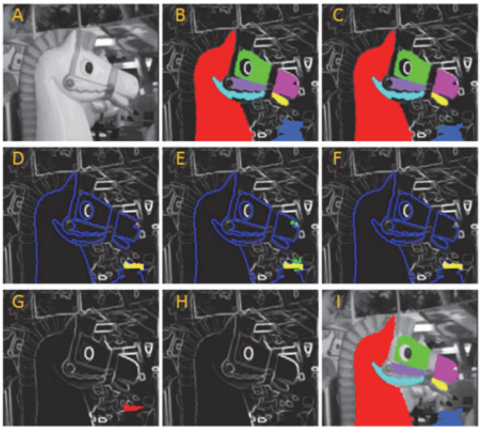

Figure 14.18. *Correction of the overlapping zones between catchment zones on a 2-steep digraph. Only the overlapping zones that are thicker than 2 pixels are retained for correction*

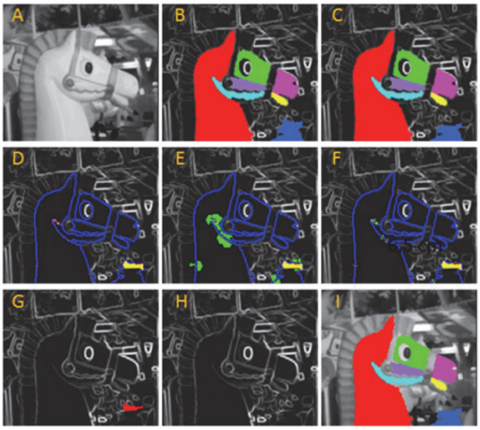

Figure 14.19. *Correction of the overlapping zones between catchment zones on a 3-steep digraph. All overlapping zones are retained for correction*

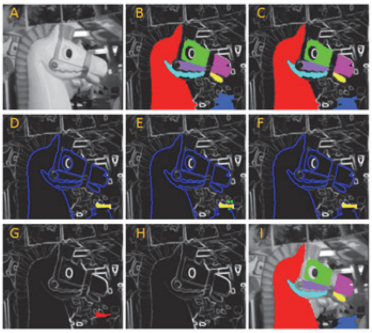

Figure 14.20. *Correction of the overlapping zones between catchment zones on a 3-steep digraph. Only the overlapping zones which are thicker than 2 pixels are retained for correction*

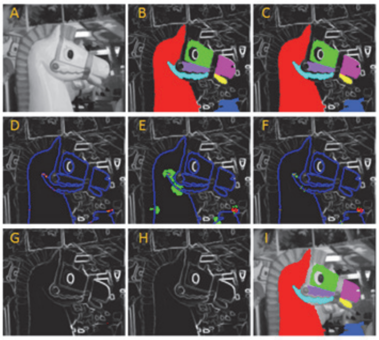

Figure 14.21. *Correction of the overlapping zones between catchment zones on a 6-steep digraph. All overlapping zones are retained for correction*

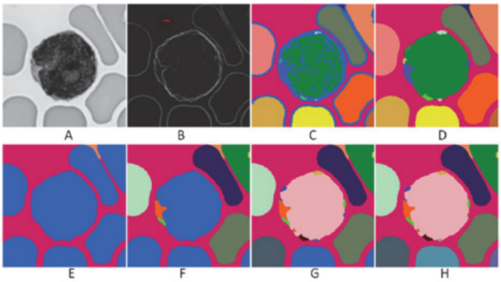

Figure 15.1.

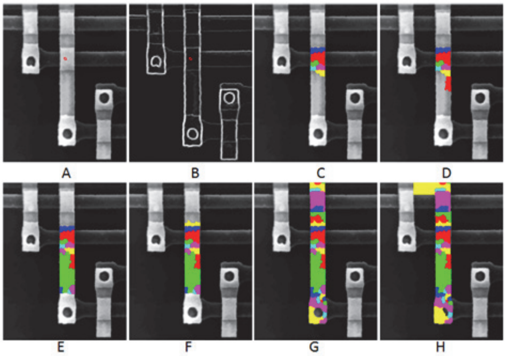

Figure 15.2. *Progressive extraction of the catchement zones starting from a seed point*

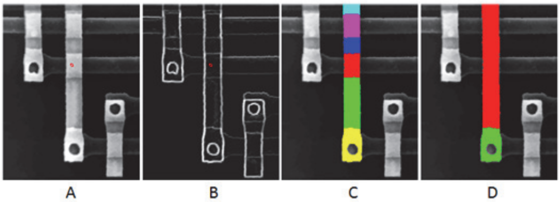

Figure 15.3. *Extraction of the regions corresponding to the highest transitions in the gradient image between regions*

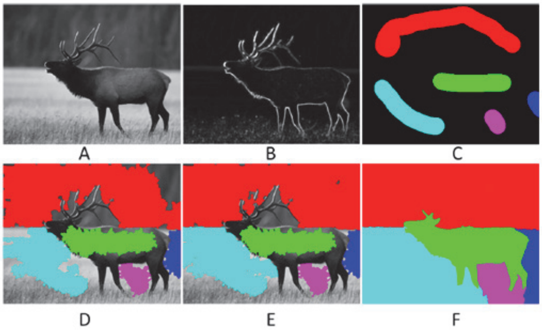

Figure 15.4. *Marker-based segmentation by a stepwise propagation of the markers: a) Image to be segmented. b) Filtered gradient image. c) Hand-drawn markers. d) Initial segmentation by propagating the labels of the markers on the* $5 - steep$ *digraph associated with the gradient image. e) After repeating seven times the step-by-step segmentation. f) After convergence*