Electromagnetism

Links to Special Relativity

Color Section

Christian Gontrand

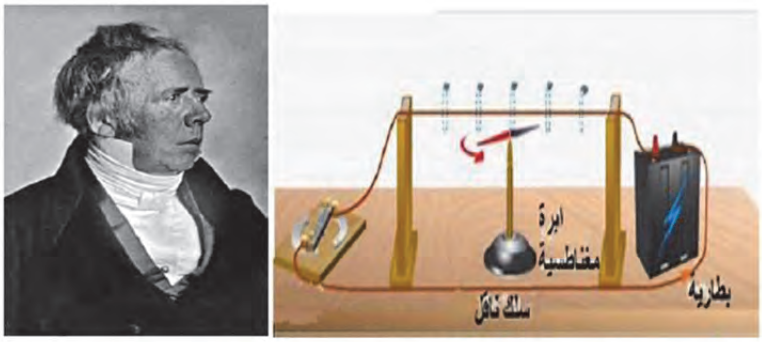


Figure 1.1a. *Ørsted and his historical experiment*

Figure 1.1b. *André-Marie Ampère took note of these results in September 1820 and quickly (within a few weeks) developed (Polymieu, Rhône, 30 km from Lyon) the theory that would allow the emergence of electromagnetism*

Figure 1.1c. *Charles-Augustin Coulomb: 1736–1806*

- INTRODUCTION - CONCEPTS OF MAGNETIC FORCE AND MAGNETIC FIELD

– **A bit of history**

Ancient period: Thales of Miletus (fourth century BC) spoke of the properties of **magnetite** which attracts iron filings

"loving stones" → magnets themselves → magnets

10th century: invention of the compass in China.

➡ The magnetic phenomenon is first manifested by the existence of forces

- magnetized materials that attract or repel
- existence of Earth's magnetic field

Figure 1.2. *Magnetism history synopsis*

19th century:

1820: Øersted's fundamental experiment, which established the *link between magnetism and electricity* and highlighted the action exerted by a wire, through which current flows, on the magnetized needle of a compass directed parallel to this wire. Biot and Savart studied the *forces of interaction* between magnets and currents.

Experiments involving identical actions of a magnet or a circuit through which current flows:

on another magnet,

on a circuit through which a current flows

onto a charged particle beam

Figure 1.3. *Magmetism history synopsis (continued)*

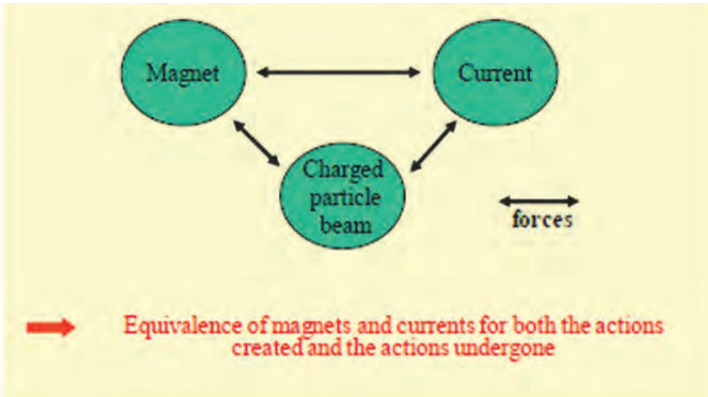


Figure 1.4. *Field sources trio and the equivalence of magnets and currents for both the actions created and the actions undergone*

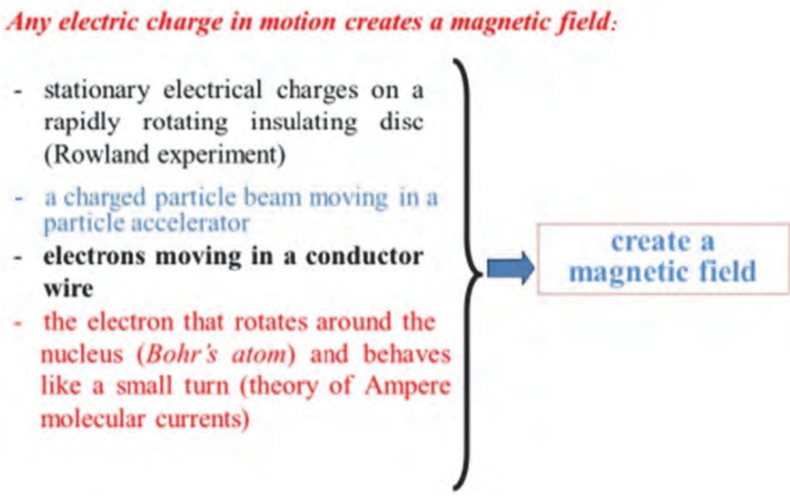


Figure 1.5. *Field sources*

- The presence of a magnet or an electric current modifies the properties of the space around it. In this space, there is a **magnetic field**.
- **Any electric charge in motion** creates a magnetic field.
- Any charge moving in an external magnetic field is subjected to **magnetic forces** perpendicular to its velocity.

Figure 1.6. *Field sources (continued)*

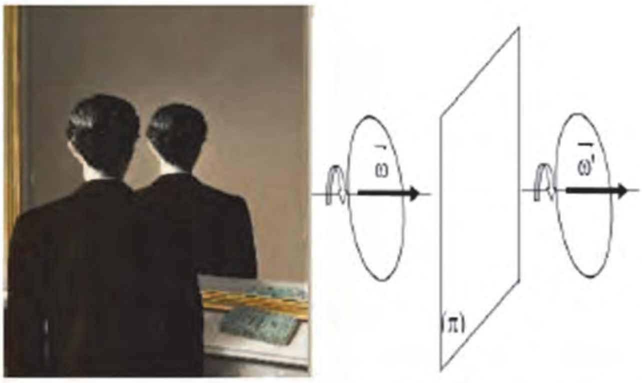


Figure 1.7. *On the left: Magritte's painting. Right: axial vectors, pseudo-vectors or Magritte's vectors*

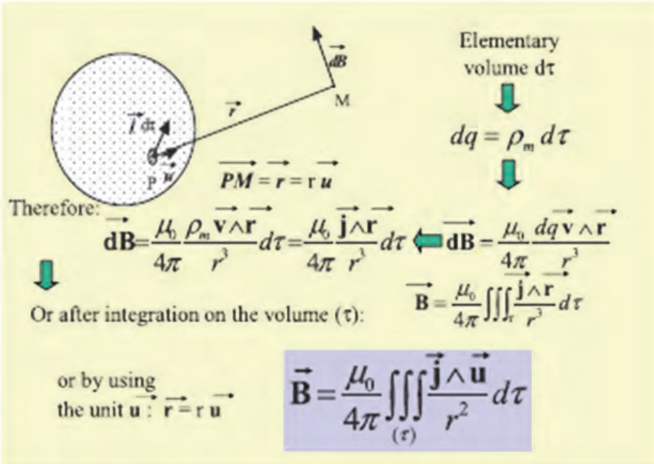


Figure 1.9. *Biot–Savart law for volume currents*

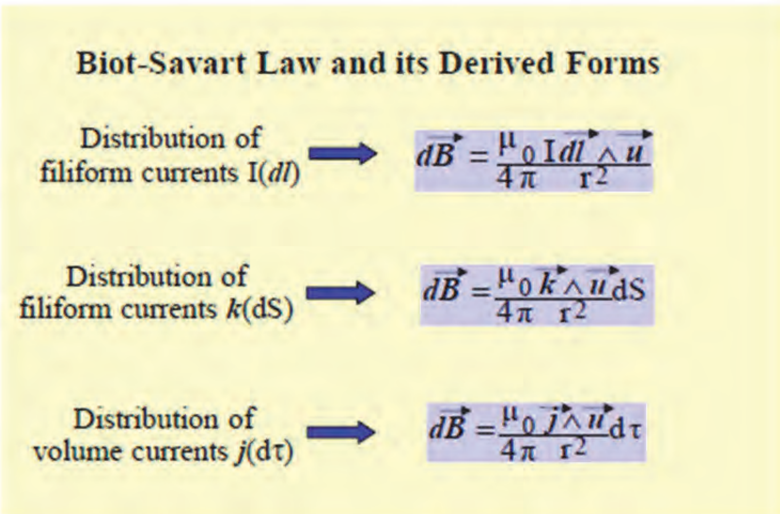


Figure 1.10. *Biot–Savart law*

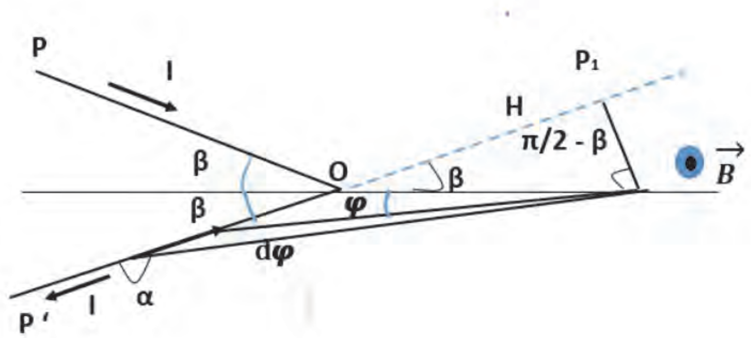


Figure 1.11. *Experiment on the Biot–Savart law*

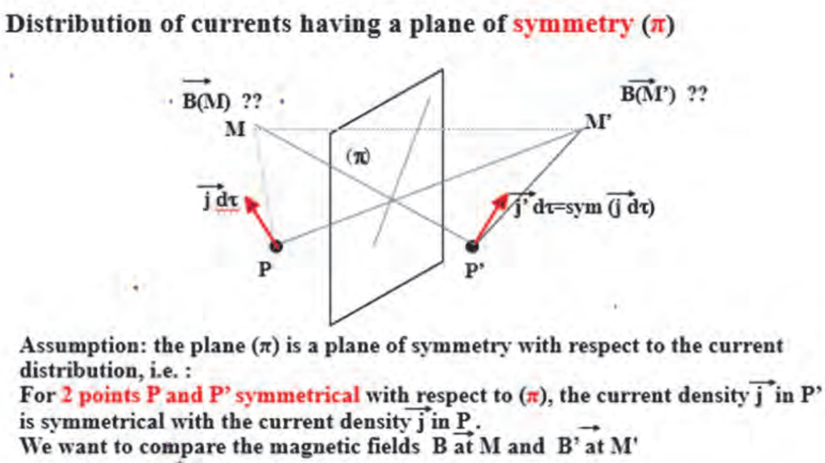


Figure 1.12. *Symmetrical currents*

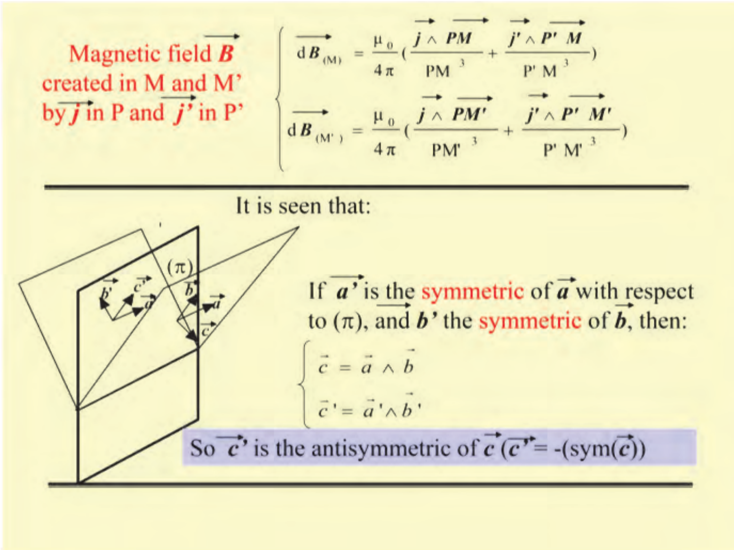


Figure 1.13. *Vector product and symmetry*

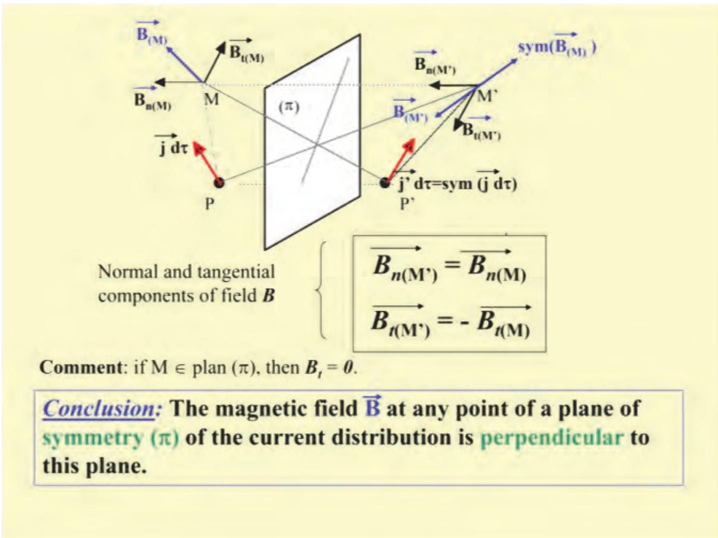


Figure 1.14a. *B perpendicular to the planes of symmetry*

- Distribution of currents with an antisymmetry plane

Assumptions: The plane (p) is an antisymmetry plane with respect to the distribution of currents, *i.e.*:

- For two points P and P' symmetrical with respect to ($\overline{\pi}$),
- The current density $\vec{j}'$ in P' is antisymmetric with the current density $\vec{j}$ in P, *i.e.* $\vec{j}'$= -sym(**j**)

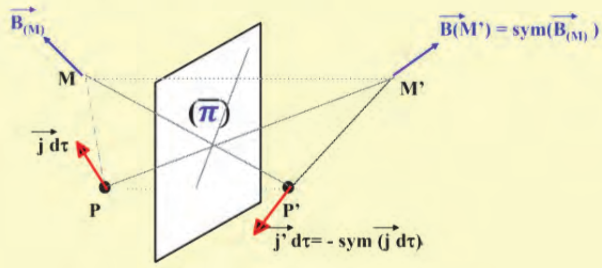


Figure 1.14b. *Symmetrical B for current density anti-symmetry planes*

Magnetic field $\quad \vec{\mathbf{B}}_{(M')} = sym\,(\vec{\mathbf{B}}_{(M)})$

Conclusion: At two points symmetrical with respect to a plane of antisymmetry ($\overline{\pi}$) of the current distribution, the magnetic fields are symmetrical.

Components of the magnetic fields
$$\begin{cases} \vec{B}_{n(M')} = -\,\vec{B}_{n(M)} \\ \vec{B}_{t(M')} = \vec{B}_{t(M)} \end{cases}$$

Note: if M $\in$ plane (π), then $\vec{B}_n = \vec{0}$

Conclusion: The magnetic field $\vec{B}$ at any point of an antisymmetry plane (π) of the current distribution is contained in this plane.

Figure 1.15. *B belongs to the anti-symmetry planes*

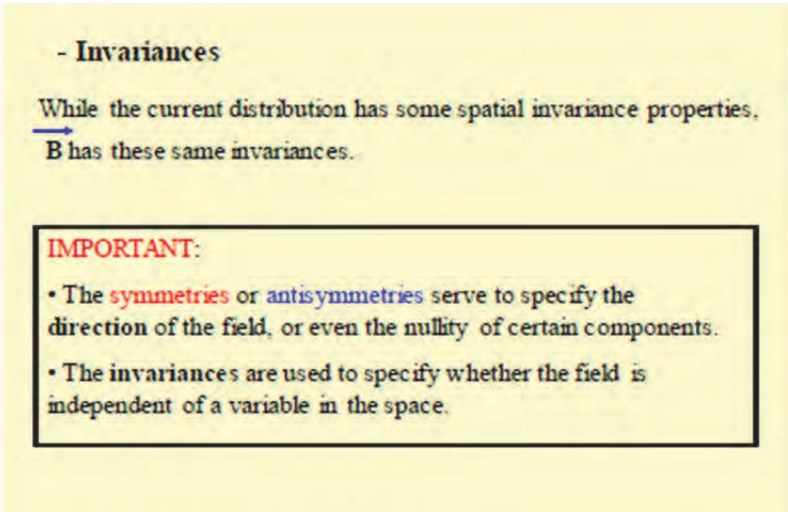


Figure 1.16. *Invariances*

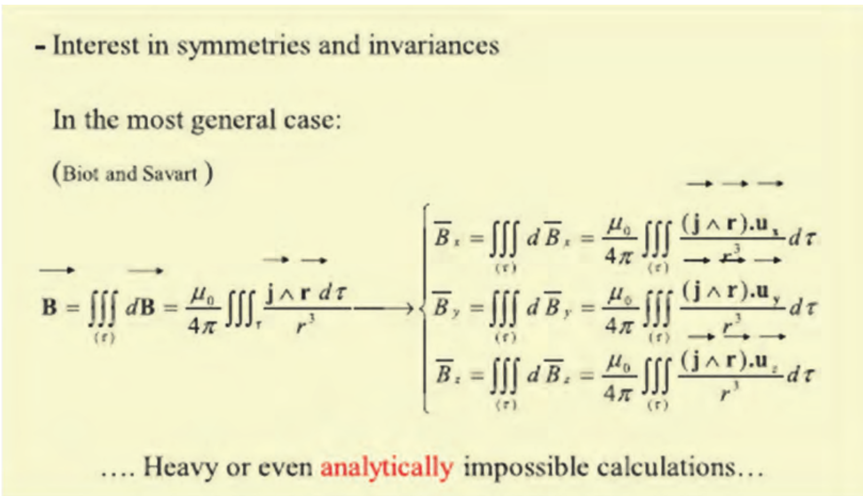


Figure 1.17. *Biot–Savart law: projections*

Prior knowledge of direction $\boldsymbol{i}$ of field $\overrightarrow{B}$ will simplify this tedious calculation. It will be sufficient to project each elementary component on this direction and to sum up these contributions

$$\begin{cases} d\overline{B} = \overrightarrow{\mathbf{dB}}.\overrightarrow{\mathbf{i}} = \dfrac{\mu_0}{4\pi}\dfrac{(\overrightarrow{\mathbf{j}}\wedge\overrightarrow{\mathbf{r}}).\overrightarrow{\mathbf{i}}}{r^3}d\tau \\ \overrightarrow{\mathbf{B}} = \overline{B}\,\overrightarrow{\mathbf{i}} = \dfrac{\mu_0}{4\pi}[\iiint_{(\tau)}\dfrac{(\overrightarrow{\mathbf{j}}\wedge\overrightarrow{\mathbf{r}}).\overrightarrow{\mathbf{i}}}{r^3}]\,\mathbf{i} \end{cases}$$

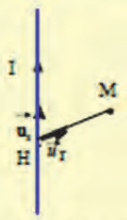
These considerations of symmetry and invariance also allow for a rapid calculation of the field by the use of the Ampère theorem.

Figure 1.18. *Biot–Savart law: projection of B*

– Examples of magnetic field calculation

By integrating the Biot-Savart law

Field created by an "infinite" wire through which an invariable current I passes



Cylindrical coordinates, well adapted to a rectilinear wire geometry, are used.

$(\mathbf{u}_r, \mathbf{u}_\theta, \mathbf{u}_z)$

Problem:

Calculation of $\overrightarrow{\mathrm{B}}$ at M at the distance r from the wire, such that HM… $\overrightarrow{HM} = r\overrightarrow{u_r}$.

The unitary vector carried by the wire having the direction of I is called $\overrightarrow{\mathbf{u}_z}$.

Figure 1.19. *Wire: calculation of the magnetic field*

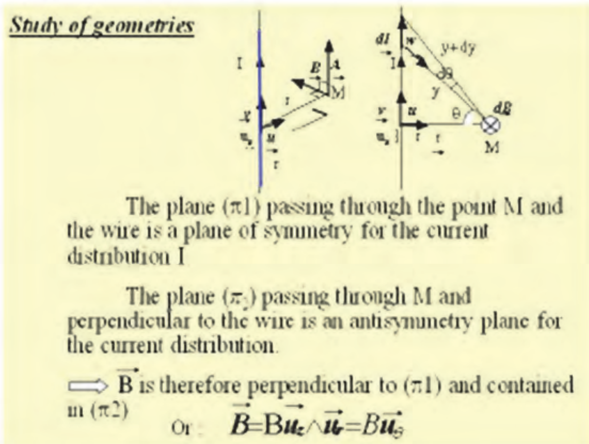


Figure 1.20. *Wire (continued)*

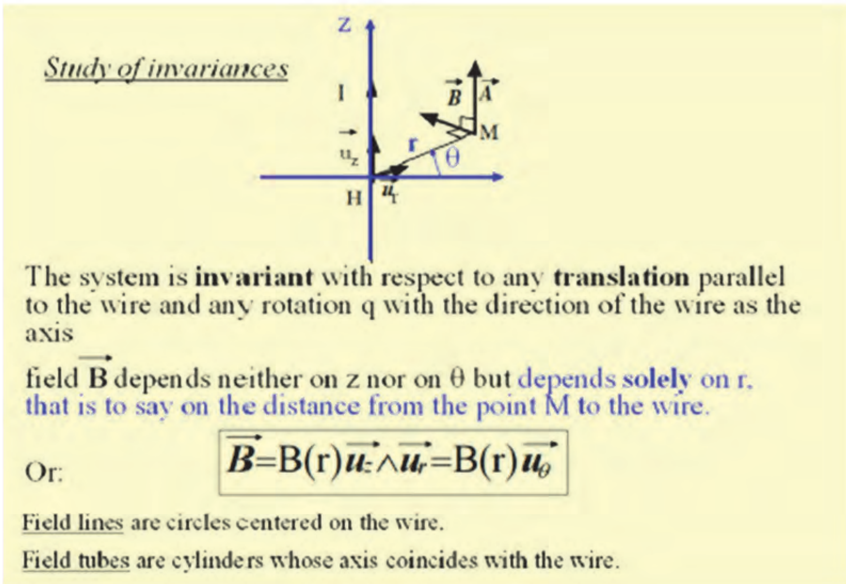


Figure 1.21. *Wire*

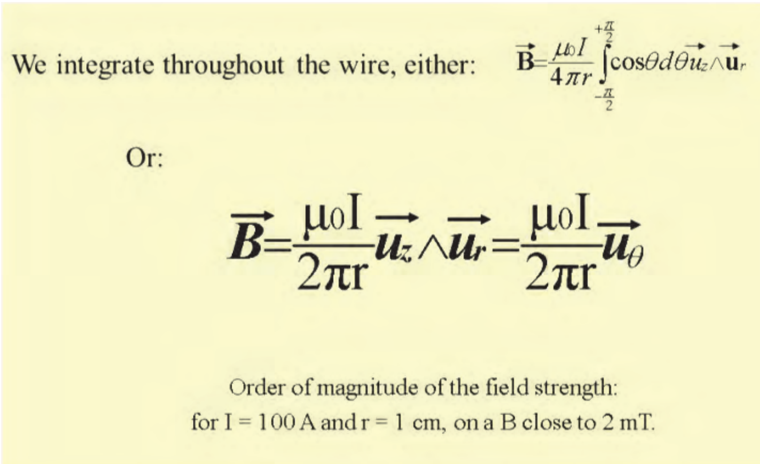


Figure 1.22. *Magnetic field created by a conductor wire*

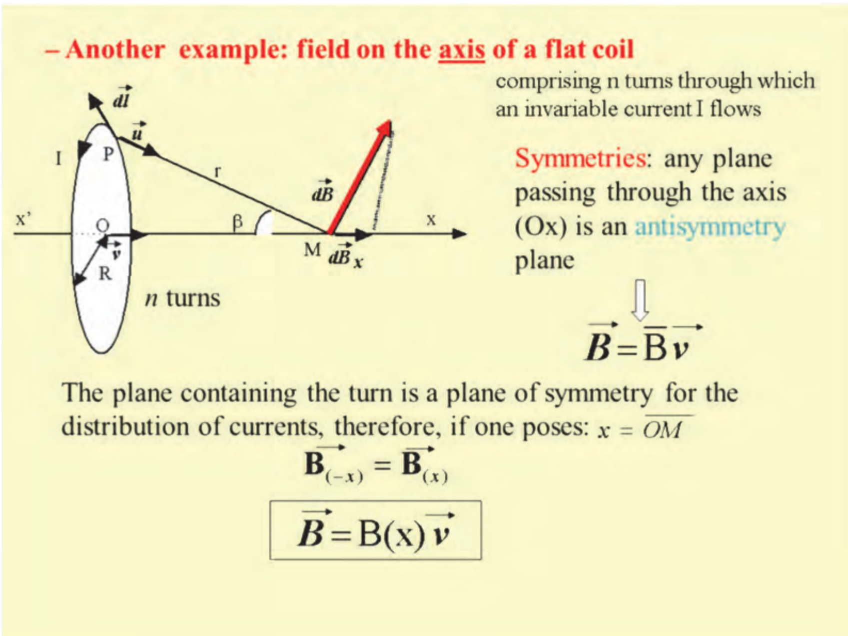


Figure 1.23. *Topography of the field created by a conductive loop*

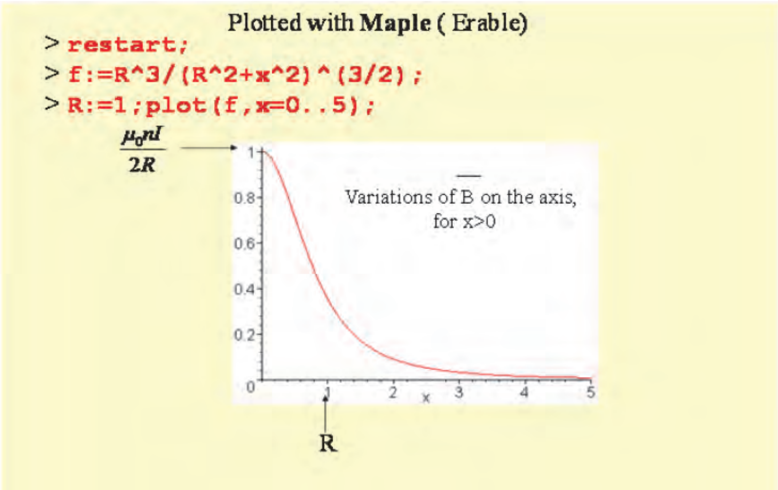


Figure 1.25. *Field B on the axis of a turn through which a current flows*

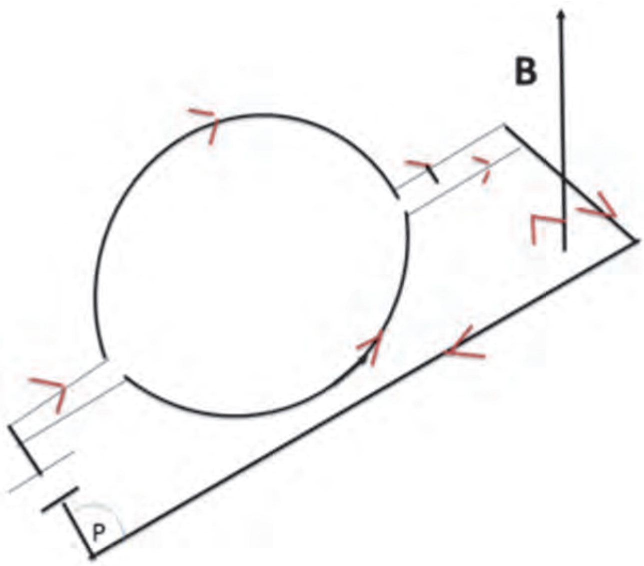


Figure 1.28. *Two half-turns*

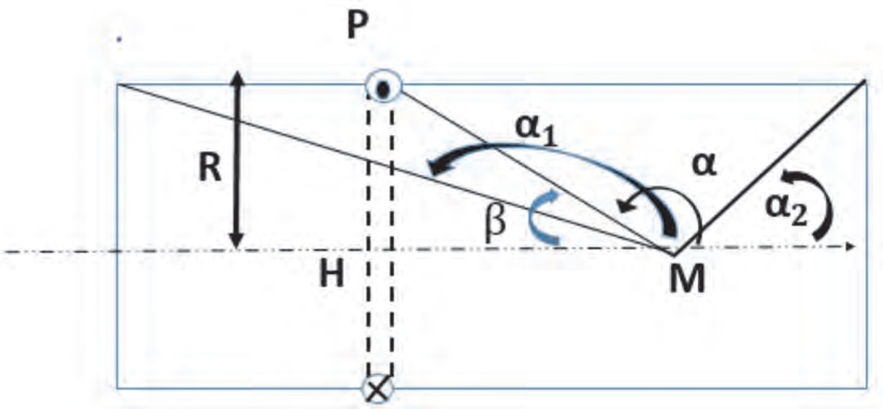


Figure 1.29. *Solenoid*

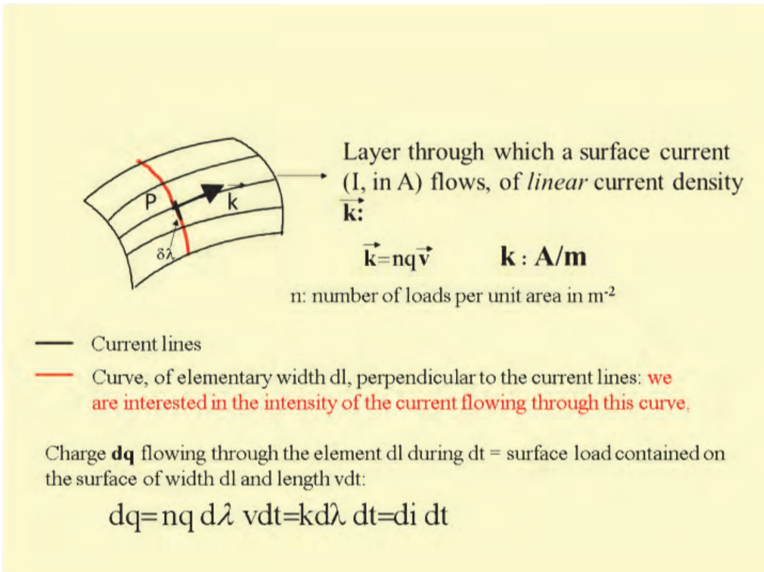


Figure 1.30. *Current layer*

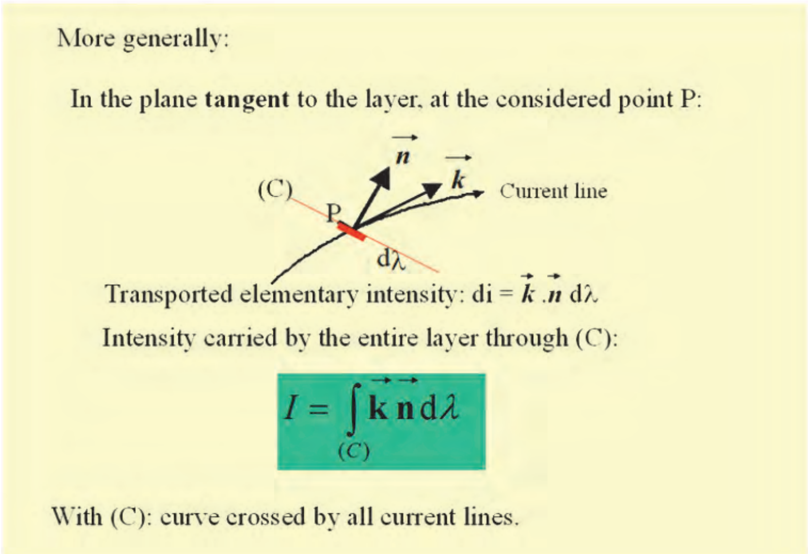


Figure 1.31. *Current layer: I function of k*

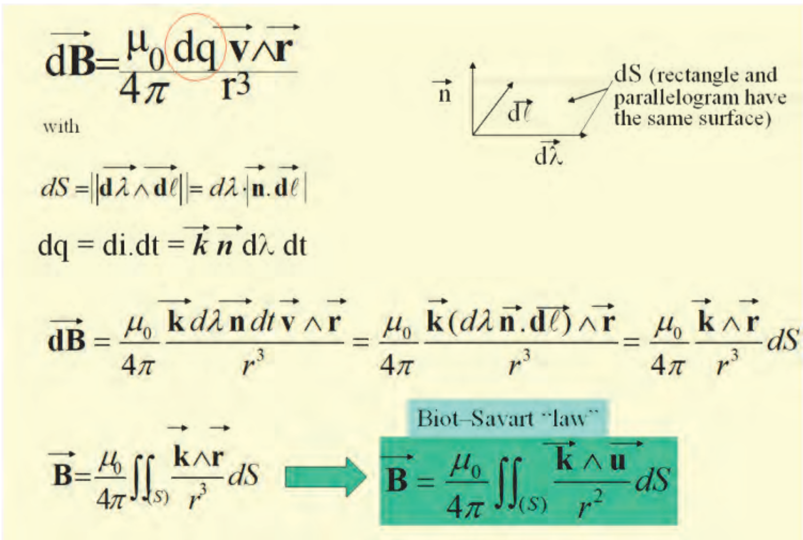


Figure 1.32. *Field for a layer*

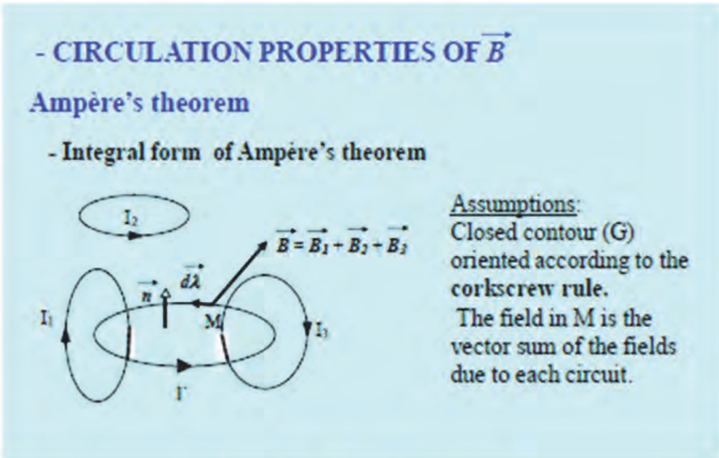


Figure 1.34. *Ampère's path*

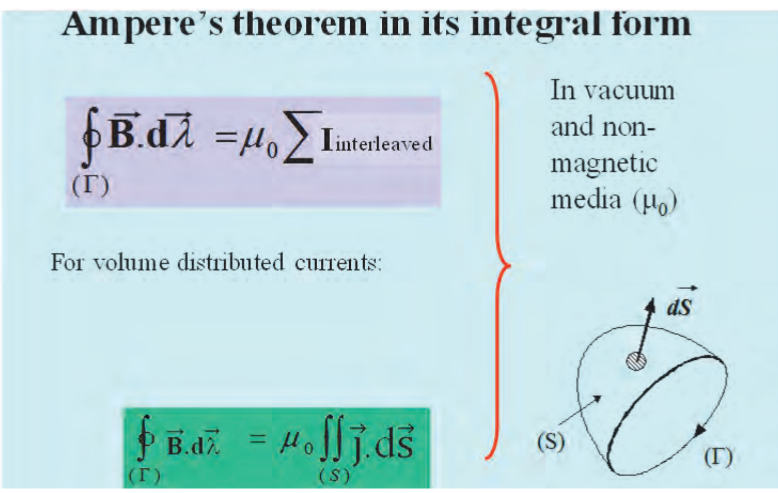


Figure 1.35. *Ampère's theorem: integral form*

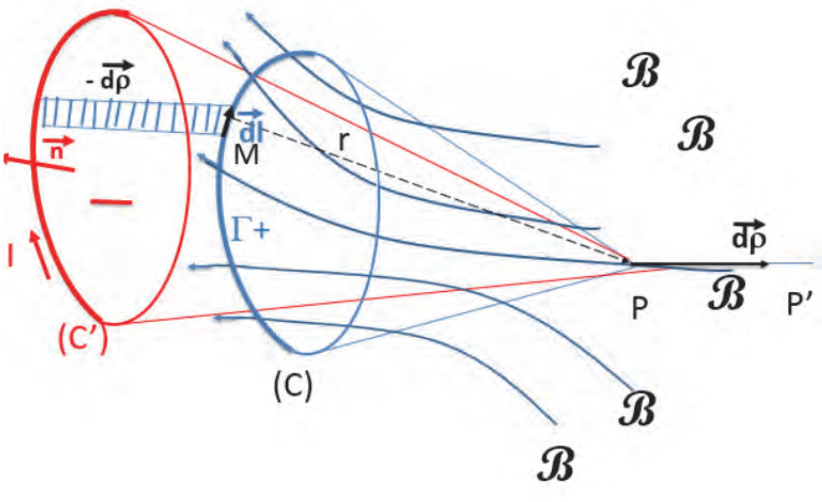


Figure 1.36. *Towards Ampère's theorem*

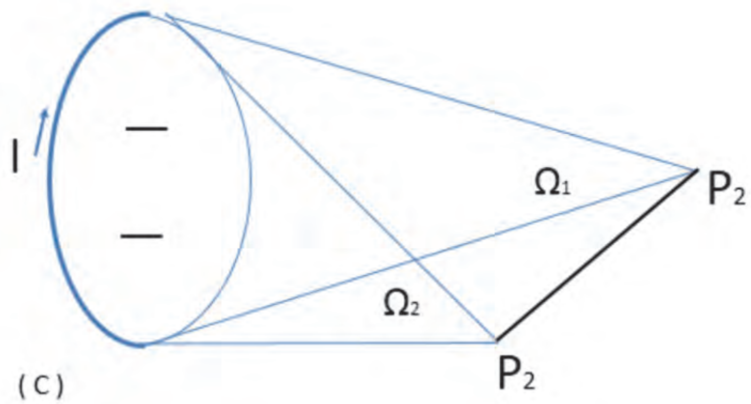


Figure 1.37. *P_1P_2 does not cross the surface S supported by the circuit*

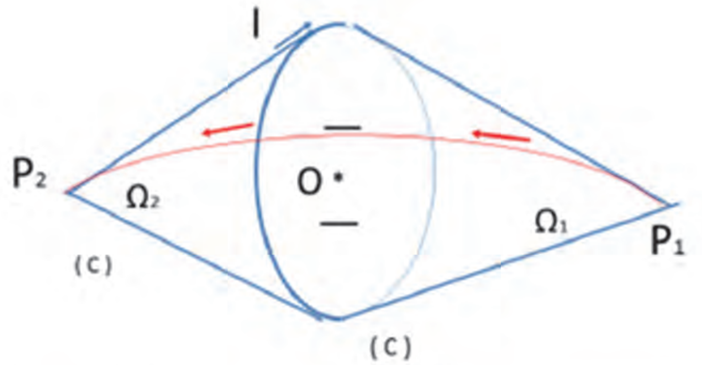


Figure 1.38. *P_1P_2 crosses the surface S supported by the circuit*

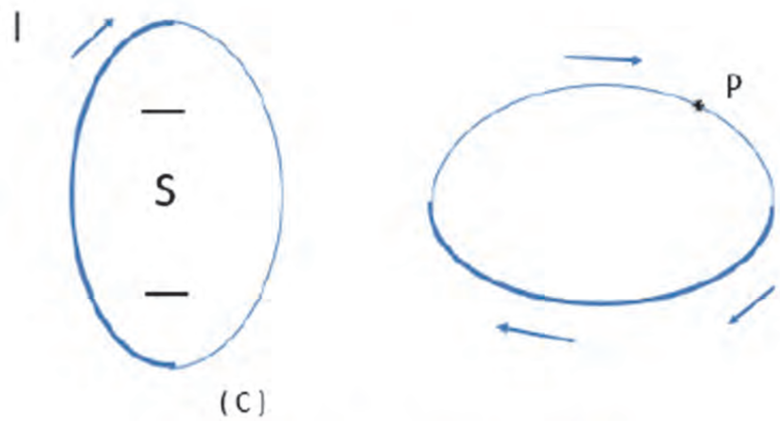


Figure 1.39. *The path does not cross the surface S supported by the circuit*

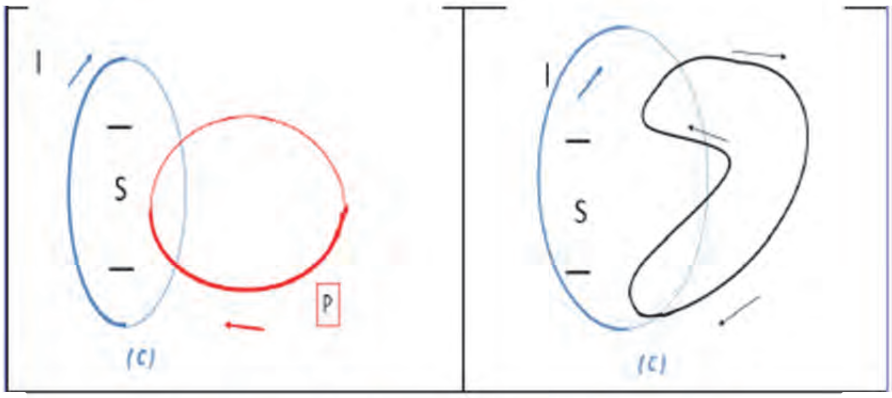


Figure 1.40. *The path crosses the surface S supported by the circuit once (on the left) and twice (on the right)*

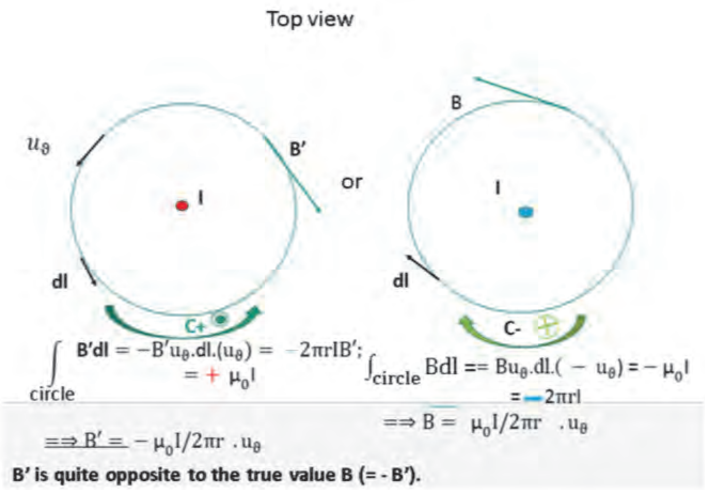


Figure 1.41. *Application of Ampère's theorem, in two ways*

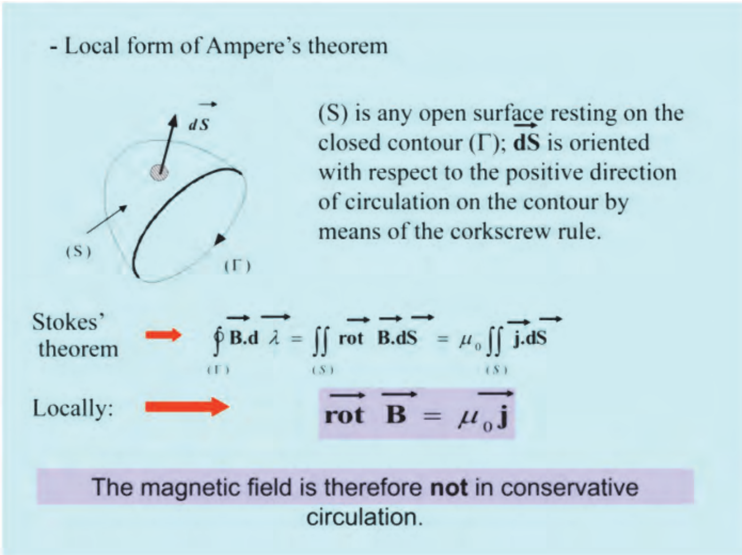


Figure 1.42. *Ampère's theorem: local form*

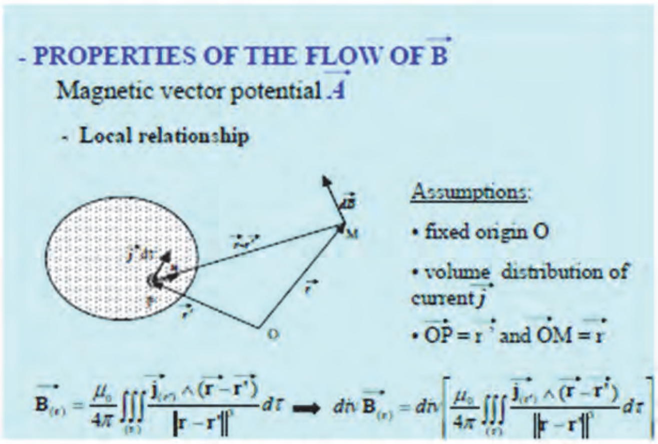


Figure 1.43. *Flow of the magnetic field*

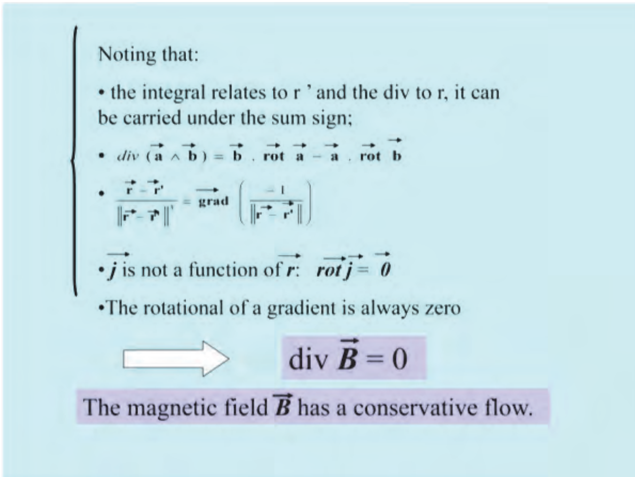


Figure 1.44. *Conservative flow – local form*

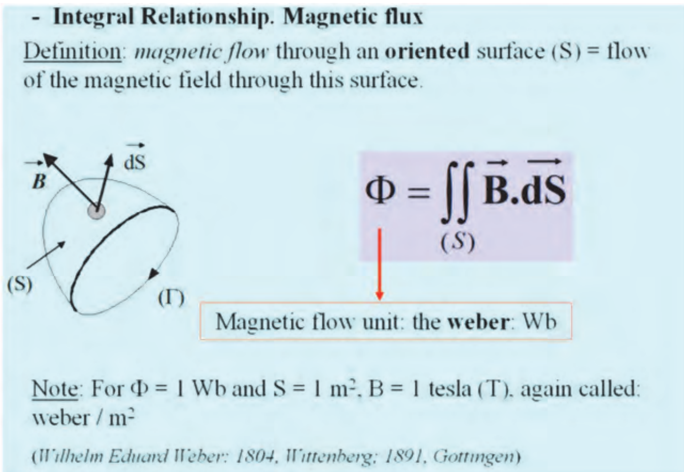


Figure 1.45. *Integral form of the field flux*

Magnetic flow conservation :

$$\mathrm{div}\,\vec{\mathbf{B}} = 0 \rightarrow \begin{cases} (S) \text{ being closed} \\ \oint\!\!\oint \vec{\mathbf{B}} \cdot \overrightarrow{\mathbf{dS}} = 0 : \text{integral form} \end{cases}$$

Consequence: $\vec{\mathbf{B}}$

- $\vec{\mathbf{B}}$ flow **constant** through any section of a **field tube**.
- $\vec{\mathbf{B}}$ flow is the same across **any** oriented surface resting on a contour (Γ): it is therefore possible to speak of flow of $\vec{\mathbf{B}}$ across the contour (Γ).

Figure 1.46. *Flow of the magnetic field – its preservation*

- Potential vector of the magnetic field

Definition

$\vec{B}$ with a conservative flow → the vector potential $\vec{A}$ is such that

$$\vec{\mathbf{B}} = \overrightarrow{\mathbf{rot}}\,\vec{\mathbf{A}}$$

the vector potential $\vec{A}$ is defined only to the gradient of any scalar function, the rotational of a gradient being zero.

*Coulomb **gauge*** *Condition*: div $\vec{A} = 0$

$\vec{A}$ is still indeterminate to within a constant vector $\vec{K}$ since the divergence of a constant vector is zero.

In practice, by analogy with electrostatics, it will be possible to place $\vec{A} = 0$ at infinity, only when the current system does not extend to infinity.

→ **Absolute vector potential $\vec{A}$**

Figure 1.47. *Vector potential*

$$\left\{\begin{array}{l} \overrightarrow{\mathbf{rot}}\,\overrightarrow{\mathbf{B}} = \mu_0 \overrightarrow{\mathbf{j}} \quad \text{Ampère's theorem; local form.} \\ \overrightarrow{\mathbf{rot}}(\overrightarrow{\mathbf{rot}}\,\overrightarrow{\mathbf{A}}) = \mu_0 \overrightarrow{\mathbf{j}} \\ \overrightarrow{\mathbf{rot}}(\overrightarrow{\mathbf{rot}}\overrightarrow{\mathbf{A}}) = \overrightarrow{\mathbf{grad}}(div\overrightarrow{\mathbf{A}}) - \overrightarrow{\Delta\mathbf{A}} = \mu_0 \overrightarrow{\mathbf{j}} \\ \text{If } \mathrm{div}\,\vec{A} = 0\text{: Lorentz. gauge} \end{array}\right.$$

$$\longrightarrow \boxed{\overrightarrow{\Delta\mathbf{A}} = -\mu_0 \overrightarrow{\mathbf{j}}}$$

Poisson's equation of magnetostatics

Figure 1.48. *Poisson's equation of vector potential*

- Integral expression of vector potential

<u>Assumptions</u>: $\vec{A}_x$, $\vec{A}_y$, $\vec{A}_z$ are the components of $\vec{A}$ in Cartesian coordinates, and α, β, γ are the unit vectors carried by the axes (Ox), (Oy), and (Oz).

$$\left.\begin{array}{l} \overrightarrow{\Delta\mathbf{A}} = \vec{\alpha}\,\Delta A_x + \vec{\beta}\,\Delta A_x + \vec{\gamma}\Delta A_x \\ \Delta A_x = \dfrac{\partial^2 A_x}{\partial x^2} + \dfrac{\partial^2 A_x}{\partial y^2} + \dfrac{\partial^2 A_x}{\partial z^2} \\ \vec{\mathbf{j}} = \vec{\boldsymbol{\alpha}}\, j_x + \vec{\boldsymbol{\beta}}\, j_y + \vec{\boldsymbol{\gamma}}\, j_z \end{array}\right\} \quad \left[\begin{array}{l} \Delta A_x = -\mu_0 j_x \\ \Delta A_y = -\mu_0 j_y \\ \Delta A_z = -\mu_0 j_z \end{array}\right.$$

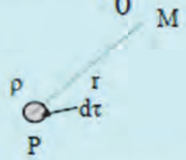
By integration it is possible to obtain the vector potential...

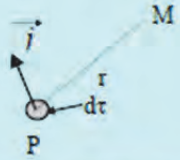
Figure 1.49. *Vector potential and current density*

A_x, A_y and A_z are scalar functions

By analogy with electrostatics where V is a scalar function of the same type:

$$\Delta V = -\frac{\rho}{\varepsilon_0} \qquad \Delta A_x = -\mu_0 \, j_x$$





$$V = \frac{1}{4\pi\varepsilon_0} \iiint_{(\tau)} \frac{\rho \, d\tau}{r}$$

$$A_x = \frac{\mu_0}{4\pi} \iiint_{(\tau)} \frac{j_x}{r} d\tau$$

Similarly for A_y and A_z

$$\vec{\mathbf{A}} = \frac{\mu_0}{4\pi} \iiint_{(\tau)} \frac{\vec{\mathbf{j}}}{r} d\tau$$

$j (A/m^2)$

Figure 1.50. *Scalar potential and vector potential*

In the case of surface currents: **k (A/m)**:

$$\vec{\mathbf{A}} = \frac{\mu_0}{4\pi} \iint_{(S)} \frac{\vec{\mathbf{k}}}{r} dS$$

In the case of filiform currents: **I (A)**:

$$\vec{\mathbf{A}} = \frac{\mu_0}{4\pi} \int_{(C)} \frac{I}{r} \vec{\mathbf{d}\ell}$$

Figure 1.51. *Vector potential*

Distribution of filiform currents I(dl) → $\overrightarrow{dA} = \frac{\mu_0}{4\pi}\frac{I\,\overrightarrow{dl}}{r}$

I(dl)

Distribution of surface currents I(dl) → $\overrightarrow{dA} = \frac{\mu_0}{4\pi}\frac{\overrightarrow{k}}{r}dS$

k(dS)

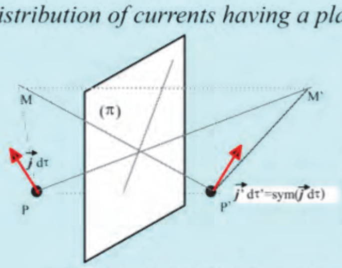
Distribution of volume currents I(dl) → $\overrightarrow{dA} = \frac{\mu_0}{4\pi}\frac{\overrightarrow{j}}{r}d\tau$

j(dτ)

Figure 1.52. *Different forms of vector potential*

Symmetry and invariance properties of vector potential $\overrightarrow{A}$

Distribution of currents having a plane of **symmetry (π)**



Assumptions : The plane (π) is a plane of symmetry with respect to the distribution of currents:
For 2 points P and P′ symmetrical with respect to (π), the current density $\boldsymbol{j}'$ at P' is symmetrical with the current density $\boldsymbol{j}$ at P.

It is desired to compare the vector potentials $\overrightarrow{A}$ in M and $\overrightarrow{A'}$ in M′ .

Vector potential $\overrightarrow{dA}$ created at M and M' by $\vec{j}$ in P and $\vec{j}'$ in P'

$$\begin{cases} \overrightarrow{dA}_{(M)} = \frac{\mu_0}{4\pi}\left(\frac{\vec{j}}{PM} + \frac{\vec{j}'}{P'M}\right)d\tau \\ \overrightarrow{dA}_{(M')} = \frac{\mu_0}{4\pi}\left(\frac{\vec{j}}{PM'} + \frac{\vec{j}'}{P'M'}\right)d\tau \end{cases}$$

Figure 1.53. *Vector potential and symmetry*

- **Continuity of the vector potential**

By analogy with electrostatics (where the scalar potential V undergoes no discontinuity as it passes through a charged surface), the **(magnetic) vector potential A does not undergo any discontinuity as it passes through a surface through which currents k pass.**

There is therefore continuity from A to the crossing of (S). $(A_2)_{(S)} = (A_1)_{(S)}$

Figure 1.54. *Vector potential and passage relationships*

Vector potential $\quad \vec{A}_{(M')} = -\,\mathrm{sym}\,(\vec{A}_{(M)})$

Vector potential components $\quad \begin{cases} \vec{A}_{n(M')} = \vec{A}_{n(M)} \\ \vec{A}_{t(M')} = -\,\vec{A}_{t(M)} \end{cases}$

Note: if M $\in$ plane (π), then $\vec{A}_t = \vec{0}$

Conclusion: The vector potential $\vec{A}$ of the magnetic field at any point of an antisymmetry plane (π) of the current distribution is perpendicular to this plane.

Figure 1.55. *Vector potential and symmetry*

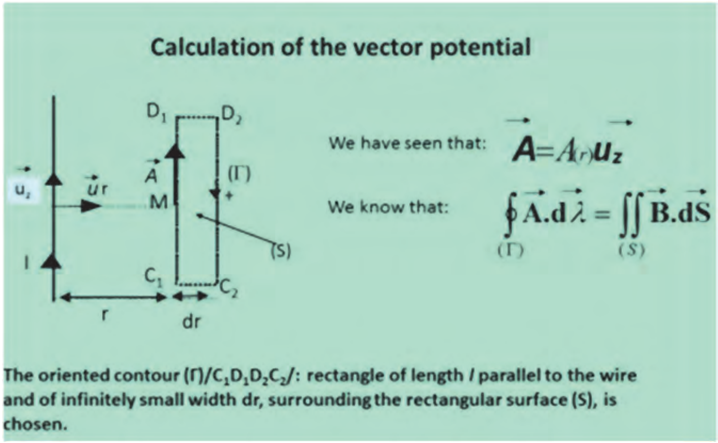


Figure 1.56. *A calculation of the vector potential (the wire)*

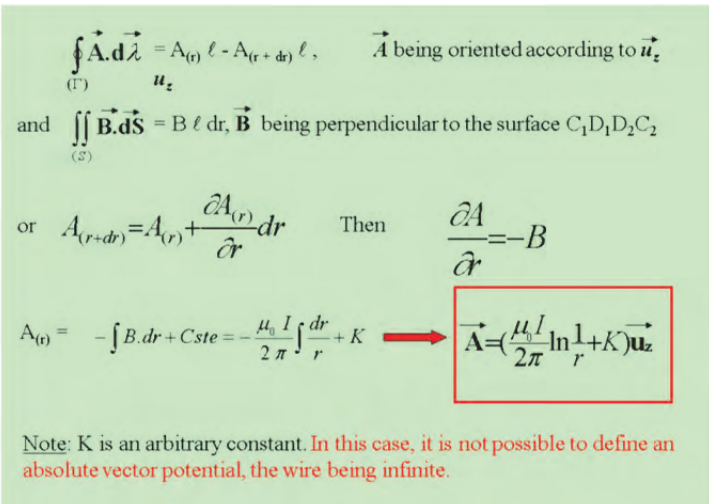


Figure 1.57. *Vector potential applied to a wire current*

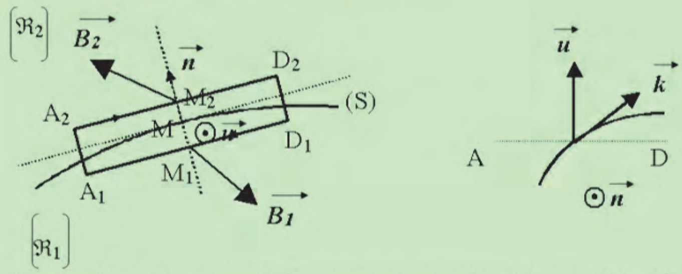
- Invariances

Same results as in electrostatics.

If the current distribution has spatial invariance properties, $\vec{A}$ has these same invariances.

Figure 1.58. *Vector potential and invariance*

- TRANSIT RELATIONSHIP



Assumptions: two regions of the empty or non-magnetic space (R_1) and (R_2) => permeability μ_0.

- through which volume currents flow
- separated by a surface (S) through which surface currents ***k**(M) pass.*

All of these currents create a magnetic field $\overrightarrow{B_1}$ at M_1 which is infinitely close to M_2, and a magnetic field $\overrightarrow{B_2}$ at M_2.

Figure 1.59. *Towards transit relationships*

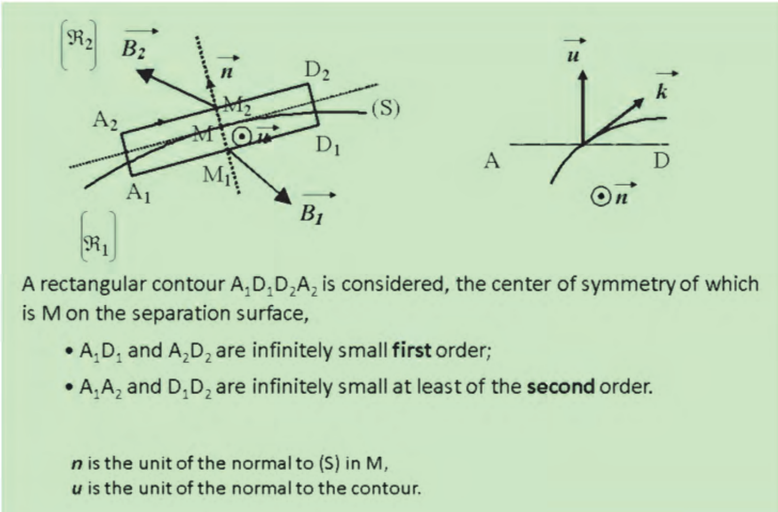


Figure 1.60. *Towards transit relationships (continued)*

Possible discontinuity of the tangential component of the magnetic field

In each region:

$$\left\{ \begin{array}{ll} \vec{\mathbf{B}}.\dfrac{\overrightarrow{\mathbf{AD}}}{AD} = B_t & \text{Tangential component of } B \text{ in M} \\ \vec{\mathbf{B}}.\vec{\mathbf{n}} = B_n & \text{Normal component of } B \text{ in M} \\ \vec{\mathbf{B}} = \vec{\mathbf{B}}_{\mathbf{n}} + \vec{\mathbf{B}}_{\mathbf{t}} & \end{array} \right.$$

Ampere's theorem is applied to the closed contour $D_1A_1A_2D_2D_1$ by calculating:
Circulation of the magnetic field along the contour, bearing in mind that the circulation on A_1A_2 and D_1D_2 may be ignored before that on A_1D_1 and A_2D_2
and the current intensity dI passing through the rectangular contour $D_1A_1A_2D_2$ which can be reduced to the contribution of surface currents $\vec{k}$.

Figure 1.61. *Possible discontinuity of the tangential component of B*

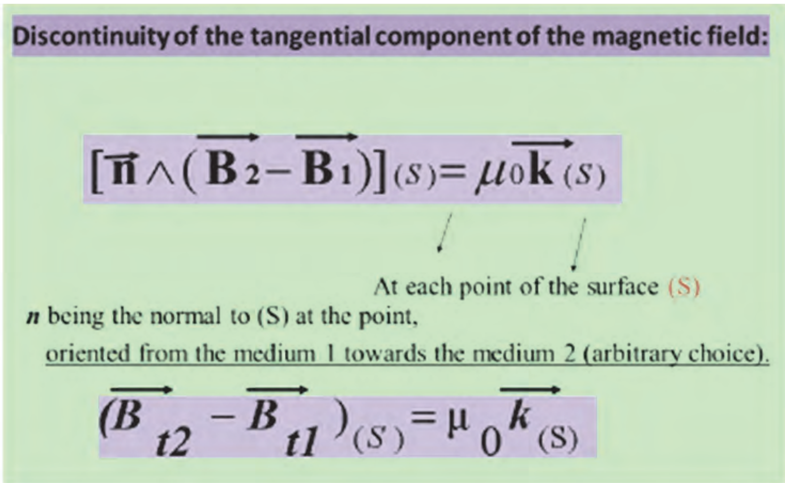


Figure 1.62. *Transit relationship for the tangential component of B*

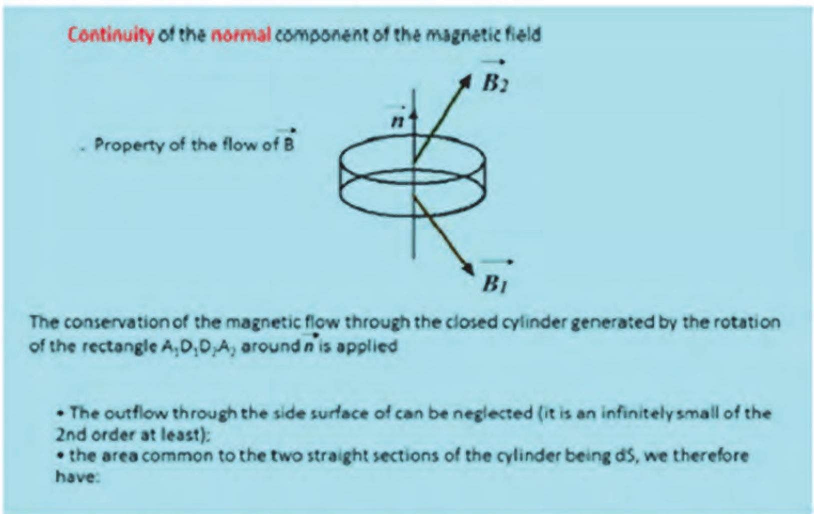


Figure 1.63. *Transit relationship for the transverse component of B*

$$\vec{\mathbf{B}}_2.\vec{\mathbf{n}}\,dS + \vec{\mathbf{B}}_1.(-\vec{\mathbf{n}})\,dS = 0$$

It is shown that

Continuity of the normal component of the magnetic field:

$$\vec{\mathbf{n}}\,.\,(\vec{\mathbf{B}}_2 - \vec{\mathbf{B}}_1)_{(S)} = 0$$

n is always the normal to (S) at the point considered,
oriented from the medium 1 towards the medium 2.

$$(B_{n2})_{(S)} = (B_{n1})_{(S)}$$

Figure 1.64. *Transit relationship for B; normal component*

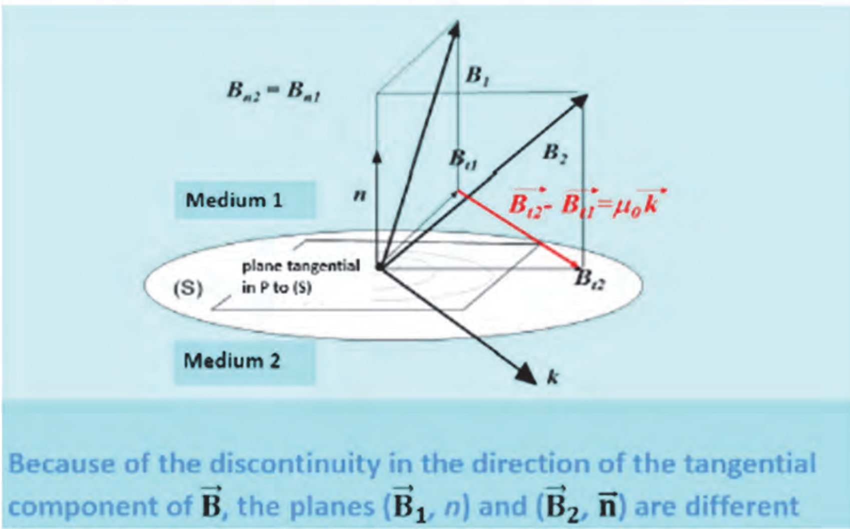


Figure 1.65. *Transit relationship on B*

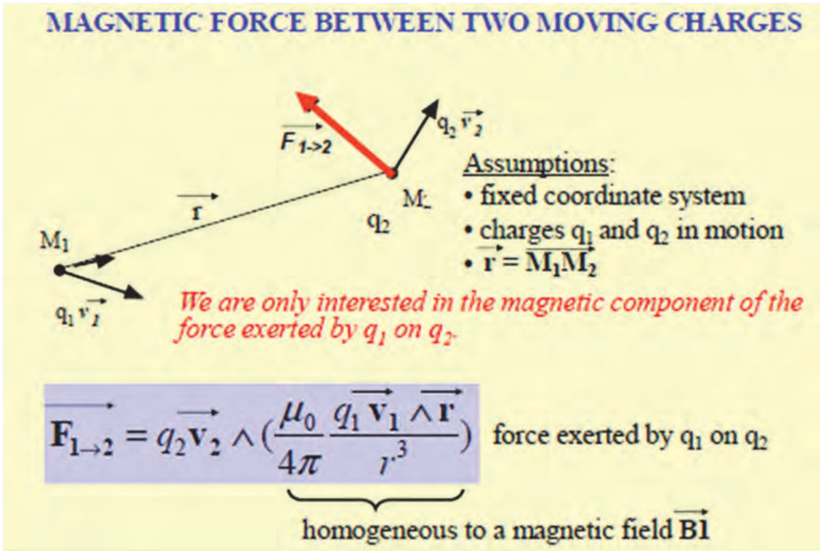


Figure 2.1a. *Field and force*

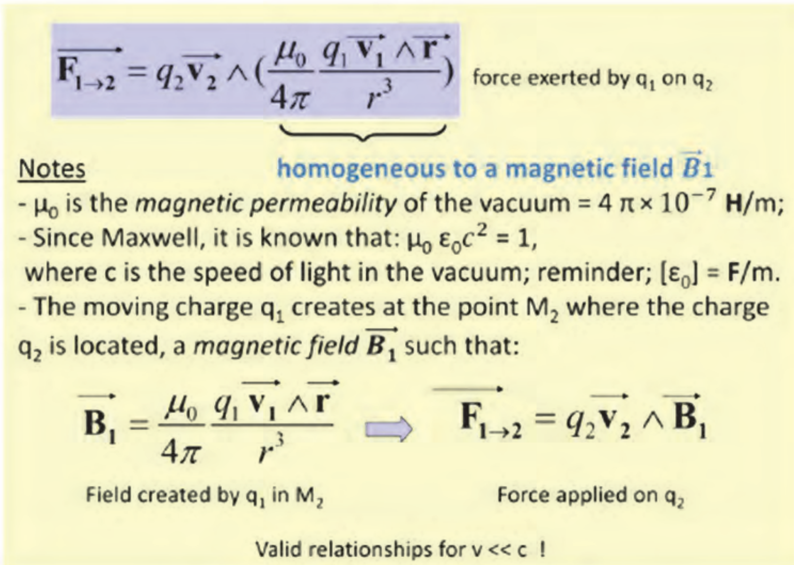


Figure 2.1b. *Towards Lorentz force*

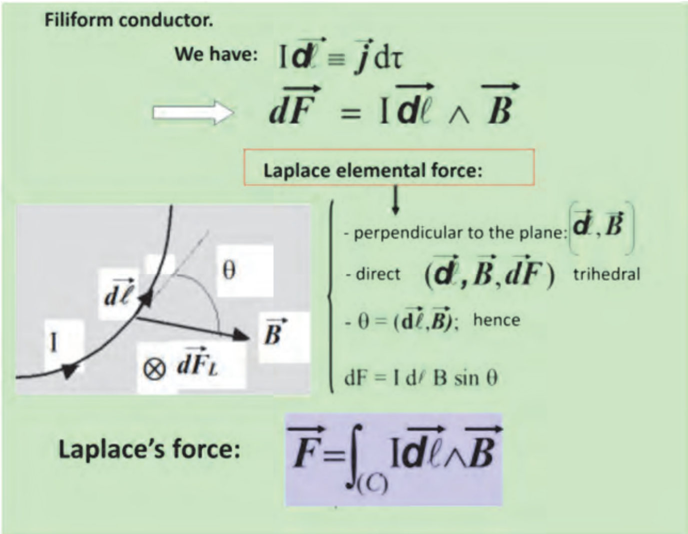


Figure 2.2. *Ampère/Laplace force for a wire*

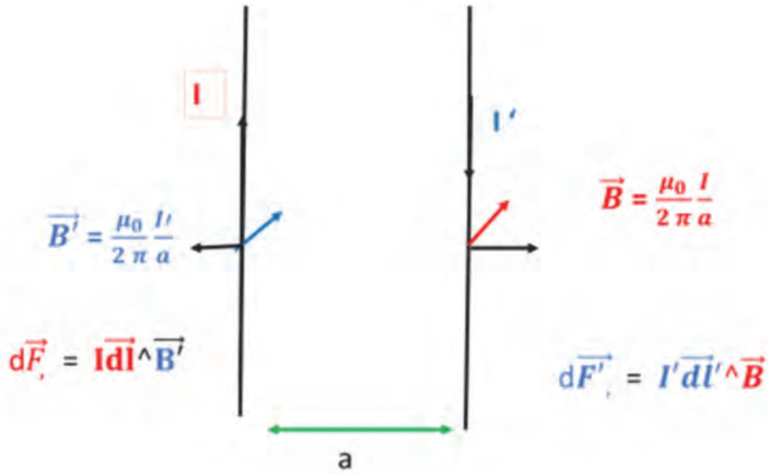


Figure 2.3. *Ampère/Laplace forces applied to two parallel conductor wires*

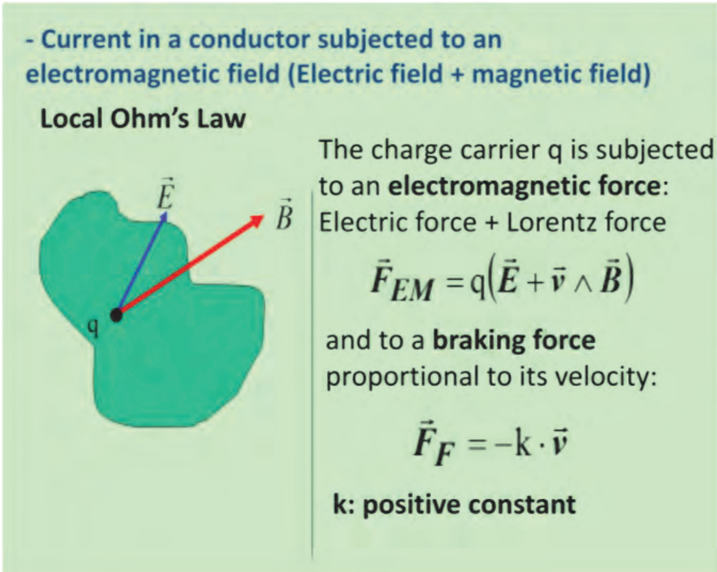


Figure 2.4. *Force in a conductor subjected to electric and magnetic fields*

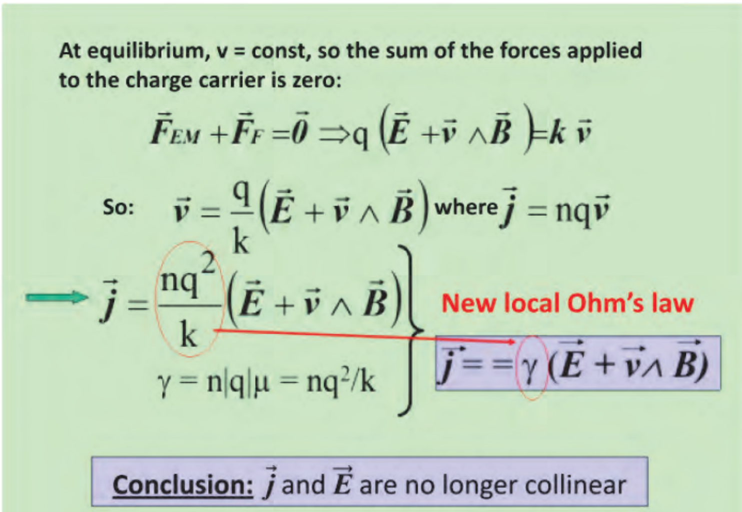


Figure 2.5. *Generalized local Ohm's law*

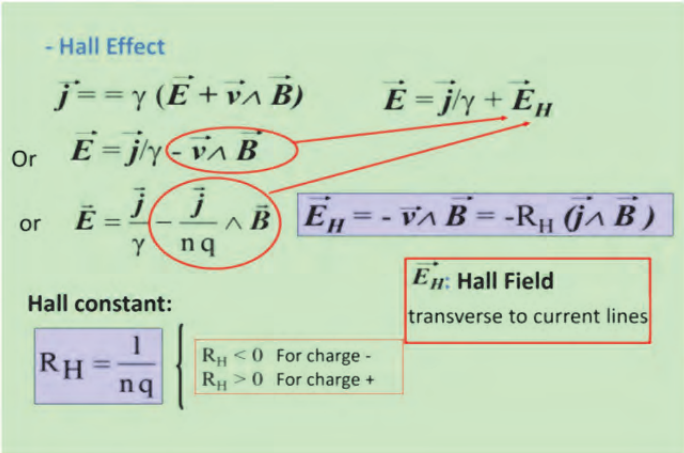


Figure 2.6. *Hall constant*

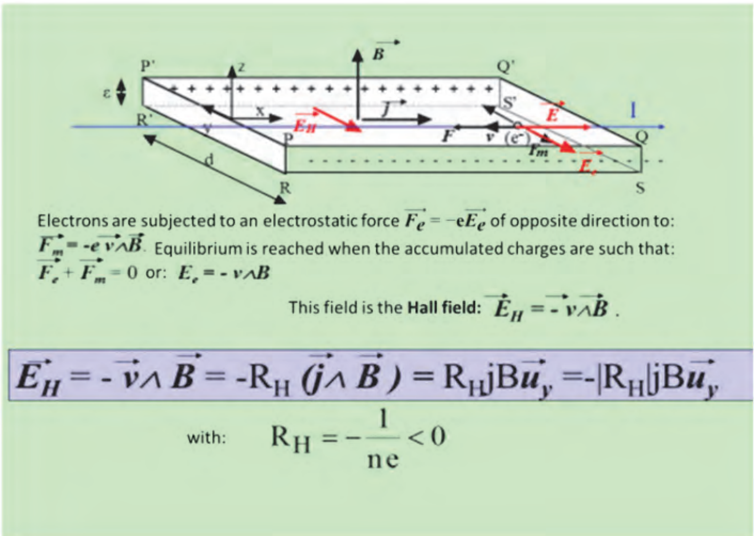
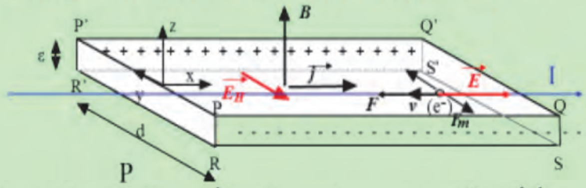


Figure 2.7. *Hall effect*

Appearance of a potential difference in a direction perpendicular to the trajectories of the carriers (here **electrons**), between the front and the rear face, equal to the circulation of the electrostatic field



$$V_{P'} - V_P = \int_{P'}^{P} \vec{E}_H \cdot \vec{d\ell} = E_H \cdot d$$

But: $I = jd\varepsilon$

$$U = V_{P'} - V_P = \frac{IB}{ne\varepsilon} = -R_H \frac{IB}{\varepsilon}$$

Notes:

- Hall's ddp sign tells us about the sign of charge carriers (in general – for metals)
- some semiconductors have a positive Hall constant, when they are P type and conduction is provided by the holes.

Figure 2.8. *Hall effect (continued)*

Applications of the Hall effect: $R_H = \frac{1}{nq}$

- **Measurement of V, I, B and ε = > value and sign of R_H for a given material, *i.e.* the *sign and density of the charge carriers*. The R_H constant is weak (and negative because of electrons) for most metals (example: -5.5×10^{-11} m^{3}C^{-1} for copper). It is much higher for semiconductors.**
- Conversely, knowing R_H characteristic of a material, the measurement of V for I and ε given will measure B (Hall effect sensor).

Figure 2.9. *Hall effect applications to metals and semiconductors*

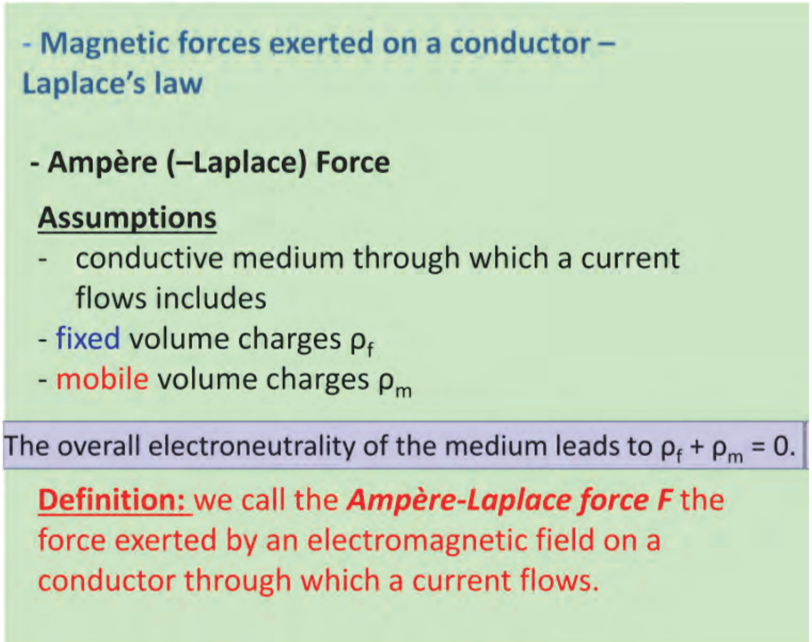


Figure 2.10. *Ampère/Laplace force*

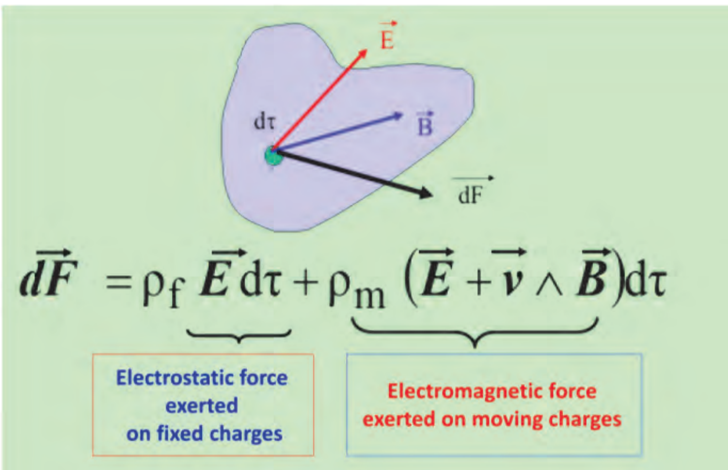


Figure 2.11. *Electromagnetic differential force*

$$\overrightarrow{dF} = \rho_f \vec{E}\, d\tau + \rho_m \left(\vec{E} + \vec{v} \wedge \vec{B}\right) d\tau$$

But: $\rho_f + \rho_m = 0$; therefore:

$$\overrightarrow{dF} = \rho_m \vec{v} \wedge \vec{B}\, d\tau = \vec{j} \wedge \vec{B}\, d\tau$$

Ampère-Laplace force exerted on the entire conductor:

$$\vec{F} = \iiint_{(\tau)} \vec{j} \wedge \vec{B}\, d\tau$$

Figure 2.12. *Ampère/Laplace force, volumetric*

Notes:

- The force exerted on the moving charges is transmitted to the conductor via the fixed ions subjected to the Hall field. Thus an element of volume d_τ of material is therefore subjected to a force:

$$\overrightarrow{dF'} = \rho_f \overrightarrow{E_H}\, d\tau = -\rho_f \vec{v} \wedge \vec{B}\, d\tau = \rho_m \vec{v} \wedge \vec{B}\, d\tau = \overrightarrow{dF}$$

- The magnetic force acting on mobile charge carriers (electrons) is balanced by the transverse force due to the Hall field E_H

- The magnetic field $\vec{B}$ that occurs here is the total magnetic field which includes the magnetic field applied to the conductor $\overrightarrow{B_0}$ and the magnetic field $\overrightarrow{B_P}$ created by the current running through this conductor.

Figure 2.13. *Total force and field*

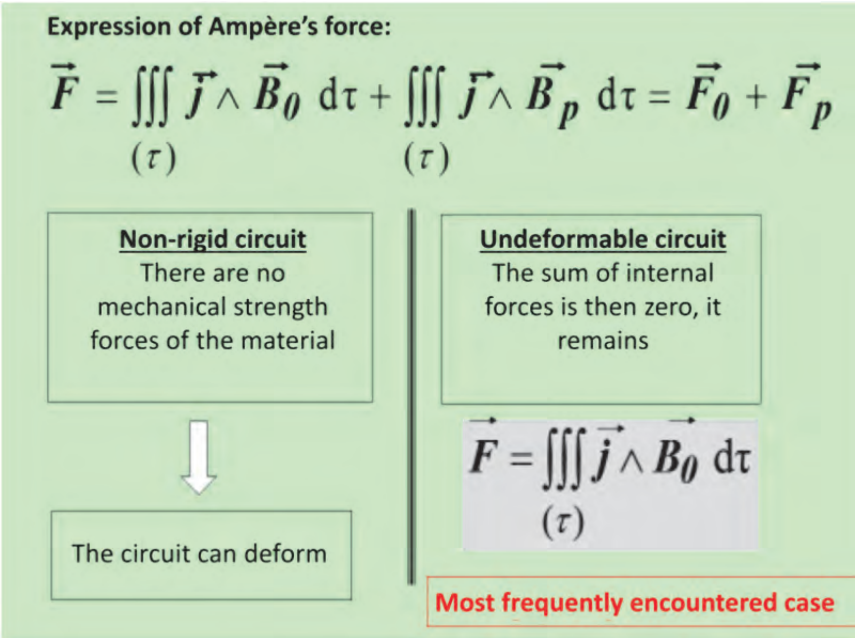


Figure 2.14. *Deformable or undeformable circuits*

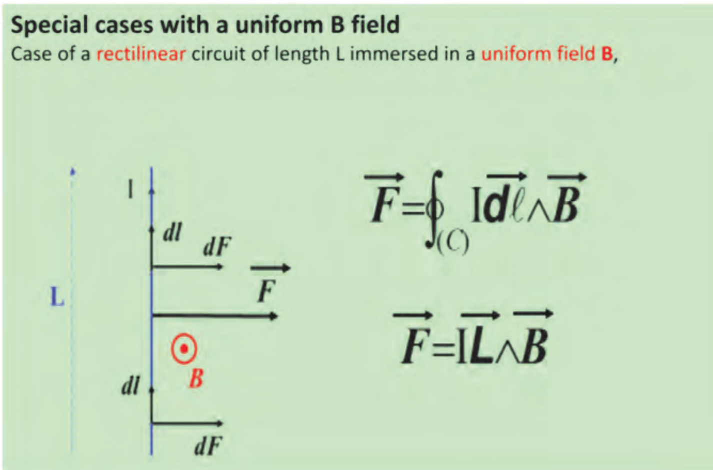
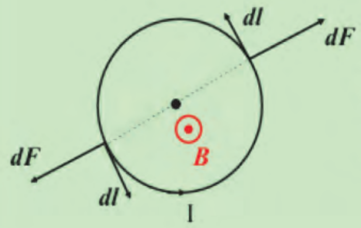


Figure 2.15. *Magnetic force applied to a wire under a magnetic field*

Case of a closed circuit placed in a uniform field **B**,
the result of the Laplace forces is zero

$$\vec{F}=\oint_{(C)} \mathrm{I}\,\vec{d\ell}\wedge\vec{B}=\mathrm{I}(\overbrace{\oint_{(C)}\vec{d\ell}}^{=0})\wedge\vec{B}=\mathbf{0}$$



The elementary forces ***dF*** cancel each other out 2 to 2, regardless of the direction of ***B***.
This does not mean that the resulting moment is zero: rotation

Figure 2.16. *Force applied to a closed circuit in a magnetic field*

Legal definition of Ampère

A portion of a straight conductor placed in a uniform field.

The Ampère is the intensity of a constant current which, held in two straight, parallel wires of infinite length, of negligible circular cross-section and placed 1 meter from each other, produces between these two conductors a force equal to 2×10^{-7} Newton per meter in length.

It follows from this definition that: $\mu_0 = 4\pi.10^{-7}$ **u.S.I.H/mN, H/m**

Figure 2.17. *A definition of Ampère*

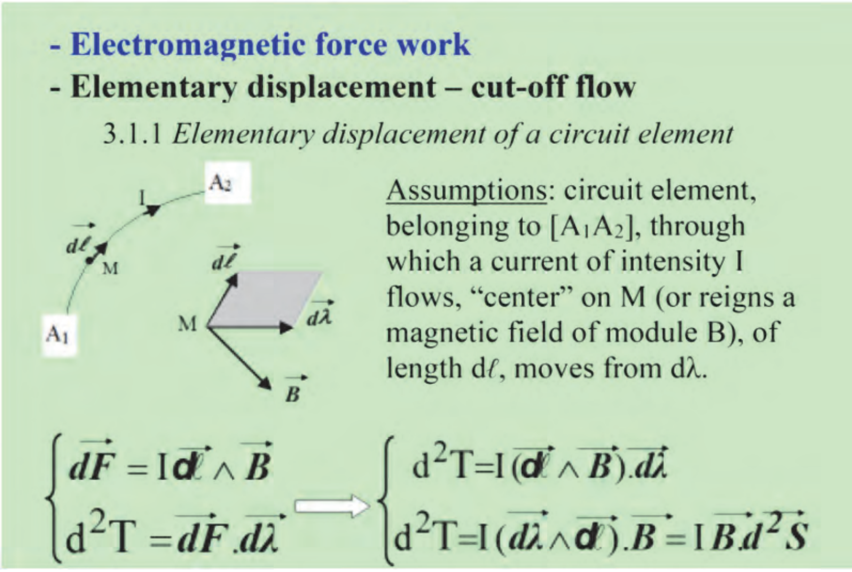


Figure 2.18. *Magnetic forces, cut-off flow; work*

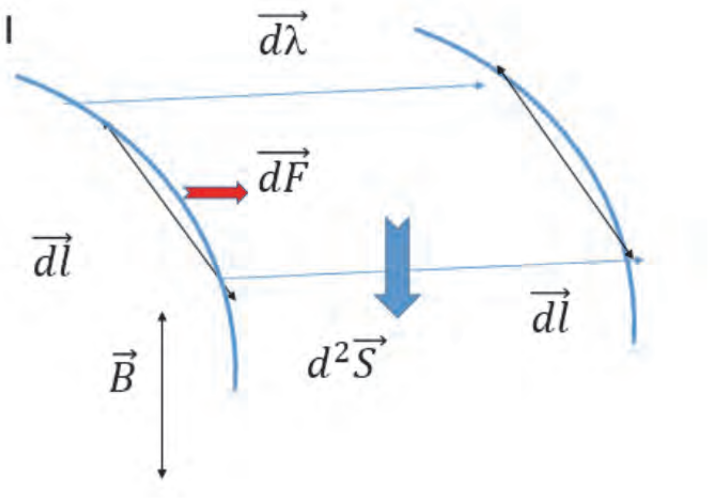


Figure 2.19. *Work and cut-off flow*

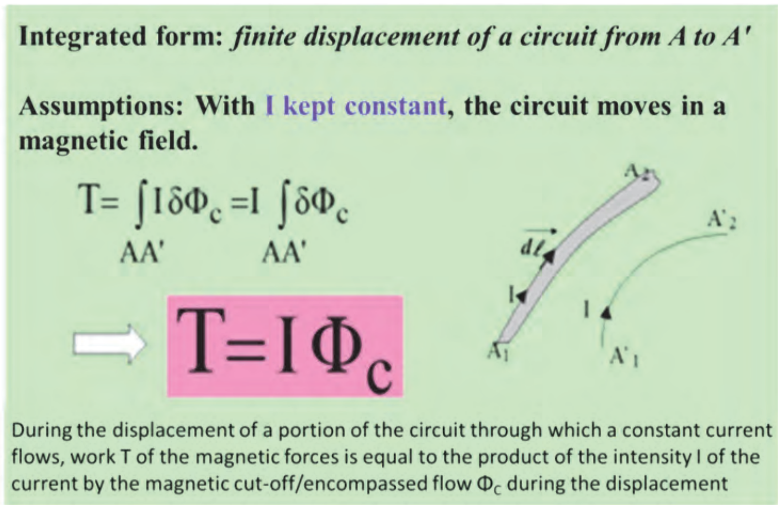


Figure 2.20. *Work via the cut flow*

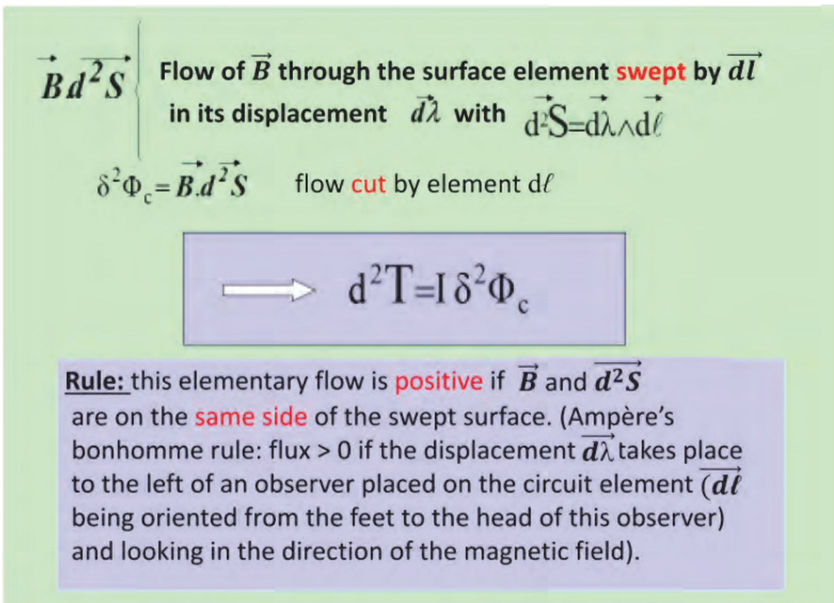


Figure 2.21. *Work and cut-off flow (encompassed)*

Validity of the study

- case of a *closed circuit* (*deformable or undeformable*)
- mobile in an *invariable* (*permanent*) field.
- we will calculate only the *work of* ***external forces****: we take into account* ***only Ampère forces*** *due to the* ***applied field***.

Note: *taking into account the eigenfield leads to estimating* the work of internal forces. If the circuit is undeformable, this work is necessarily zero.

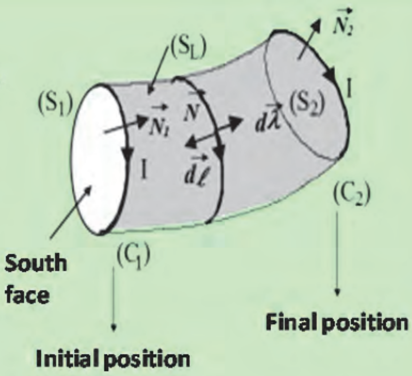
Figure 2.22. *Introduction to Maxwell's theorem*

Applied $\vec{B}$ field

According to the properties of the magnetic field, the flow of B through any closed surface is zero

$(\mathrm{div}\vec{B} = 0)$.

We call (C_1) the initial position of the circuit (C) and (C_2) its final position



Assumptions: constant current I. The circuit scans a surface (S_L). A closed surface (Σ) is delimited by adding to (S_L) a surface (S_1) resting on (C_1) and a surface (S_2) resting on (C_2). These surfaces are oriented by the corkscrew rule.

(S_1) is oriented locally by $\vec{N}$ as: $\vec{d\lambda} \wedge \vec{d\ell}$

Figure 2.23. *Displacement of a closed circuit in a magnetic field*

Conservation of the flow of $\vec{B}$ **leaving the closed surface** (Σ)

$$-\Phi_1 - \Phi_C + \Phi_2 = 0$$

Either: $\Phi_C = \Phi_2 - \Phi_1$ or: $T = I\,\Phi_c$

$$\Rightarrow \quad T = I\,(\Phi_2 - \Phi_1)$$

Maxwell's theorem

The work T of magnetic forces exerted by field **B** applied to a circuit through which a constant current **I** flows and which field moves in an invariable magnetic field is equal to the product of the intensity I of the current by the variation of the magnetic flux entering through the south face of the circuit.

Figure 2.24. *Maxwell's theorem*

Maxwell's theorem

$$T = I\,(\Phi_2 - \Phi_1)$$

Elementary form of Maxwell's theorem

For an elementary displacement of a circuit through which an invariable current flows, I such that the variation of the magnetic flux is dΦ, the work of magnetic forces is dT such that:

$$dT = I\,d\Phi$$

Figure 2.25. *Differential form of Maxwell's theorem*

Consequence: **Maximum Flow Rule**

- rigid circuit through which a current flows and placed in a magnetic field and retained by an external operator;
- if the operator releases the circuit, it moves under the action of the magnetic forces applied to it;
- since this displacement can take place only in the direction of the forces applied, these forces perform a motor work (T>0). This results in $\Phi_2 > \Phi_1$. *The circuit moves under the effect of electromagnetic forces in the direction that causes an increase in the flow entering through its south face.*
- the equilibrium position of the circuit is reached when the flow that passes through the circuit is maximum.

Figure 2.26. *Maximum flow rule*

- When a conductor (filiform or massive), through which an invariable current (I, $\boldsymbol{j}$) flows, placed in an invariable outer magnetic field $\vec{B}$ it is subjected to a torsor of magnetic forces called **Ampère/Laplace Force Torsor**:

This torsor is composed of:

- **a resulting force**
- **a couple of forces given by the resulting moments**

- If the conductor is free to move, it moves (transverse displacement, rotation) under the action of this force torsor, and finds a balanced position when the torsor cancels out.
- The displacement is such that it tends to **minimize magnetic flow of $\vec{B}$** through the circuit

How can magnetic forces and their work during a circuit displacement be calculated?

Figure 2.27. *Forces/torsors/flows*

Application: **Calculation of the torsor of magnetic forces exerted by an invariable field on a rigid circuit**

- **Case of a transverse displacement**
- **Case of a rotation**

Laplace magnetic force torsor:

- a resulting force $\vec{F}$
- a resulting moment $\vec{\Gamma}$

Figure 2.28. *Magnetic force torsor*

Assumptions:

- a rigid circuit (C) through which a current I flows and which is placed in an invariable magnetic field $\vec{B}$;
- the flow Φ which passes through this circuit depends on the parameters fixing the position of the circuit in space:
  - 3 position parameters of a circuit point (x,y,z)
  - and 3 parameters of movement of the circuit around this point (ϴ, ϕ, ω).

Elemental displacement of the circuit $\Phi \rightarrow \Phi + d\,\Phi$

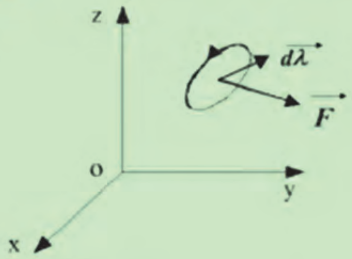
Hence: dT = IdΦ (elementary form of Maxwell's theorem)

Figure 2.29. *Introduction to Maxwell's theorem (elementary form)*

Case of a transverse displacement

$\vec{F}$ is the result of the torsor of magnetic forces exerted on the circuit

$$\vec{F}\begin{cases} F_X \\ F_Y \\ F_Z \end{cases}$$



During an elementary transverse displacement $\overrightarrow{d\lambda}$ (dx, dy, dz) of the circuit, the elementary work dT of magnetic forces will be:

$$dT = \overrightarrow{F}.\overrightarrow{d\lambda} = I\, d\Phi$$

Figure 2.30. *Magnetic work and flux; transverse displacement*

$$d\Phi = \frac{\partial \Phi}{\partial x} dx + \frac{\partial \Phi}{\partial y} dy + \frac{\partial \Phi}{\partial z} dz$$

Hence: $$F_X dx + F_Y dy + F_Z dz = I\left(\frac{\partial \Phi}{\partial x} dx + \frac{\partial \Phi}{\partial y} dy + \frac{\partial \Phi}{\partial z} dz\right)$$

This must be true regardless of the displacement $\overrightarrow{d\lambda}$ (dx, dy, dz); therefore:

Or: $$F_X = I\frac{\partial \Phi}{\partial x}, \quad F_Y = I\frac{\partial \Phi}{\partial y}, \quad F_Z = I\frac{\partial \Phi}{\partial z}$$

Transverse Displacement: $$\boxed{\vec{F} = I\, \overrightarrow{grad}\, \Phi}$$

Figure 2.31. *Force as a function of flow variation*

In the case of rotation about an axis (Δ):

- θ: parameter fixing the position of the circuit for rotation about any axis (Δ),
- elemental rotation of the circuit about this axis
- $\vec{\Gamma}$: moment resulting from the torsor of magnetic forces with respect to this axis (Δ).

The elementary work dT of magnetic forces during this virtual displacement will be:

$dT = \Gamma\, d\theta = I d\Phi$ and the flow variation: $d\Phi = \frac{\partial \Phi}{\partial \theta} d\theta$

rotation θ about an axis (Δ) $\Gamma = I \frac{\partial \Phi}{\partial \theta}$

Figure 2.32. *Work, torque, flow, associated with a rotation*

Conclusion

The evaluation of the magnetic flux Φ entering through the south face of a circuit as a function of the position parameters (x, y, z) and of the angular parameters marking its orientation in (θ, φ, etc.), helps determine the result of the torsor of magnetic forces, resultant force, and resultant moments with respect to each axis of rotation.

In transverse displacement: $\vec{F} = I\, \overrightarrow{grad}\, \Phi$

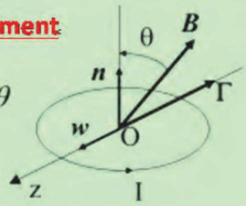
In rotation θ about an axis (Δ): $\Gamma = I \frac{\partial \Phi}{\partial \theta}$

Figure 2.33. *Force and torque*

Forces undergone in transverse displacement

$$\Phi = N\iint_{(S)} \vec{B}\cdot d\vec{S} = N\iint_{(S)} \mathrm{B}\,\mathrm{dS}\cos\theta = NBS\cos\theta$$

$$\overrightarrow{F} = \mathrm{I}\,\overrightarrow{grad\Phi}$$



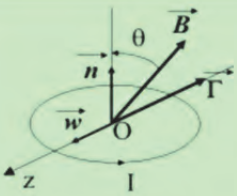
Since the field $\vec{B}$ is uniform, during a transverse displacement, θ does not vary, so Φ does not vary, and thus $\overrightarrow{grad\Phi} = \vec{0}$: hence $\vec{F}$ is zero.

Consistent with:

$$F = \oint_{(C)} \mathrm{I}\,d\ell \wedge B = \mathrm{I}(\overbrace{\oint_{(C)} d\ell}^{=0}) \wedge B = 0$$

The result of the magnetic forces is zero: $\vec{F} = \vec{0}$

Figure 2.34. *Flow, torque, in rotation*



Forces undergone during rotation

$$\Phi = N\iint_{(S)} \vec{B}\cdot d\vec{S} = N\iint_{(S)} \mathrm{B}\,\mathrm{dS}\cos\theta = NBS\cos\theta$$

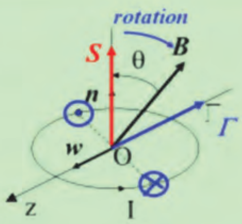
$$\Gamma = \mathrm{I}\frac{\partial\Phi}{\partial\theta}$$

- Any rotation that does not cause a variation in the angle θ between $\vec{B}$ and $\vec{n}$ and does not cause a variation in the flow Φ.
Thus, any rotation about $\vec{n}$ or about the direction of $\vec{B}$ does not lead to a variation of Φ.

- Since these two axes of rotation are chosen, the third angular reference mark for fixing the position of the coil in space is that corresponding to a rotation θ about the direction Oz of unit $\vec{w}$ orthogonal to $\vec{n}$ and to $\vec{B}$.

Figure 2.35a. *Flow and torque associated with a coil*

According to the *maximum* flux rule, the coil will tend to rotate in such a way that $\vec{n}$ takes the direction of $\vec{B}$ ($\theta = 0$, $\cos\theta = 1$, Φ max = NBS).
The magnetic torque $\vec{\Gamma}$ to which the coil is subjected is a torque of rotation through an angle θ with respect to the axis Oz.



The moment of the torque $\vec{\Gamma}$ with respect to the point O is a vector orthogonal to $\vec{n}$ and $\vec{B}$, directed in the opposite direction to $\vec{w}$.

We know that: $\Gamma = I\, d\Phi/d\theta$

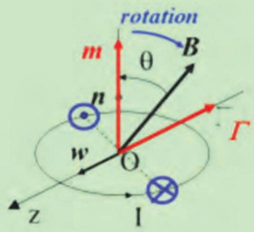
But: $\Phi = NBS\cos\theta$ Therefore: $\Gamma = -INSB\sin\theta$

$$\boxed{\vec{\Gamma} = NI\, \vec{S} \wedge \vec{B} \quad \text{with:} \quad \vec{S} = S\, \vec{n}}$$

Figure 2.35b. *Torque associated with a coil*

Introducing the **magnetic moment $\vec{m}$** of the coil:

$$\boxed{\vec{m} = NI\, \vec{S}}$$



The resulting moment $\vec{\Gamma}$ of forces exerted by an invariable and uniform field $\vec{B}$ on a turn or coil of magnetic moment $\vec{m}$ is:

$$\boxed{\vec{\Gamma} = \vec{m} \wedge \vec{B}}$$

Figure 2.36. *Magnetic torque*

Generalization: Action of a magnetic field on a current loop (or magnetic dipole)

Potential energy of interaction with a magnetic field or potential magnetostatic energy

Current loop of magnetic moment $\vec{m}$ placed in a non-uniform external magnetic field $\vec{B}$ (function of the coordinates of the space).

For a loop of "**very small dimensions**", $\vec{B}$ can be considered to be **uniform** over the surface of the loop (or $\vec{B}$ very slightly variable on the scale of the dimensions of the loop)

Let dT be the elementary work magnetic forces during a displacement of the loop leading to a flux variation dΦ.

The **potential magnetostatic energy E_p** of the loop is defined by:

$$dEp = - dT$$

Figure 2.37. *Differential potential energy*

But $dT = I\, d\Phi$ **Elementary form of Maxwell's theorem**

with $\Phi = \iint_{(S)} \vec{B}\cdot d\vec{S} = \vec{B}.\iint_{(S)} dS\vec{n} = \vec{B}.S\vec{n}$ **because *B* is considered uniform on the loop surface S**

or $dT = I\, d\Phi = d(IS\,\vec{n}.\vec{B}) = d(\vec{m}.\vec{B})$

$$dE_p = -\, dT$$

so $dE_p = -\, d(\vec{m}.\vec{B})$

or $$E_p = -\,\vec{m}.\vec{B}$$

Figure 2.38. *Potential energy associated with the magnetic moment*

– Magnetic force acting on the loop in a non-uniform field:

$$\delta \mathbf{T} = - dE_p = \boldsymbol{F} \,.\, \boldsymbol{d\lambda} = F_x dx + F_y dy + F_z dz$$

So: $F_x = \frac{\partial E_p}{\partial x}$, $F_y = \frac{\partial E_p}{\partial y}$, $F_z = \frac{\partial E_p}{\partial z}$

or: $\boxed{\vec{F} = - \overrightarrow{grad}(E_p) = \overrightarrow{grad}\,(\vec{m}.\vec{B})}$

$$F_x = \left(\vec{m}.\frac{\partial \vec{B}}{\partial x}\right), \quad F_y = \left(\vec{m}.\frac{\partial \vec{B}}{\partial y}\right), \quad F_z = \left(\vec{m}.\frac{\partial \vec{B}}{\partial z}\right)$$

Figure 2.39. *Magnetic force versus potential energy*

Analogy with the electrostatic dipole

The torsor of magnetic forces exerted by an invariable field $\vec{B}$ on a current loop of magnetic moment $\vec{m}$, also called magnetic dipole, is:

In **any (non-uniform)** invariable field

Potential electrostatic energy	Potential magnetostatic energy
$E_p = - \vec{p}.\vec{E}$	$E_p = - \vec{m}.\vec{B}$
Electric force	Magnetic force
$\vec{F} = -\overrightarrow{grad}\,(E_p) = \overrightarrow{grad}\,(\vec{p}.\vec{E})$	$\vec{F} = -\overrightarrow{grad}\,(E_p) = \overrightarrow{grad}\,(\vec{m}.\vec{B})$

In an **uniform** invariable field

Moment of electrical torque:	Moment of magnetic torque:
$\vec{\Gamma} = \vec{p} \wedge \vec{E}$	$\vec{\Gamma} = \vec{m} \wedge \vec{B}$

Figure 2.40. *Torque: electrostatic/magnetostatic analogy*

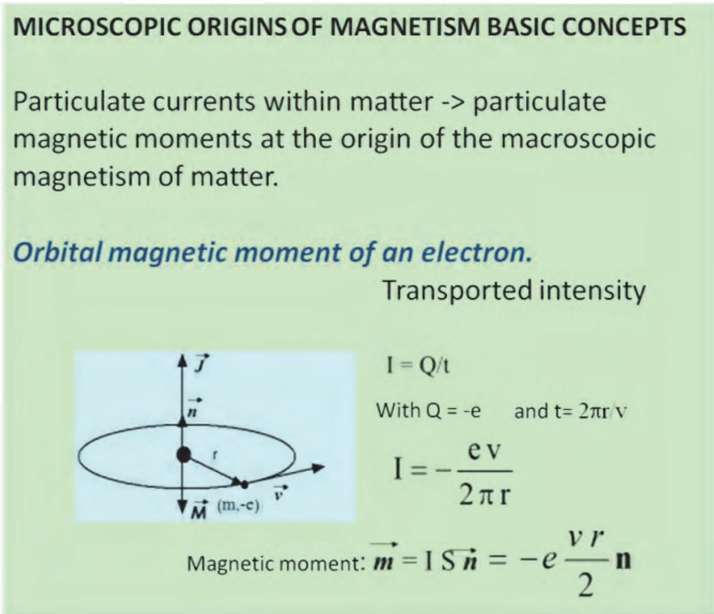


Figure 3.1. *Magnetic moment*

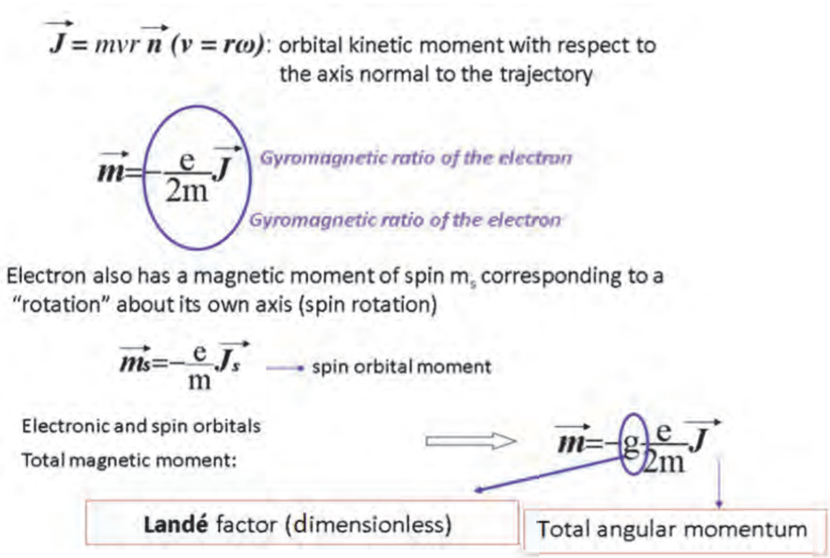


Figure 3.2. *Magnetic versus kinetic moment*

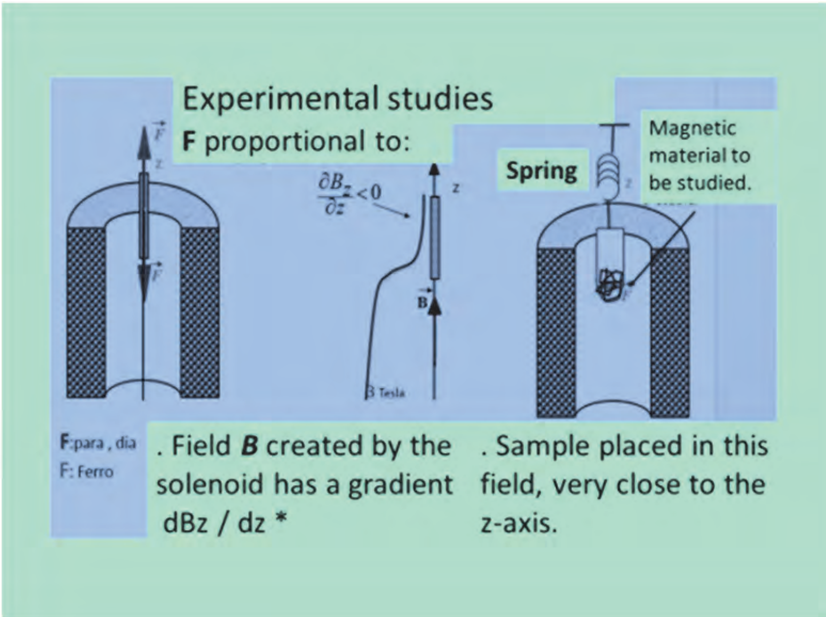


Figure 3.3. *Matter in a magnetic field*

Diamagnetic media

Calculation of the new orbital magnetic moment m of the electron:

$$\vec{m} = e\frac{\omega\, r^2}{2}\vec{n} = \frac{er^2(\omega_0 + \Delta\omega)}{2}\vec{n} = \frac{er^2\omega_0}{2}\vec{n} + \frac{er^2\Delta\omega}{2}\vec{n}$$

$$\vec{m} = \vec{m}_0 + \Delta\vec{m}$$

Or: $\Delta\vec{m} = \frac{er^2\Delta\omega}{2}\vec{n}$ with $\Delta\omega = -\,eB/2m$

Therefore: $$\Delta\vec{m} = -\frac{e^2r^2}{4m}\vec{B}$$

- Variation of ***m*** is proportional to ***B*** and in the opposite direction.
- This results in the appearance of macroscopic magnetization ***M*** in a direction opposite to ***B***.

Figure 3.4. *Characteristic of diamagnetism*

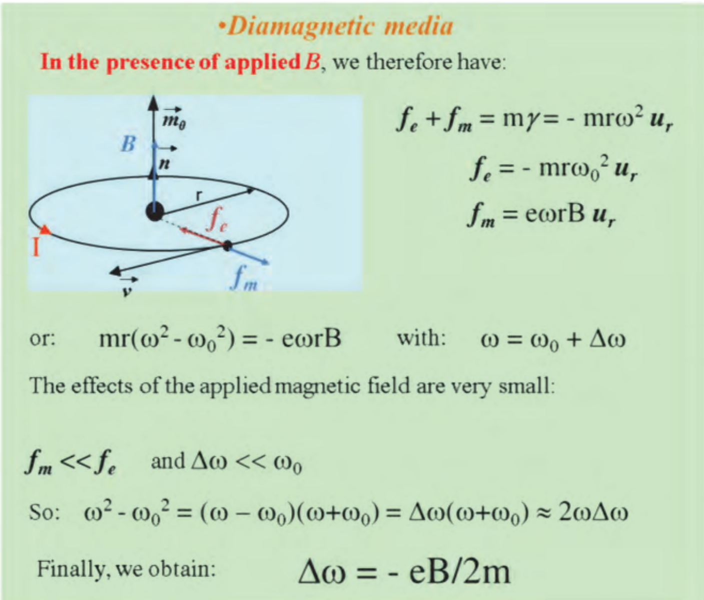


Figure 3.5. *Diamagnetism: influence of field B on the angular velocity of an electron*

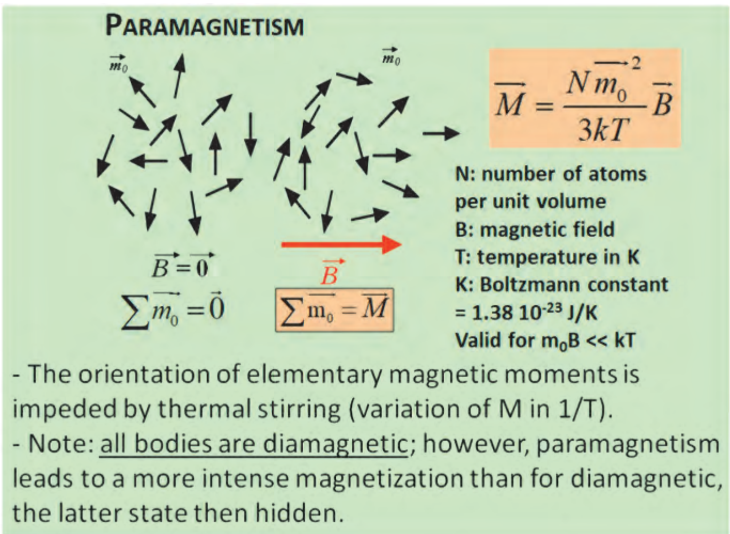


Figure 3.6. *Paramagnetism: magnetic moment as a function of B*

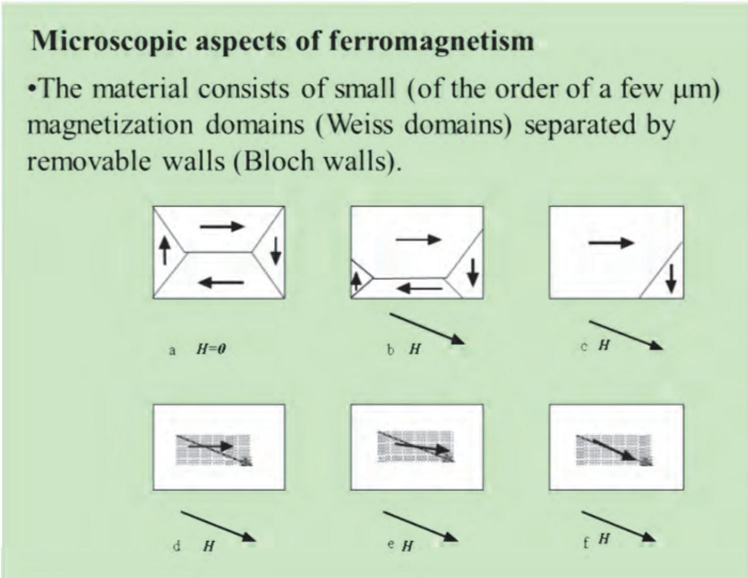


Figure 3.7. *Ferromagnetism: microscopic aspect*

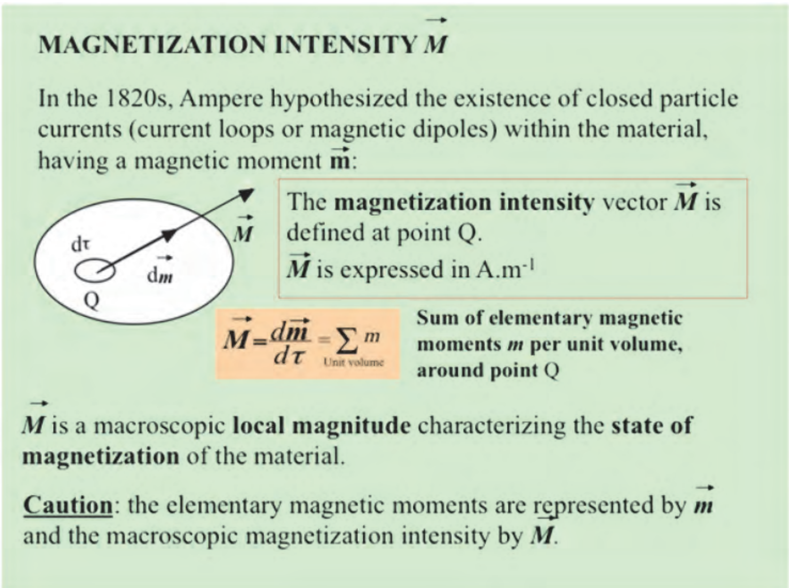
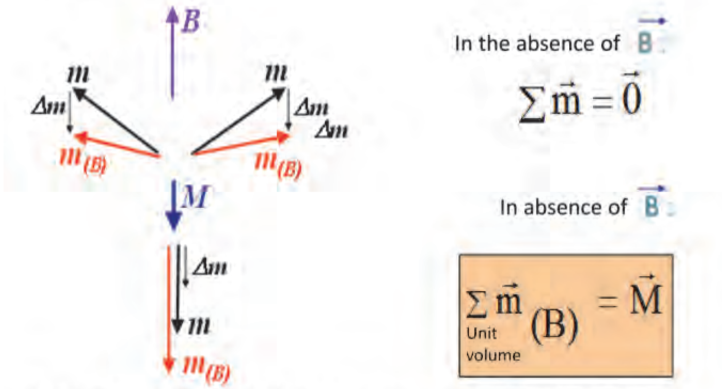


Figure 3.8. *Intensity of magnetization*

The variation ***Δm*** of ***m*** is identical for all electrons in the electron cloud of an atom or molecule.



This results in the appearance of **macroscopic magnetization *M* in a direction opposite to *B***.

Figure 3.9. *Origin of macroscopic magnetization*

We therefore integrate:

$$\overrightarrow{\mathbf{A'}} = \frac{\mu_0}{4\pi} \iiint_{(\tau)} \vec{M} \wedge \overrightarrow{\mathbf{grad}}_Q \left(\frac{1}{r}\right) d\tau$$

the fundamental result of which is:

$$\overrightarrow{\mathbf{A'}} = \frac{\mu_0}{4\pi} \iiint_{(\tau)} \frac{\mathbf{rot}\ \vec{M}}{r} d\tau + \frac{\mu_0}{4\pi} \iint_{(S)} \frac{\vec{M}}{r} \wedge \vec{\mathbf{N}}\, dS$$

The corresponding magnetic field is then :

$\overrightarrow{B'} = \overrightarrow{rot\ A'}$

Figure 3.12. *Calculated vector potential and magnetic field*

By analogy:

$$\vec{\mathbf{A}}' = \frac{\mu_0}{4\pi}\iiint_{(\tau)} \frac{\overrightarrow{\mathbf{rot}}\,\vec{M}}{r}\,d\tau + \frac{\mu_0}{4\pi}\iint_{(S)} \frac{\vec{M}}{r} \wedge \vec{\mathbf{N}}\,dS$$

$$\vec{\mathbf{A}}' = \frac{\mu_0}{4\pi}\iiint_{(\tau)} \frac{\vec{\mathbf{j}}'}{r}\,d\tau + \frac{\mu_0}{4\pi}\iint_{(S)} \frac{\vec{\mathbf{k}}'}{r}\,dS$$

Magnetization currents:

Volume: $\overrightarrow{j'} = \overrightarrow{rot}\,\overrightarrow{M}$

Surface: $\overrightarrow{k'} = \overrightarrow{M} \wedge \overrightarrow{N}$

Figure 3.13. *Magnetization, volume and surface currents*

<u>Conclusion</u>:

The magnetic field created by a magnetized substance is analogous to that which would be due, in a vacuum, to:

- *a distribution of surface currents of magnetization*

  $\vec{k}' = \vec{M} \wedge \vec{N}$

- *a volume velocity distribution of magnetization*

  $\vec{j}' = \overrightarrow{rot}\,\vec{M}$

$\vec{M}$ being the local magnetization intensity of the material.

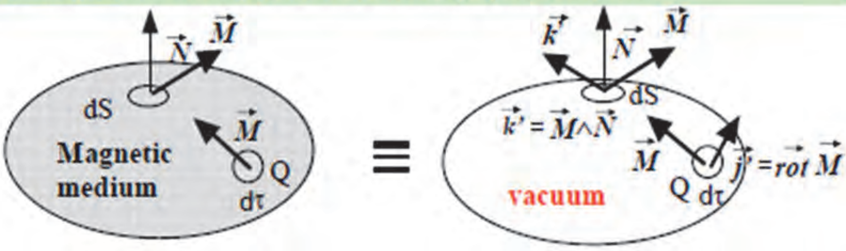


Figure 3.14. *Volumetric and surface magnetization currents*

FUNDAMENTAL PROPERIETES OF THE MAGNETIC FIELD IN THE MATERIAL.

If the total field $\vec{B} = \vec{B'} + \vec{B_0}$ created by **all** true currents created by the **magnetized medium**

$$\vec{\mathbf{A}} = \frac{\mu_0}{4\pi}\iiint_{(\tau)} \frac{\vec{\mathbf{j}} + \vec{\mathbf{rot}}\,\vec{M}}{r}\,d\tau + \frac{\mu_0}{4\pi}\iint_{(S)} \frac{\vec{\mathbf{k}} + \vec{M}\wedge\vec{\mathbf{N}}}{r}\,dS$$

- $\vec{j}$ and $\vec{k}$ being the local densities of conduction currents

Figure 3.15. *The vector potential as a function of magnetization currents*

Flow property:

$\vec{B} = \vec{rot}\,\vec{A}$ ⇒ $\vec{B}$ is a conservative flow

$$\oiint_{(S)} \vec{\mathbf{B}}.\vec{\mathbf{dS}} = \mathbf{0}$$

$$\mathrm{div}\,\vec{B} = 0$$

At the crossing of a surface (S) of discontinuity between two media:

$$\vec{n}.\,(\vec{B}_2 - \vec{B}_1)_{(S)} = 0$$

Continuity of the normal component of $\vec{B}$

$\vec{n}$ normal at (S) oriented from medium 1 to the medium 2

Figure 3.16. *Flow of B in the magnetized material*

Circulation properties of the magnetic field

• In **Ampère's theorem**, we take into account the true currents (of conduction) as well as the linked currents

$$\oint_{(C)} \vec{B}\,\vec{dl} = \mu_0 \iint_{(S)} (\vec{j} + \vec{j'})\,\vec{dS} \qquad \vec{rot}\,\vec{B} = \mu_0\,(\vec{j} + \vec{rot}\,\vec{M})$$

$$\vec{rot}\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \vec{j} \quad \Rightarrow \quad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

Definition of $\vec{H}$: **magnetic excitation**, such as: $\vec{rot}\,\vec{H} = \vec{j}$.
$\vec{H}$ is expressed as $\vec{M}$ in A/m.

Figure 3.17. *H: magnetic excitation*

• **New formulation of Ampère's theorem in the media**:

Local form: $\vec{rot}\,\vec{H} = \vec{j}$

Integral form:

$$\oint_{(C)} \vec{H}.\vec{d\ell} = \sum \text{algebraic of } \mathbf{I} \text{ interleaved conduction by (C)}$$

•The flow of currents I through the closed circuit (C) is counted algebraically (I is counted positively if its direction is that of the normal oriented to a surface (S) relying on (C)).

Figure 3.18. *Local Ampère's theorem for magnetic matter*

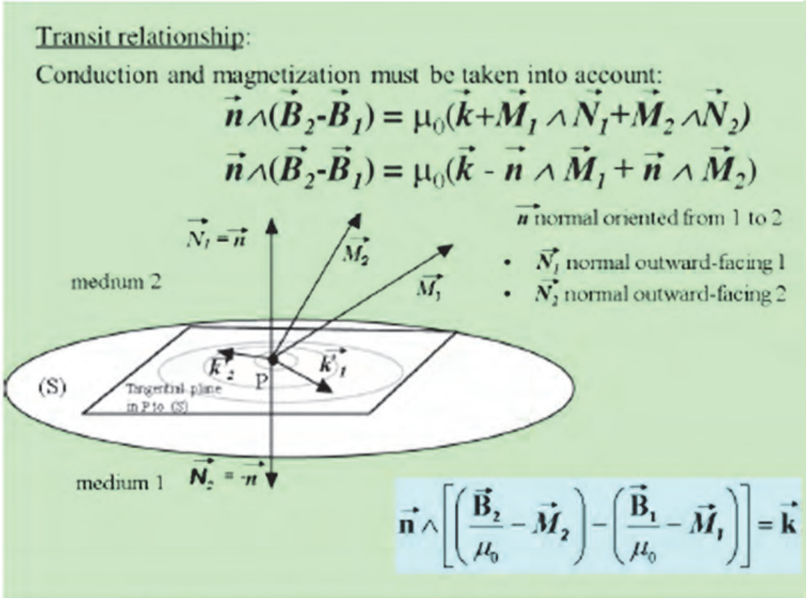


Figure 3.19. *Transit relationships for B tangential*

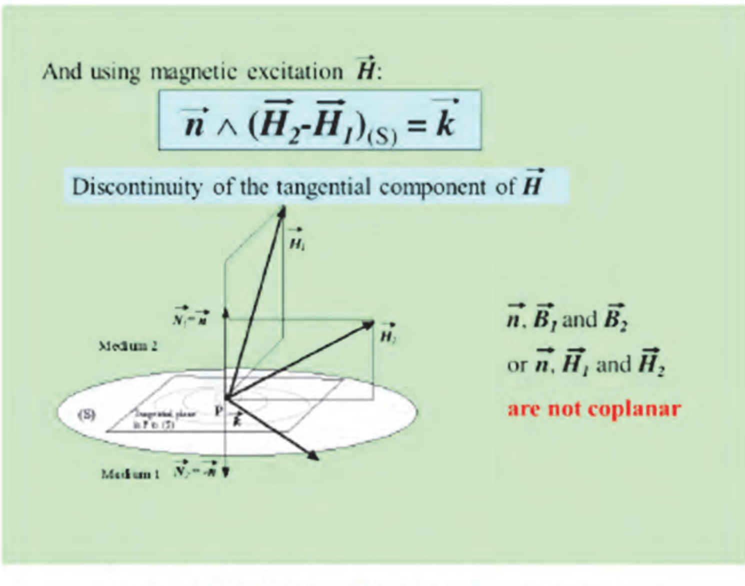


Figure 3.20. *Transit relationships for transverse H*

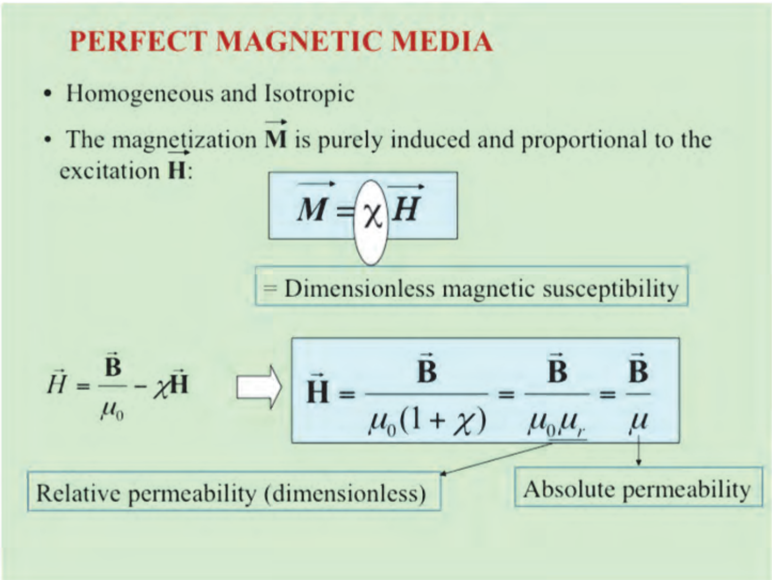


Figure 3.21. *HLI media*

PROPERTIES of VARIOUS MEDIA

Media	χ	Influence of the temperature
Diamagnetic Gaseous N_2 Copper Bismuth Superconductor	 -4×10^{-8} -9.4×10^{-6} -1.7×10^{-4} -1	χ Independent of T Critical T Example: Ba2Cu3O7, Tc = 95 K
Dispersed paramagnetic Gaseous O_2 Air	$+2\times10^{-6}$ $+3.7\times10^{-7}$	$\chi = \frac{C}{T}$ (Curie's law) Positive constant C
Condensed paramagnetic Liquid O_2 Aluminum	$+3.1\times10^{-3}$ $+2.1\times10^{-5}$	$\chi = \frac{C}{T-T_0}$ (Curie-Weiss's law) Real constant To

χ very weak; then: $\mu = \mu_0\mu_r = \mu_0(1+\chi)$ close to μ_0

Table 3.2. *Some perfect media*

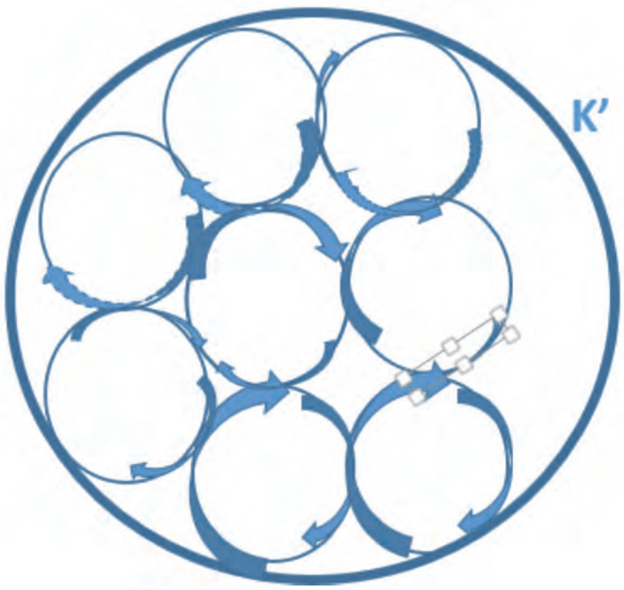


Figure 3.22. *Cancellation of volume currents*

Summary

Vacuum		HLI media	
$div\vec{B} = 0$	$\oiint \vec{B}.\overrightarrow{dS} = 0$	$div\vec{B} = 0$	$\oiint \vec{B}.\overrightarrow{dS} = 0$
$\overrightarrow{rot}\vec{B} = \mu_0 \vec{j}$	$\oint \vec{B}.\overrightarrow{dl} = \mu_0 \sum I_{\text{Inter-leaved}}$	$\overrightarrow{rot}\vec{B} = \mu \vec{j}$	$\oint \vec{B}.\overrightarrow{dl} = \mu \sum I_{\text{Inter-leaved}}$
$\vec{n}.\left(\overrightarrow{B_2} - \overrightarrow{B_1}\right) = 0$		$\vec{n}.\left(\overrightarrow{B_2} - \overrightarrow{B_1}\right) = 0$	
$\vec{n} \wedge \left(\overrightarrow{B_2} - \overrightarrow{B_1}\right) = \mu_0 \vec{k}$		$\vec{n} \wedge \left(\frac{\overrightarrow{B_2}}{\mu_2} - \frac{\overrightarrow{B_1}}{\mu_1}\right) = \vec{k}$	

Table 3.3. *Magnetic field equations for perfect materials and vacuum*

Notes :

Diamagnetic and paramagnetic materials can be considered as perfect materials.

Diamagnetic :
$$\Delta m = -\frac{e_2 \langle r \rangle^2}{4m} \mathbf{B}$$

Magnetization $\vec{M}$ is proportional to $\vec{B}$, so to $\vec{H}$ and of opposite direction to $\vec{B}$

Paramagnetic:
$$\vec{M} = \frac{N \vec{m}_0^{\,2}}{3kT} \vec{\mathbf{B}}$$
Same direction as $\vec{B}$

$$\mu_r = 1 + \chi \quad \begin{cases} > 0 \text{ paramagnetic} \\ < 0 \text{ diamagnetic} \end{cases}$$

Figure 3.23. *On the sign of permeability*

For a **diluted** paramagnetic body, $\vec{M}$ is proportional to $\vec{H}$:

$$\vec{M} = C \frac{\vec{H}}{T} \quad \text{with} \quad C = \frac{N m_0^2}{3k} \qquad \chi = \frac{C}{T}$$

For **condensed** paramagnetic substances, the "molecular" field due to neighboring atoms must be taken into account:

$$\vec{H} \rightarrow \vec{H} + \vec{H}_m = \vec{H} + \lambda \vec{M}$$

$$\vec{M} = C \frac{\vec{H} + \lambda \vec{M}}{T} \quad \text{or} \quad \vec{M} = \frac{C}{T - \lambda C} \vec{H}$$

$$\vec{M} = \frac{C}{T - \theta} \vec{\mathbf{H}} = \chi \vec{\mathbf{H}} \qquad \chi = \frac{C}{T - \theta}$$

Curie temperature

Figure 3.24. *Curie temperature*

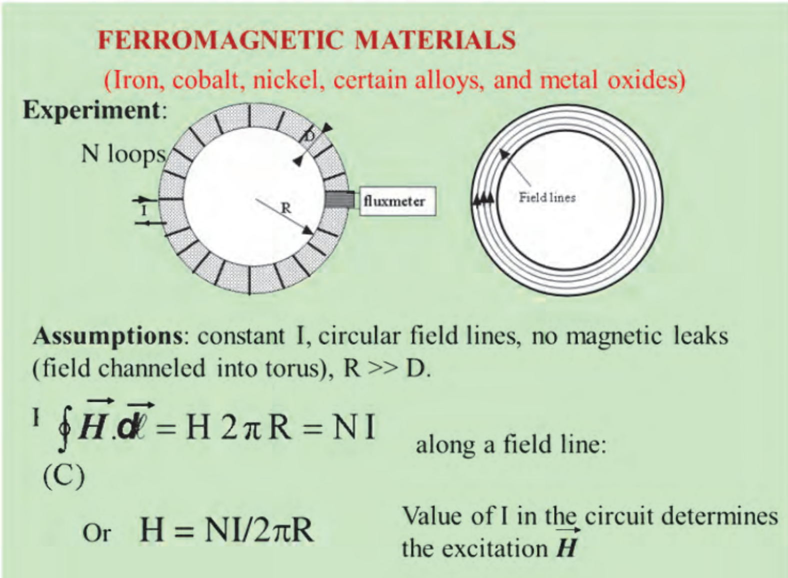


Figure 3.25. *Magnetic excitation in ferromagnetic materials*

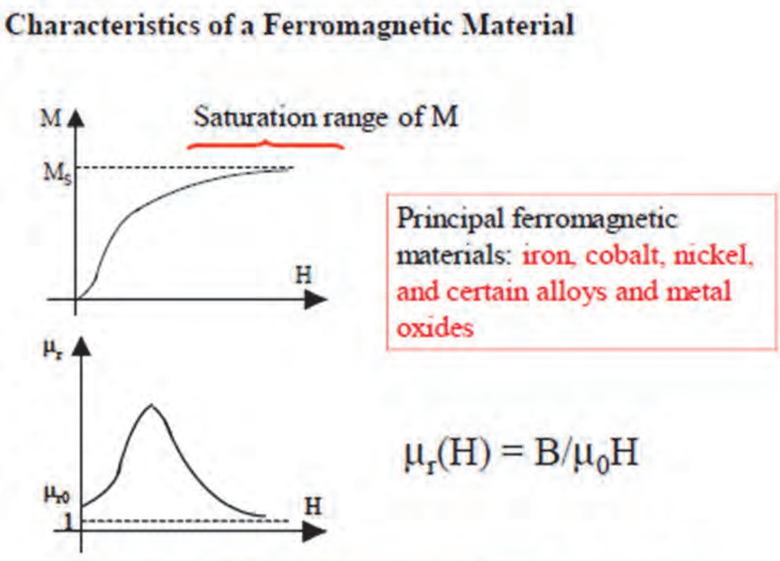


Figure 3.27. *Saturation phenomena in ferromagnetic materials*

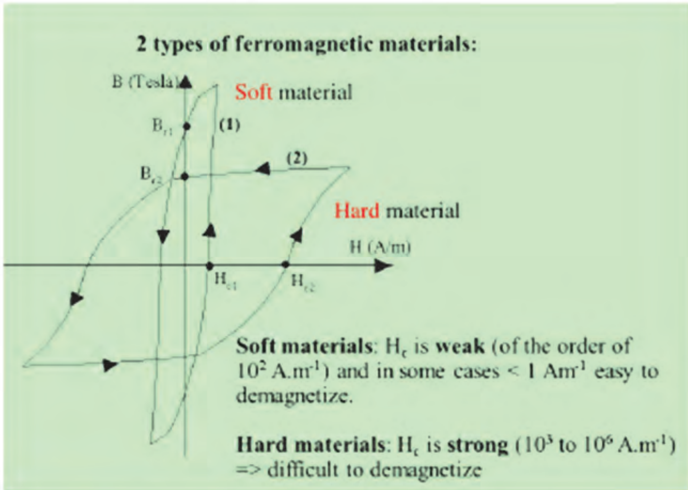


Figure 3.28. *Typology of ferromagnetic materials*

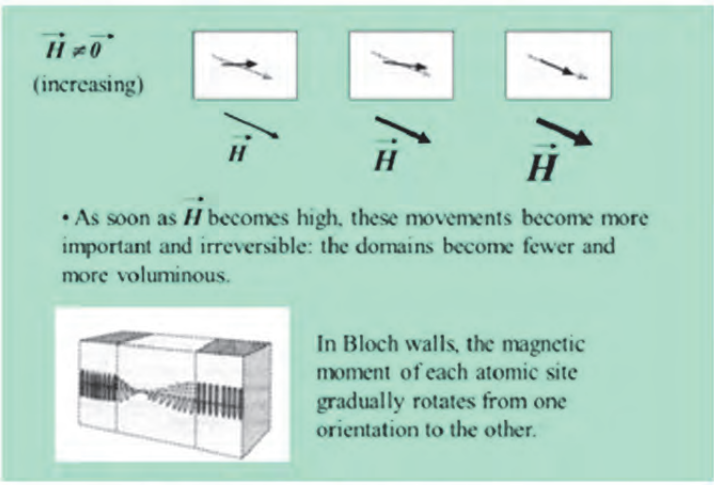


Figure 3.29. *Bloch walls/Weiss domains*

Principal ferromagnetic materials:

Iron, cobalt, nickel, certain alloys and metal oxides

Applications:

Soft materials: transformers, electromagnets, motors

Hard materials: permanent magnets

Figure 3.30. *Some applications of ferromagnetics*

STEADY STATE:

$\boldsymbol{j}, \boldsymbol{k} \longrightarrow \boldsymbol{B}, \boldsymbol{M}, \boldsymbol{H}$

No relationship between ***E*** and ***B***

TIME-VARYING STATES:

$\boldsymbol{j}(\mathrm{t}), \boldsymbol{k}(\mathrm{t}) \longrightarrow \boldsymbol{B}(\mathrm{t}), \boldsymbol{M}(\mathrm{t}), \boldsymbol{H}(\mathrm{t})$??

Is there a relationship between ***E*** and ***B***?

3 new phenomena:

- electromagnetic **induction**
- **propagation** phenomenon
- **capacitive** phenomenon

Figure 4.1. *Current–field relationships, in quasi-steady state*

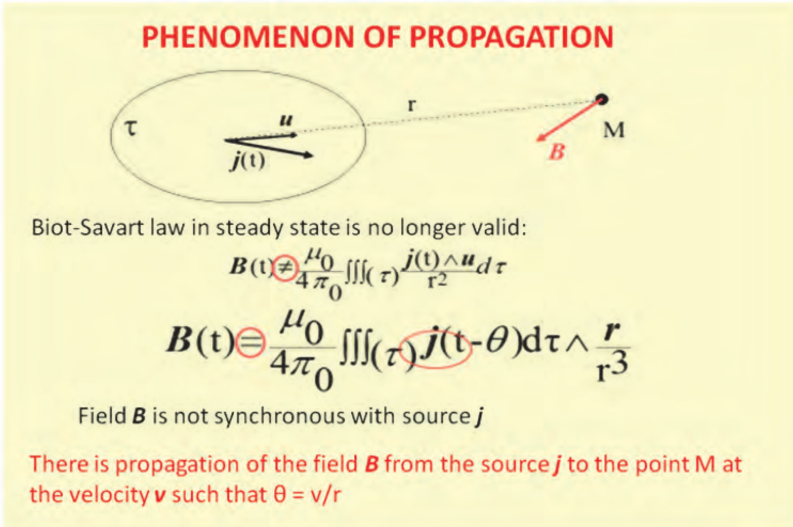


Figure 4.2. *Magnetic field propagation*

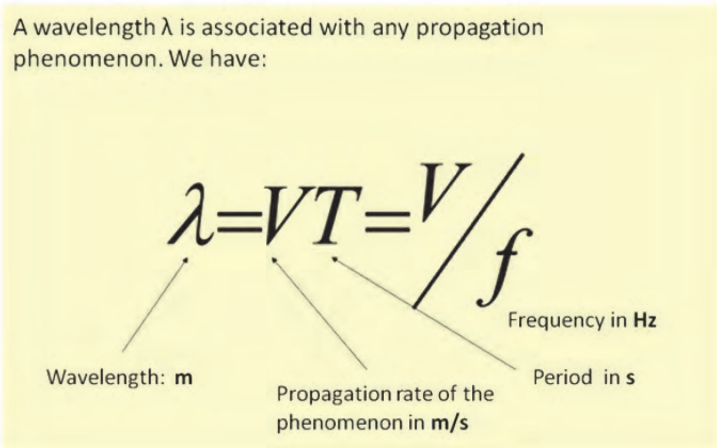


Figure 4.3. *Wavelength versus period, versus frequency*

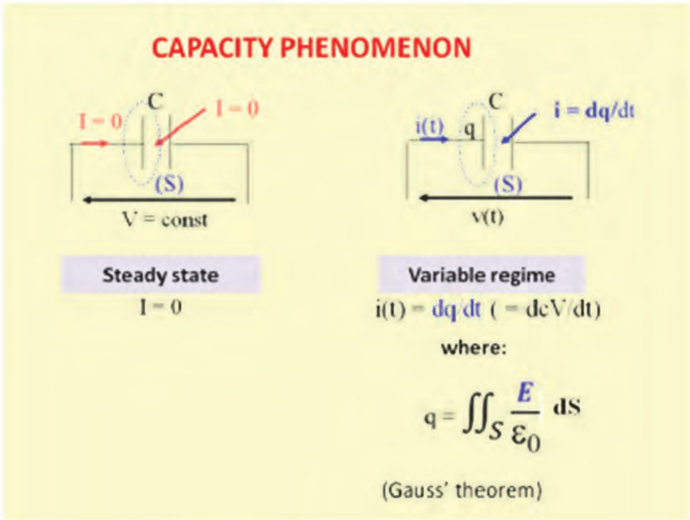


Figure 4.4. *Capacitive effect.*

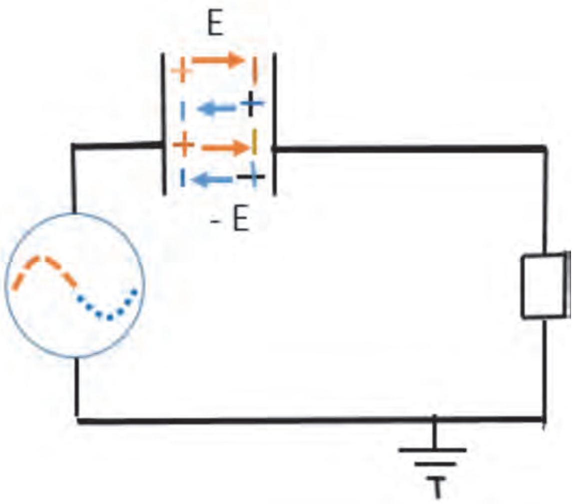


Figure 4.5. *Capacitor in a variable regime; E = const, perpendicular to plate elctrodes, at t given, throughout the capacitor (indefinite)*

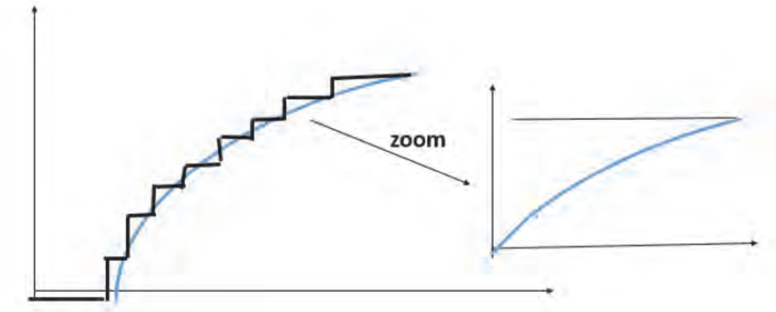


Figure 4.6. *Assimilation of a continuous curve, by a staircase function*

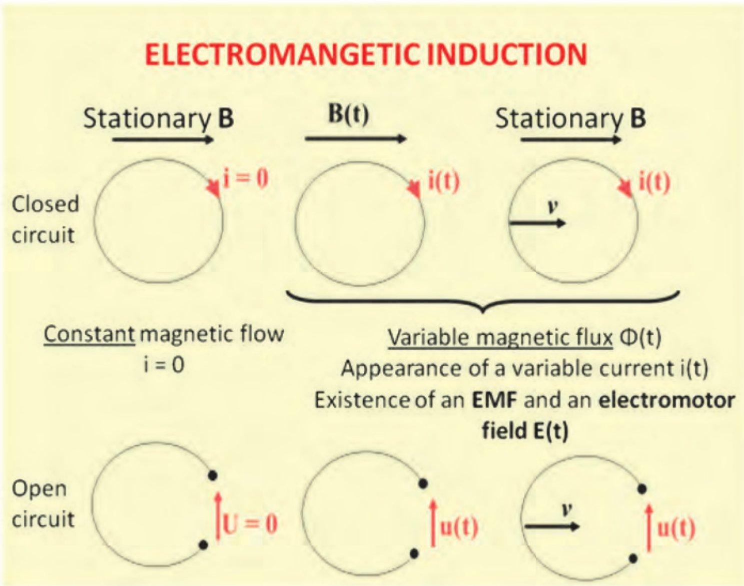


Figure 4.7. *Induction: open and close circuits*

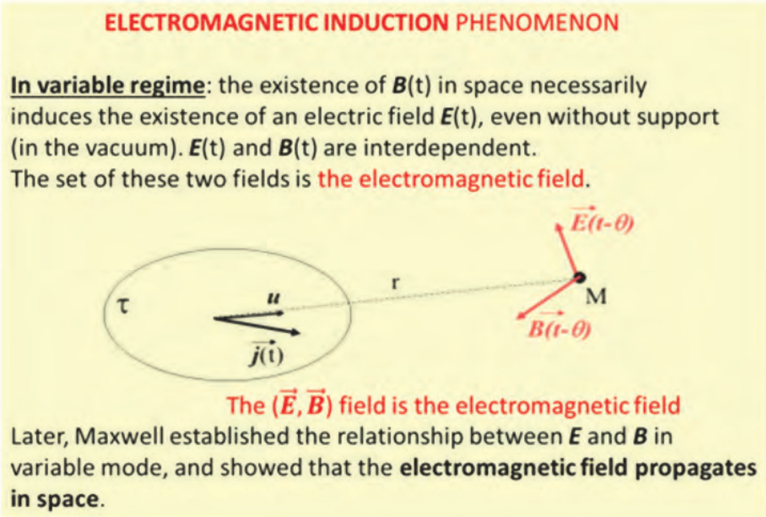


Figure 4.8. *Induction: open and close circuits*

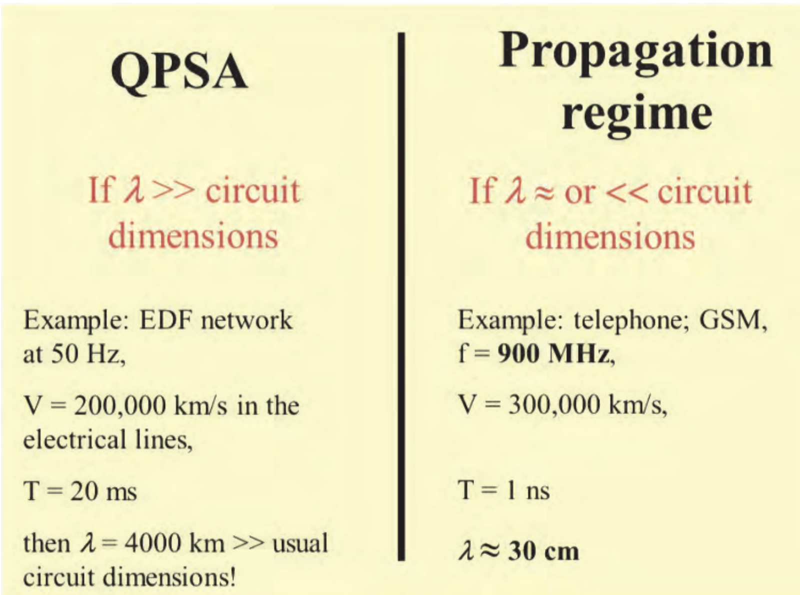


Figure 4.9. *Influence of the wavelength*

Consequences of QPSA:

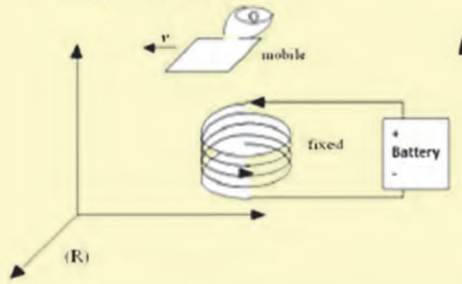
- charges and currents vary "slowly enough" to use the same expressions of $\vec{E}$ and $\vec{B}$ as those established in stationary state
- vector j remains at *conservative* flow, i.e.:

$$\oiint_{(\Sigma)} \vec{j}.d\vec{\Sigma} = 0 \qquad \text{or} \qquad div(\vec{j}) = 0$$

The intensity of the current is everywhere the same in any branch of a circuit. The conduction current lines are closed, except in the case of capacitors that are in QP mode crossed by currents while the current lines are physically cut. But there is the **current** associated **with dE/dt** between reinforcements!

Figure 4.10. *Current density is in a conservative flow*

- **Electromagnetic induction**
  - **Experiments**



Motional induction e

Steady B

A *mobile* or deforming circuit that **cuts the field lines** of a fixed circuit creating a field is the seat of an *induction EMF*.

(unless the field B is uniform and the circuit moves in translation, the flow not varying)

Figure 4.11. *Induction: mobile circuit*

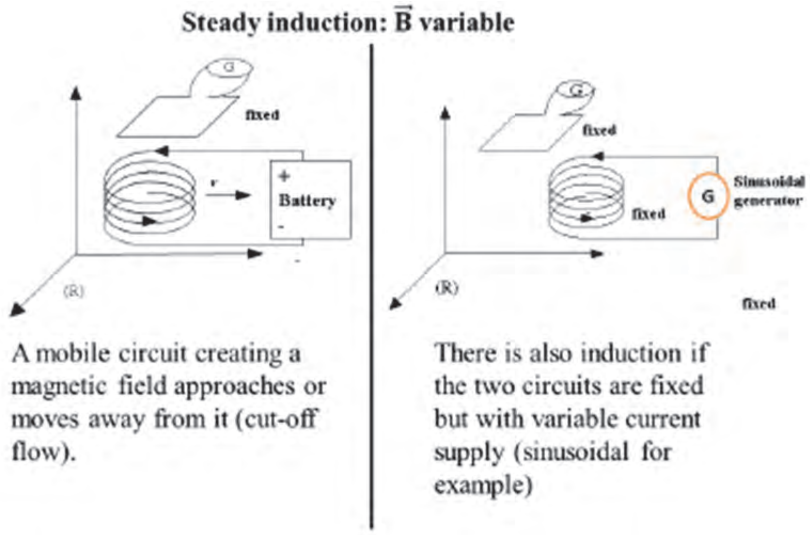


Figure 4.12. *Static induction: B(t)*

Experimental observations:

- the current **induced** in the whorl is all the more intense the higher the speed of movement of the whorl or coil.

- the direction of the current reverses if the displacement changes direction.

- The induced current is all the higher the faster the temporal variations of $\vec{B}$ (high f).

Figure 4.13. *Important experimental observations*

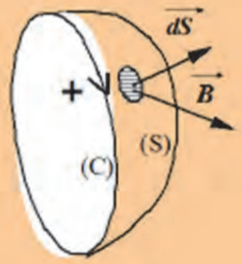
FARADAY-LENZ law

Assumptions:

• closed circuit (C) with a positive direction of orientation,

• oriented normal (corkscrew rule) has any surface (S) resting on the circuit

• If an induced current appears, it means that the charge carriers are subjected to an electromotive force e(t).



e(t) as a function of Φ(t) ⟹ **Faraday-Lenz law**

Figure 4.14. *Towards the Faraday–Lenz law*

• It is possible to induce in the circuit (C) an EMF e(t) by varying the flow Φ of the magnetic field B through (C) by:

• Flow of the magnetic field : $\Phi = \iint_{(S)} \vec{\mathbf{B}}.\vec{\mathbf{dS}}$

• a displacement of the circuit (C) with respect to the external system creating the magnetic field or a deformation of the circuit (*motional or Lorentz induction*)

• A variation of the magnetic field over time (fixed inductor $\boldsymbol{B}$(t) or mobile inductor), the circuit (C) remaining fixed (*static or Neumann induction*).

Faraday-Lenz Law:

$$\boxed{e(t) = -\frac{d\Phi}{dt}}$$

V Wb s

Figure 4.15. *Faraday–Lenz law*

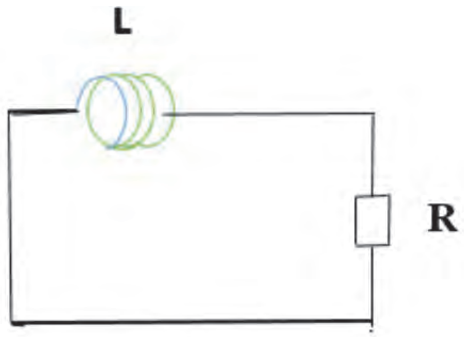
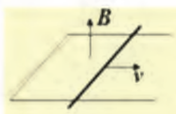


Figure 4.16. *Possible induction in an "RL" circuit*

Important notes:

1) *The Faraday-Lenz law is valid for any closed, fixed, or mobile circuit in the considered coordinate system,*

provided that it is of ***constant material constitution****.*



2) The Lenz-Faraday's law is algebraic*.*

If e > 0, it will generate a current in the positive direction chosen on the circuit, if e < 0, in the opposite direction.

The sign in the Faraday-Lenz law translates the fact that the effects of the induced current will be such that they will oppose the causes that gave rise to it:

- If the circuit is stationary, this current will itself create a magnetic field such that its flow through the circuit opposes the variation of the inductor flow.
- If the circuit is mobile, Ampère/Laplace forces due to this current will oppose the displacement of the circuit.

Figure 4.17. *Some comments on Lenz–Faraday's law*

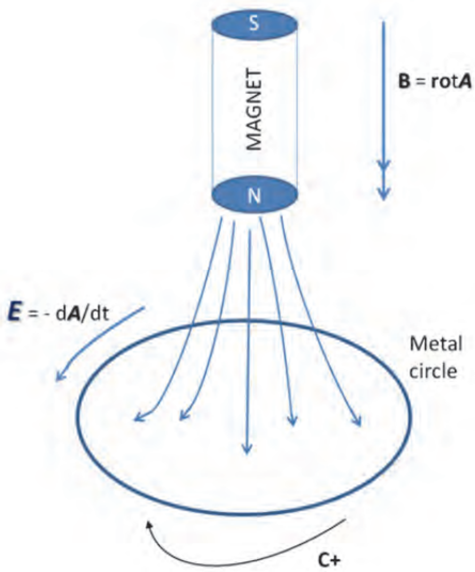


Figure 4.18. *Magnetic induction*

CAUSE OF FLOW VARIATION

Motional induction (Lorentz)

$$e = -\left(\frac{\partial \Phi}{\partial x}\frac{dx}{dt} + \frac{\partial \Phi}{\partial y}\frac{dy}{dt} + \frac{\partial \Phi}{\partial z}\frac{dz}{dt} + \ .\right)...$$

The variation of the flow is due to a displacement of the induce relative to the inductor.

Static induction (Neumann)

$$e = -\frac{d\Phi}{dt}$$

•the inductor field ***B***(t) varies, the induced circuit remaining fixed:

• either the inductor moves,

• or the inductor is supplied by variable currents

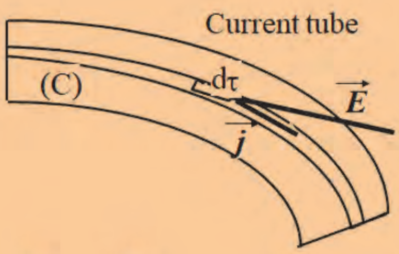
Figure 4.19. *Static or Neumann induction and motional or Lorentz induction*

Induced electromotive force

- We will first show that e(t) is equal to the circulation of the electric field $\vec{E}$ in the circuit.

- $\vec{v}_i$ is the velocity of momentum of the charge carriers in the conductor and q their load

- In a conductor subjected to a magnetic field, $\vec{j}$ and $\vec{E}$ are non-collinear



$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}_{\mathbf{i}} \wedge \vec{\mathbf{B}}\right) = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$$

$$\vec{\mathbf{F}}_1 = q\,\vec{\mathbf{E}} \qquad \vec{\mathbf{F}}_2 = q\,\vec{\mathbf{v}}_{\mathbf{i}} \wedge \vec{\mathbf{B}}$$

Figure 4.21. *Electric and Lorentz forces*

- The energy dW provided by the system producing the variable electromagnetic field to a charge **carrier** during time dt is:

$$dW = \vec{\mathbf{F}}.\vec{\mathbf{v}}_{\mathbf{i}}\,dt = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}_{\mathbf{i}} \wedge \vec{\mathbf{B}}\right).\vec{\mathbf{v}}_{\mathbf{i}}\,dt = q\,\vec{\mathbf{E}}.\vec{\mathbf{v}}_{\mathbf{i}}\,dt$$

For a volume dτ of conductor comprising **N charge carriers** per unit volume, the total work provided will be:

$$dW = N\,d\tau\,q\,\vec{\mathbf{E}}.\vec{\mathbf{v}}_{\mathbf{i}}\,dt = (N\,q\,\vec{\mathbf{v}}_{\mathbf{i}})\vec{\mathbf{E}}\,d\tau\;dt = \vec{\mathbf{j}}.\vec{\mathbf{E}}\,d\tau\,dt$$

- dτ being a current tube element, it is possible to write, with $\vec{u}$ being a unit vector carried by the current line:

$$\vec{j} = \mathrm{j}.\vec{u} \quad , \quad d\tau = \vec{dS}.\vec{u}\,d\ell$$

Figure 4.22. *Work of electromagnetic forces*

$$dW = j(\vec{\mathbf{u}}\cdot\vec{\mathbf{E}})\cdot(\mathbf{d}\vec{\mathbf{S}}\cdot\vec{\mathbf{u}})\cdot d\ell\cdot dt = d\ell\cdot(\vec{\mathbf{u}}\cdot\vec{\mathbf{E}})\cdot(j\cdot\mathbf{d}\vec{\mathbf{S}}\cdot\vec{\mathbf{u}})\cdot dt$$

$$dW = \mathbf{d}\vec{\ell}\cdot\vec{\mathbf{E}}\cdot di\cdot dt$$

(di being the elemental intensity carried by the current tube)

The total energy dW supplied during time dt to the entire closed circuit (C) constituted by the current tube will therefore be:

$$\delta W = di\cdot dt\oint_{(C)}\vec{\mathbf{E}}\cdot\mathbf{d}\vec{\ell}$$

This means that an EMF is produced in the closed circuit such that:

$$\delta W = e_{(t)}\,di\,dt$$

$$e_{(t)} = \oint_{(C)}\vec{\mathbf{E}}\cdot\mathbf{d}\vec{\ell} = \oint_{(C)}\frac{\vec{F}_1}{\mathrm{q}}\cdot\mathbf{d}\vec{\ell}$$

•Non-zero circulation of $\boldsymbol{E}$ along the closed circuit (C)

•The origin of e(t) lies in the energy supplied by the EM field, via the driving force exerted on the charge carriers

Figure 4.23. *Electromotive force*

• **Local form of the Maxwell-Faraday relationship (deduced from Stockes' theorem):**

Integral form:

$$\oint_{(\Gamma)}\vec{\mathbf{E}}\cdot\mathbf{d}\vec{\lambda} = -\iint_{(S)}\frac{\partial\vec{\mathbf{B}}}{\partial \mathrm{t}}\cdot\mathbf{d}\vec{\mathbf{S}}$$

Based on Stokes' theorem!

$$\oint_{(\Gamma)}\vec{\mathbf{E}}\cdot\mathbf{d}\vec{\lambda} = \iint_{(S)} rot\vec{E}\cdot\mathbf{d}\vec{\mathbf{S}}$$

Maxwell-Faraday relationship in its local form

$$\overrightarrow{\mathbf{rot}}\,\overrightarrow{\mathbf{E}} = -\frac{\overrightarrow{\partial\mathbf{B}}}{\partial t}$$

Figure 4.24. *Local form of Lenz's law*

– Electromagnetic field E – Electric induction field E_i

Using the magnetic vector potential $\boldsymbol{A}$ such that $\boldsymbol{B} = \boldsymbol{rot}\,\boldsymbol{A}$:

$$e_{(t)} = \oint_{(C)} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\ell} = -\frac{d}{dt}\iint_{(S)} \vec{\mathbf{B}} \cdot \mathbf{d}\vec{\mathbf{S}} = -\frac{d}{dt}\iint_{(S)} \mathbf{r\vec{o}t}\,\vec{\mathbf{A}} \cdot \mathbf{d}\vec{\mathbf{S}} = -\frac{d}{dt}\oint_{(C)} \vec{\mathbf{A}} \cdot \mathbf{d}\vec{\ell}$$

or:

$$e_{(t)} = \oint_{(\Gamma)} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\lambda} = \oint_{(\Gamma)} -\frac{\partial \vec{\mathbf{A}}}{\partial t} \cdot \mathbf{d}\vec{\lambda}$$

•This does not generally mean that because $E = -\frac{\partial A}{\partial t}$ can be the sum of several fields, some of which are conservative circulation.

Figure 4.25. *Influence of the Neumann vector (- dA/dt)*

The electromagnetic field $\boldsymbol{E}$ therefore consists of two terms:

$$\vec{\mathbf{E}} = \underbrace{-\mathbf{gr\vec{a}d}(V)}\ \underbrace{-\frac{\partial \vec{\mathbf{A}}}{\partial t}}$$

Component deduced from Coulombian expression of the field created by charges but where the distribution of these charges is variable over time. Its circulation is conservative.

•**The induction electric field $\mathbf{E_i}$** induced by the temporal variations of B and therefore of A. This field has a non-conservative circulation. This component is the only one that appears in the presence of only B variable.

In case of no charge: $\vec{E} = \vec{\mathbf{E}}_i = -\frac{\partial \vec{\mathbf{A}}}{\partial t}$

In case of steady state with charges: $\vec{E} = -\overrightarrow{\mathrm{grad}\mathrm{V}}$

$$\vec{\mathbf{E}}_i = -\frac{\partial \vec{\mathbf{A}}}{\partial t}$$

Figure 4.26. *Electromagnetic field and potentials*

• **Mobile circuit in an electromagnetic field**

•**General expression of induced e.m.f**

•Assumptions:

• mobile conductor in the coordinate system (R) with velocity $\vec{v}$

• and subjected to the electromagnetic field $(\vec{E}, \vec{B})$;

• $\vec{v}_i$ is the velocity of momentum of the charge carriers in the conductor relative to the conductor.

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + (\vec{\mathbf{v}} + \vec{\mathbf{v}}_{\mathbf{i}}) \wedge \vec{\mathbf{B}}\right) = \vec{\mathbf{F}}_1' + \vec{\mathbf{F}}_2'$$

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \wedge \vec{\mathbf{B}}\right) + q\left(\vec{\mathbf{v}}_{\mathbf{i}} \wedge \vec{\mathbf{B}}\right) = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$$

$$e_{(t)} = \oint_{(C)} \frac{\vec{\mathbf{F}}_1'}{q} . \mathbf{d}\vec{\ell} = \oint_{(C)} (\vec{\mathbf{E}} + \vec{\mathbf{v}} \wedge \vec{\mathbf{B}}) . \mathbf{d}\vec{\ell}$$

Figure 4.27. *Fields; mobile under B*

– Electromagnetic field – electromotive field

The electromagnetic field **E** in the reference frame (R) of the laboratory is always defined by:

$$\vec{\mathrm{E}} = -grad\,\mathrm{V} - \frac{\partial \vec{\mathrm{A}}}{\partial t} = -grad\,\mathrm{V} + E_i$$

$$e(t) = \oint_{(\Gamma)} E_i . \mathrm{d}\lambda + \oint_{(\Gamma)} (\mathbf{v} \wedge \mathbf{B}) . \mathrm{d}\lambda$$

The quantity **v^B** which has the dimension of an electric field is called the **electromotor field $\mathbf{E_m}$**. This quantity is not a constituent part of the EM field in the laboratory coordinate system (R).

Figure 4.28. *Induction and electromotor fields*

Finally, the **induction emf** e(t) can be written in the form:

$$e(t) = \oint_{(\Gamma)} E_i . d\lambda + \oint_{(\Gamma)} E_m . d\lambda$$

With, in V/m) :

$$\vec{\mathbf{E}}_i = - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

$$\vec{E}_m = \vec{v} \wedge \vec{B}$$

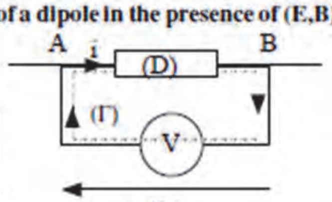
Figure 4.29. *Associated EMF, induction and electromotor fields*

Special cases and comment…

a) Measurement of the voltage at the terminals of a dipole in the presence of (E,B)

$$U_{AB} = \int_{AVB} \vec{\mathbf{E}} . \vec{\mathbf{d}\boldsymbol{\lambda}}$$

field to which the constituent elements of the apparatus are subjected.



On the *closed* circuit (Γ): $\oint_{(\Gamma)} \vec{\mathbf{E}} . \vec{\mathbf{d}\boldsymbol{\lambda}} = -\frac{d\Phi}{dt} = -U_{AB} + \int_{ADB} \vec{\mathbf{E}} . \vec{\mathbf{d}\boldsymbol{\lambda}}$ U_{AB}

$$\boxed{-U_{AB} = \int_{BVA} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\boldsymbol{\lambda}}}$$

But, for the resistance resistor r:

$$\int_{ADB} \vec{\mathbf{E}} . \vec{\mathbf{d}\boldsymbol{\lambda}} = r i$$

Therefore: $$U_{AB} = r i + \frac{d\Phi}{dt}$$

Φ being the flow of B through (G) oriented in the direction indicated by the figure…

Figure 4.30. *Pouillet's law*

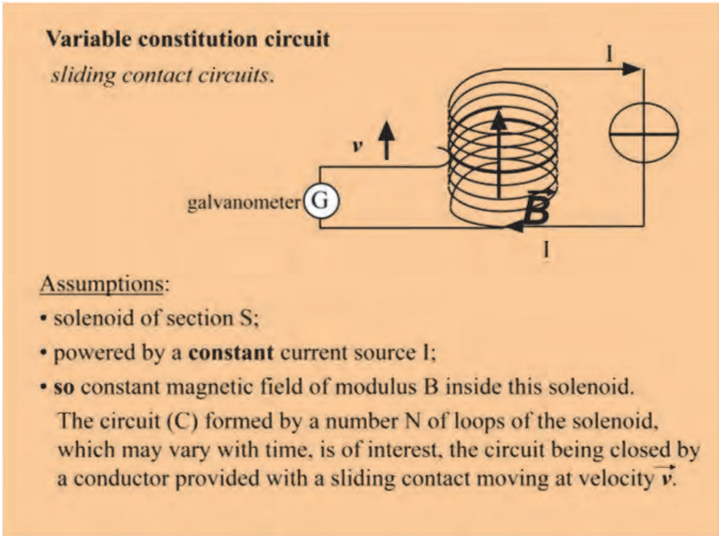


Figure 4.31. *A typical circuit*

Total entwined flow by (C): $\Phi = N(t).B.S$

Be careful not to hastily (wrongly) apply the Faraday-Lenz relationship:

$$\xcancel{e = -\frac{d\Phi}{dt} = -\frac{dN(t)}{dt} B \cdot S}$$

…because B is not variable and the moving part of the circuit moves in an area where B is zero. The circuit is not of fixed constitution.

…so $$e(t) = -\iint_{(S)} \frac{\partial \vec{\mathbf{B}}}{\partial t} \cdot \mathbf{d}\vec{\mathbf{S}} + \oint_{(\Gamma)} (\vec{\mathbf{v}} \wedge \vec{\mathbf{B}}) \cdot \mathbf{d}\vec{\lambda} = 0$$

Figure 4.32. *A typical circuit (continued)*

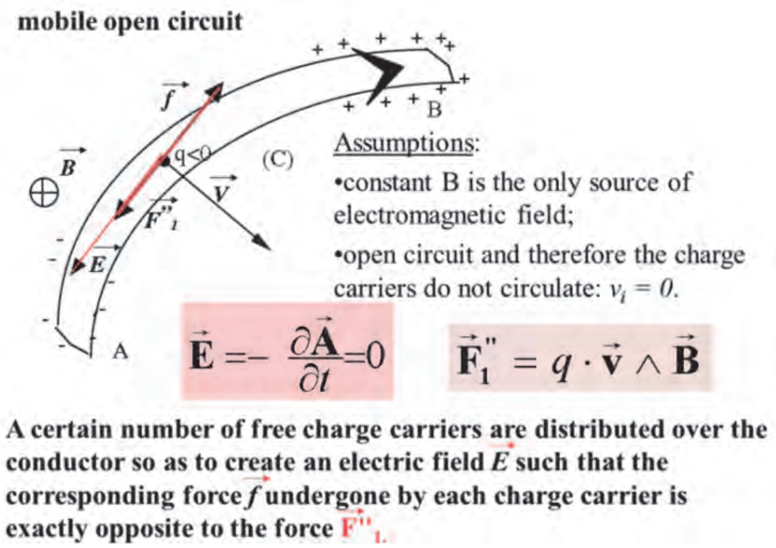


Figure 4.33. *Mobile open circuit*

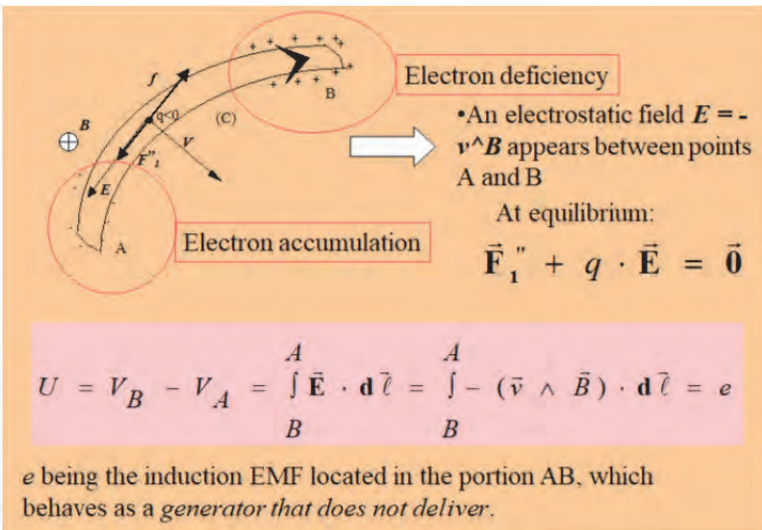


Figure 4.34. *Mobile open circuit (continued and end)*

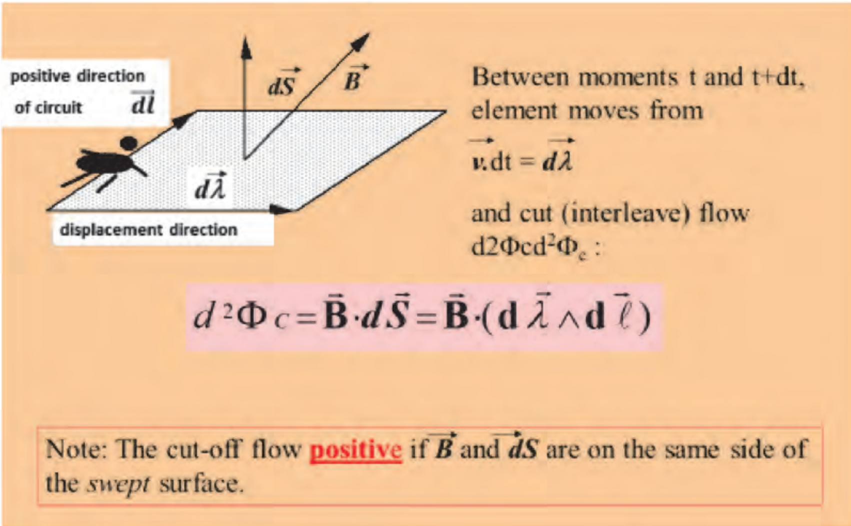


Figure 4.35. *Cut-off flow*

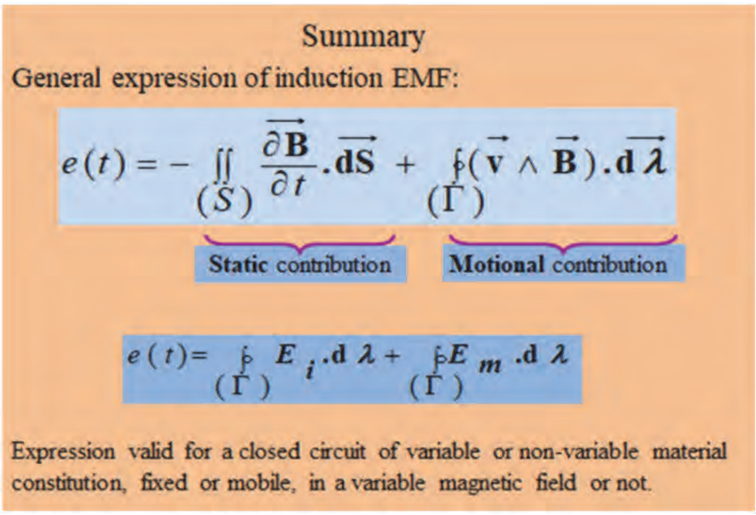


Figure 4.37. *EMF in a circuit induced by flow variation*

specific cases:

CLOSED CIRCUIT

- *Static* or *Neumann* induction (*function of time!*): fixed circuit in a variable magnetic field

$$e(t) = -\underbrace{\iint_{(S)} \frac{\partial \vec{\mathbf{B}}}{\partial t} . \vec{\mathbf{dS}}}_{\text{Static contribution}}$$

Figure 4.38. *EMF function of B(t) variant*

Motional or Lorentz induction:
Mobile circuit in a stationary magnetic field

$$e(t) = \underbrace{\oint_{(\Gamma)} (\vec{\mathbf{v}} \wedge \vec{\mathbf{B}}) . \mathbf{d}\vec{\lambda}}_{\text{Motional contribution}}$$

Figure 4.39. *Motional (Lorentz) EMF*

MOBILE OPEN CIRCUIT IN A STATIONARY FIELD

$$e(t) = \underbrace{\int_{(C)} (\overrightarrow{\mathbf{v}} \wedge \overrightarrow{\mathbf{B}}).\mathbf{d}\vec{\lambda}}_{\text{Motional contribution}} = -\frac{d\Phi_C}{dt}$$

potential difference appearing
at the terminals of the circuit

Figure 4.40. *EMF: mobile open circuit*

Property	*Integral formualism*	*Local formualism*	*Passing equation*
Flow	$\oiint_{(S)} \boldsymbol{B}.\boldsymbol{dS} = 0$	$\mathrm{div}\, B = 0$	$\boldsymbol{n}(\boldsymbol{B_2} - \boldsymbol{B_1})_{(S)} = 0$
Circulation	$\oint H \,.\, d\lambda = \mu_0 \sum_{\text{algebraic}} \mathrm{I}_{\text{[illegible]}}$	$\boldsymbol{rot}\ \boldsymbol{H} = \boldsymbol{j}$	$\boldsymbol{n} \wedge (\boldsymbol{H_2} - \boldsymbol{H_1})_{(S)} = \boldsymbol{k}_{(S)}$

Medium relation: $\vec{\mathrm{H}} = \dfrac{\vec{\mathbf{B}}}{\mu_0} - \vec{\mathbf{M}}$

Figure 4.41. *Flow, circulation: result*

- **Maxwell's equations in quasi-permanent regime**

- For the magnetic field

Integral and local equations are all the same as in stationary states, since the approximation of quasi-permanent states amounts to conceding that in each moment, the system, although subject to quantities variable in time, is in a state of equilibrium.

Figure 4.42. *Maxwell's equations: result*

- **For the electric field**

Flow relationships remain unchanged, for the same reasons as those mentioned for quantities $\vec{B}$ and $\vec{H}$. Relationships concerning the circulation of $\vec{E}$ integrate the phenomenon of electromagnetic induction. However, the relationship concerning transit equations remains unchanged

Figure 4.43. *Maxwell's equations: result (continued)*

We write: $\phi = \iint_{(S)} \boldsymbol{B}.d\boldsymbol{S} = \mathrm{Li}$

L is called the self inductance of the circuit (**C**).

Let $\mathrm{L} = \frac{\phi}{\mathrm{i}}$

L is expressed in Henry (**H**).

L is an *always positive* quantity, characteristic of the circuit (C), depending only on the geometric parameters of the circuit and the permeability μ of the medium in which it is located.

Figure 4.44. *Definition of inductance*

Self-induction electromotive force:

Circuit of constant material constitution (Faraday-Lenz law), rigid (constant L), in a perfect medium:

$$e(t) = -\frac{d\Phi}{dt} = -\frac{d(Li)}{dt}$$

Or :

$$\boxed{e(t) = -L\frac{di}{dt}}$$

- If i increases, di/dt>0, e<0: the EMF induction opposes the EMF of the flow generator i and tends to decrease i.
- If i decreases, cdi/dt<0, e>0: the EMF induction is added to the EMF of the flow generator i and tends to increase i.
- If i is constant, di/dt = 0 is e = 0, no induction.

Figure 4.46. *Self-induction: EMF*

Φ_{12} flux created by (C_1) and cut-off by the n_2 loops of (C_2):

$$\Phi_{12} = n_2\varphi_{12} = n_2\iint_{(S_2)} B_1 . dS_2 \quad \text{proportional to } i_1$$

We write: $\Phi_{12} = M_{12} i_1$

Likewise

Φ_{21} flux created by (C_2) and cut off by the n_1 loops of (C_1):

$$\Phi_{21} = n_1\varphi_{21} = n_1\iint_{(S_1)} B_2 . dS_1 \quad \text{proportional to } i_2$$

We write: $\Phi_{21} = M_{21} i_2$

Figure 4.48. *Mutual induction; coupling coefficient*

•We let (C_1) remain fixed and we move (C_2) away to infinity at constant currents: Work T_2 of magnetic forces undergone by (C_2):
$T_2 = i_2\,(0 - \Phi_{12}) = -\,i_2\,\Phi_{12} = -M_{12}\,i_1\,i_2$

•We let (C_2) remain fixed and we move (C_1) away to infinity at constant currents: Work T_1 of magnetic forces undergone
•$T_1 = i_1\,(0 - \Phi_{21}) = -\,i_1\,\Phi_{21} = -M_{21}\,i_1\,i_2$

•The relative displacement of the 2 circuits being the same, the principle of equality of action and reaction leads to the equality of works T_1 and T_2, namely:

$$M_{12} = M_{21} = M$$

•M is the mutual inductance of the 2 circuits.
•M is expressed in Henry (H).
•M depends only on the shape, the relative position of the 2 circuits and the medium.
•M is positive if the mutual induction flow is added to the eigenflow.

Figure 4.49. *Mutual induction (end)*

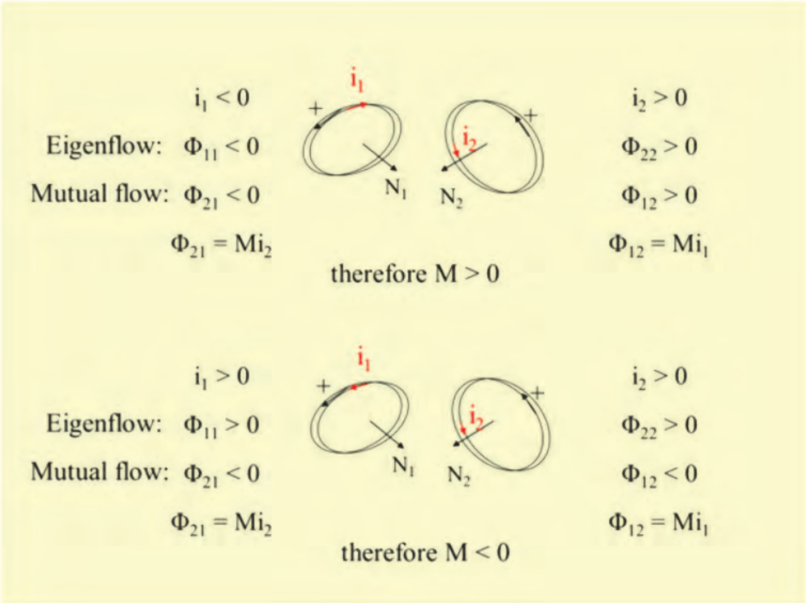


Figure 4.50. *Mutual: its sign depends on the winding direction of wires*

– **Total electromotive force of induction (eigenflow + mutual)** Rigid circuits, perfect media

<u>Mutual induction EMF</u>:

If i_2 varies, in (C_1): $e_{21} = -\frac{d\Phi_{21}}{dt} = -M\frac{di_2}{dt}$

If i_1 varies, in (C_2): $e_{12} = -\frac{d\Phi_{12}}{dt} = -M\frac{di_1}{dt}$

<u>Mutual induction EMF are added to the EMF eigen inductions</u>.

If i_1 and i_2 vary:

In (C_1): $e_1 = e_{11} + e_{21} = -L\frac{di_1}{dt} - M\frac{di_2}{dt}$

In (C_2): $e_2 = e_{22} + e_{12} = -L\frac{di_2}{dt} - M\frac{di_1}{dt}$

Figure 4.51. *Total EMF*

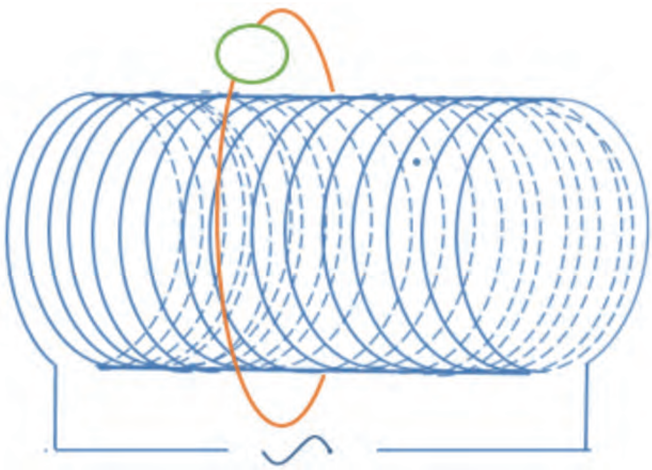


Figure 4.54. *A solenoid inducing a current in a loop surrounding it*

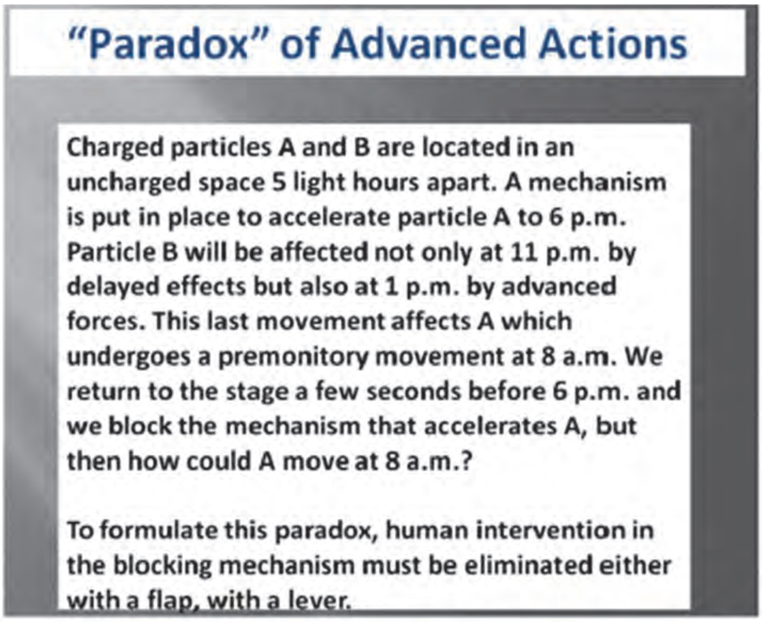
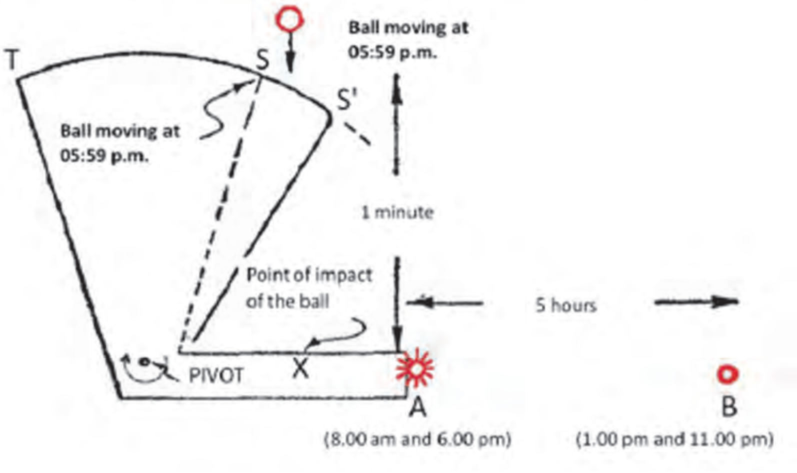


Figure 5.1. *Paradox of advanced effects (R. Feynman)*



To solve this problem, we divide it into two parts: the effect of A's past on its future and of the future on its past. There is no solution because the action of the future on the past was supposed to be discontinuous. However, the influence of the future on the past depends continuously on the configuration of the future. There is then a solution, where the 2 curves intersect, of coordinates: Vc optimal: Valve Open and Closed.

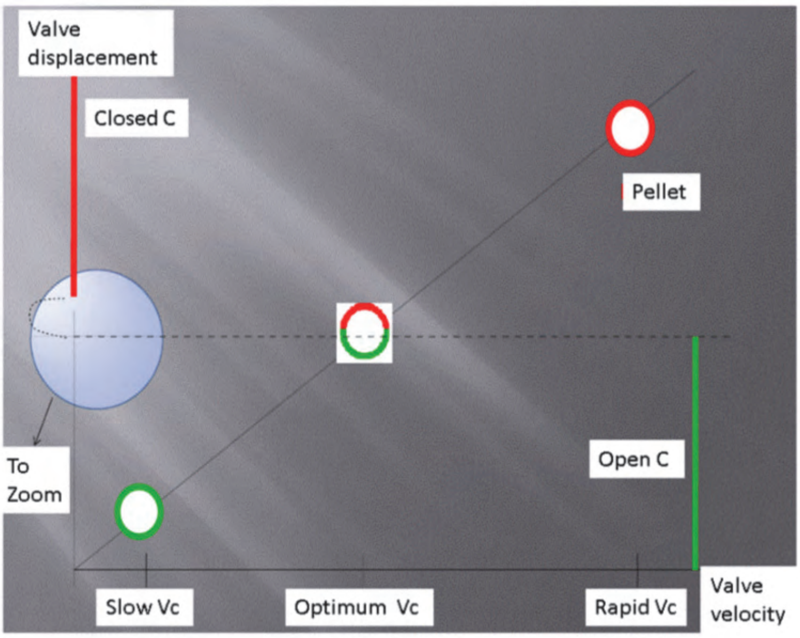


Figure 5.2. *Advanced effects: the action of the shutter on the pellet – interaction of the past and the future – is continuous (dotted line) and the action and reaction curves intersect*

1 minute before 6 p.m., A received a counterclockwise shock and a stronger shock in the opposite direction at 6 p.m.

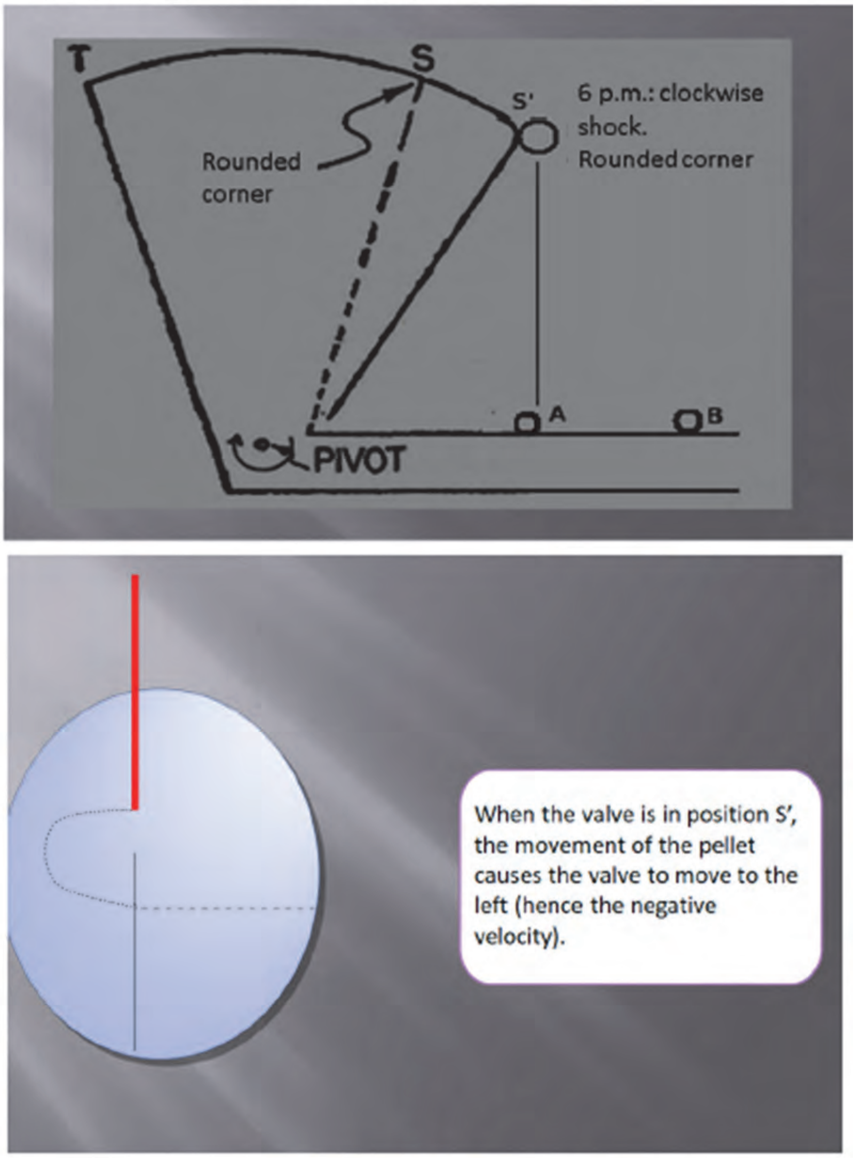


Figure 5.3. *Zoom in on Figure 5.2*

Accelerations received by A at these two moments are transmitted, in attenuated form, to B at 1 p.m., and again to A in an even more attenuated manner.

So A receives a clockwise shock one minute before 8 a.m. and at 8 a.m., a pulse in the opposite direction. The resulting rotational momentum is in the increasing clockwise direction. The chain of action and reaction is therefore resolved.

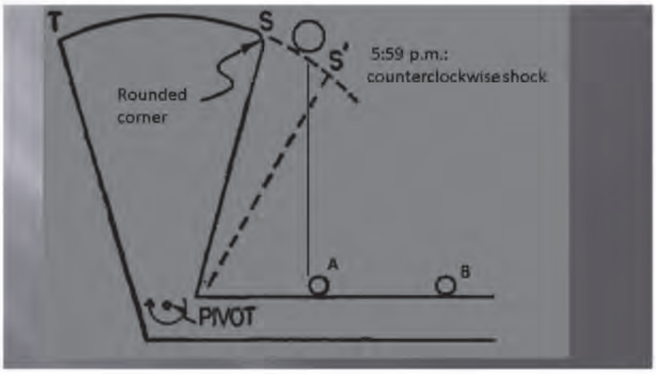


Figure 5.4. *Action–reaction*

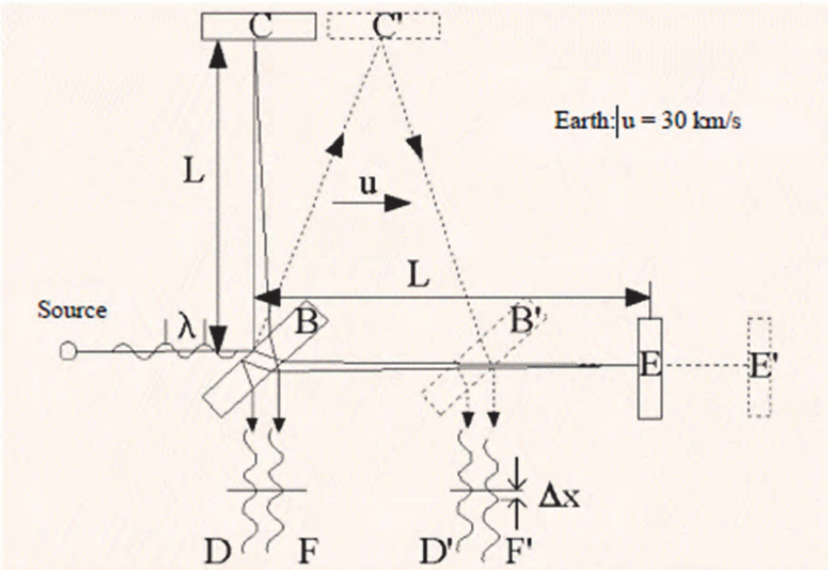


Figure 5.5. *Interferometer*

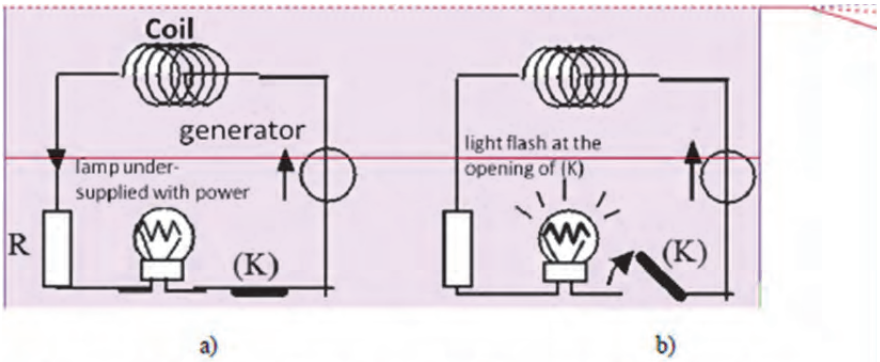


Figure A2.1. *Energy conservation, when turning on and turning off a switch*

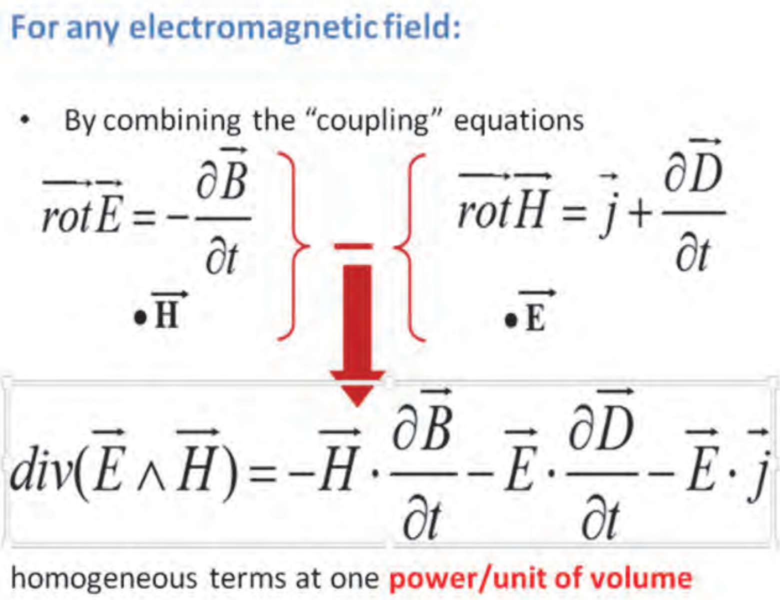


Figure A2.2. *Divergence related to electromagnetic power* balance of powers for a volume (τ) limited by a surface (S):

$$\iiint_{(\tau)}\left(div(\vec{E}\wedge\vec{H})+\vec{E}\cdot\vec{j}+\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}+\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}\right)d\tau=0$$

$$\boxed{=\iint_{(S)}(\vec{E}\wedge\vec{H})\cdot\vec{dS}}\qquad\boxed{=\frac{\vec{j}^{\,2}}{\gamma}}\qquad\boxed{=\frac{\partial\left(\frac{\vec{B}^{2}}{2\mu}\right)}{\partial t}}\qquad\boxed{=\frac{\partial\left(\varepsilon\frac{\vec{E}^{2}}{2}\right)}{\partial t}}$$

= Power radiated by O.E.M.

= Joule effect

= variation in static magneto energy density/t

= variation in static electro energy density/t

= electromagnetic energy density variation/t

Figure A2.3. *Power balance*

POYNTING VECTOR

$$=\iint_{(S)}(\vec{E}\wedge\vec{H})\cdot\vec{dS}=\iint_{(S)}\vec{R}\cdot\vec{dS}$$

= Vector flow $\vec{R}=\vec{E}\wedge\vec{H}$

(= Poynting Vector) through (S);

$\boxed{\vec{R}}$ gives the <u>intensity</u> and <u>direction</u> of the radiated electromagnetic energy

Figure A2.4. *Poynting vector*

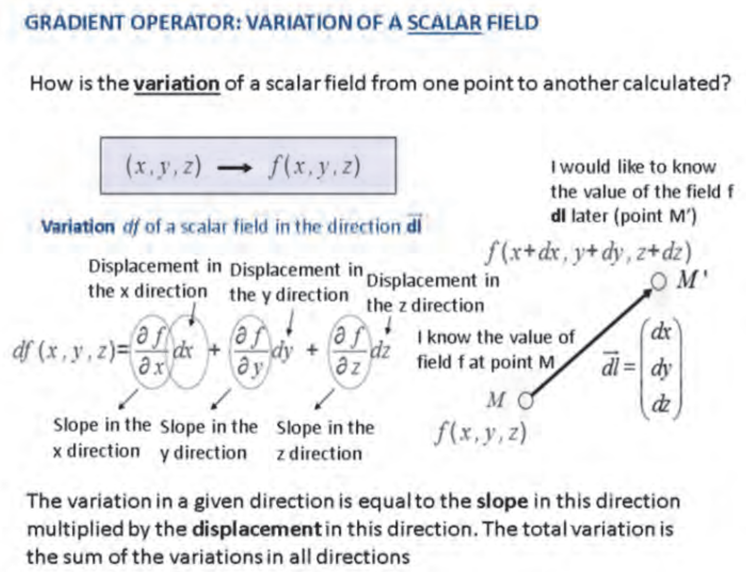


Figure A3.3. *Gradient*

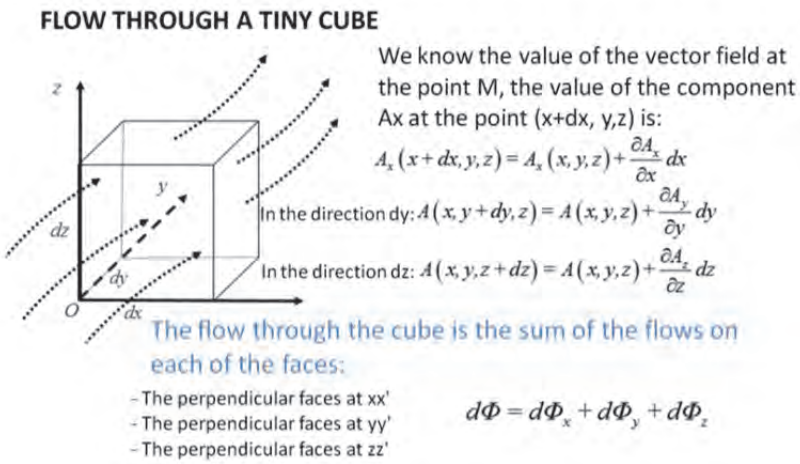


Figure A3.4a. *Flow*

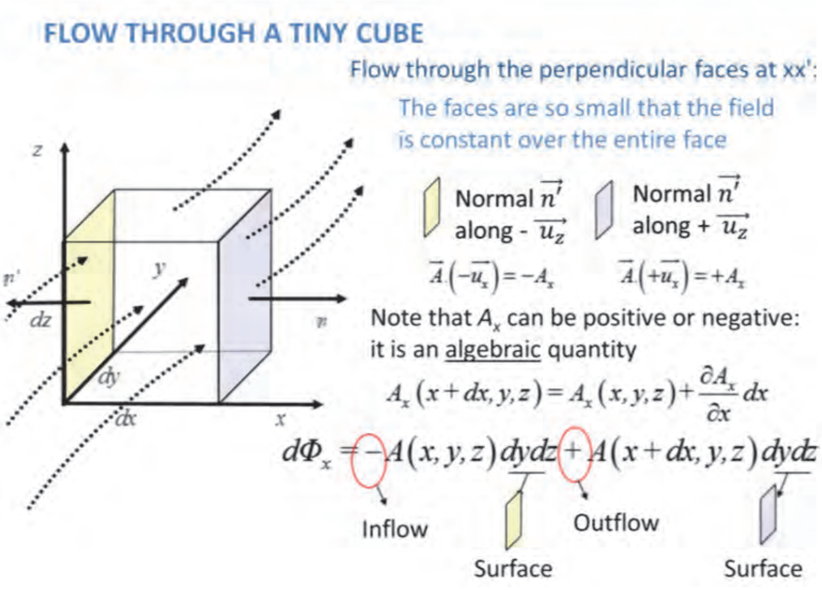


Figure A3.4b. *Flow (continued)*

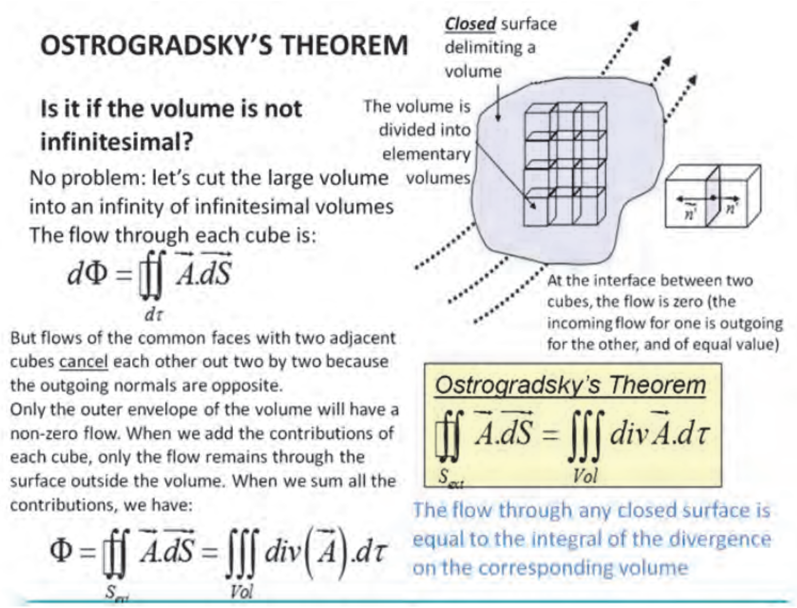


Figure A3.5. *Ostrogorsky's theorem*

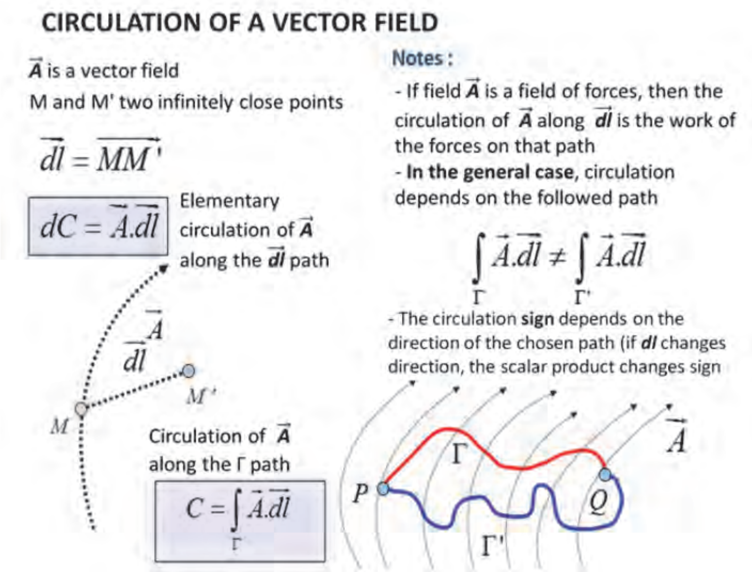


Figure A3.6. *Vector circulation*

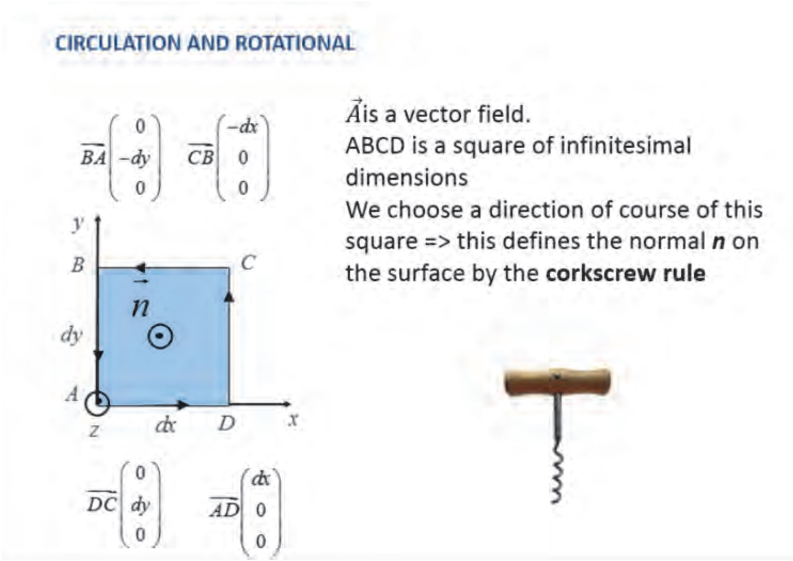


Figure A3.7. *Corkscrew circulation and paradigm*

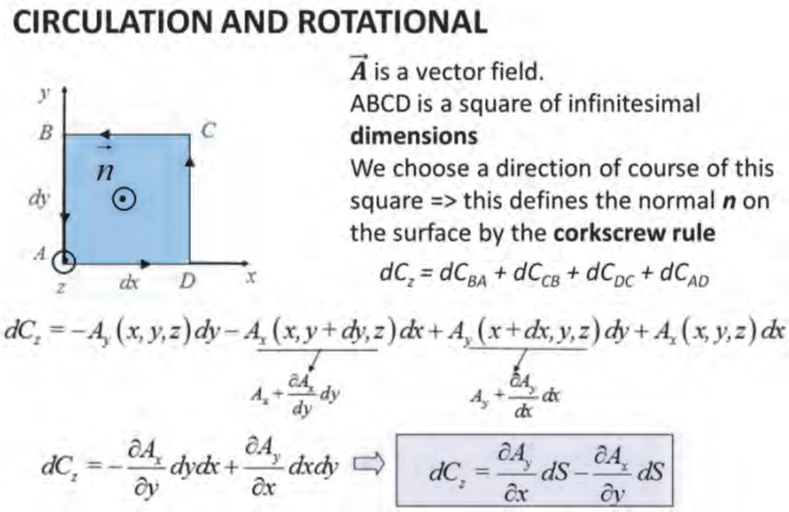


Figure A3.8. *Circulation around a surface*

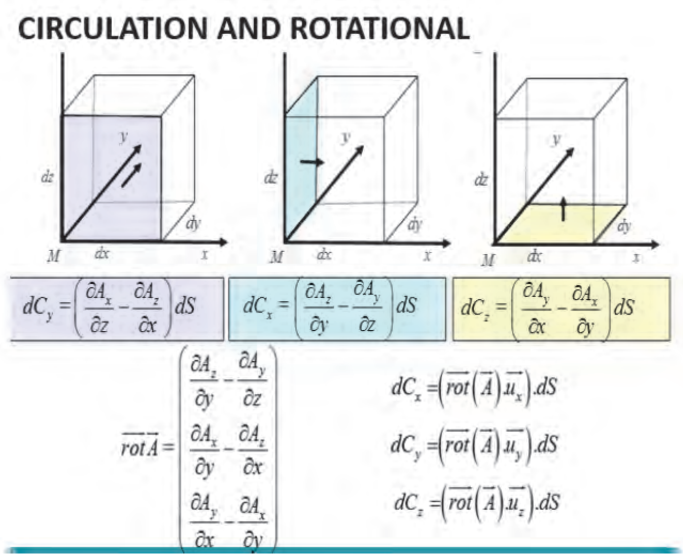


Figure A3.9. *Circulation and rotational*

THE ROTATIONAL IS A CIRULATION PER UNIT OF SURFACE

$$\overrightarrow{rot}\,\vec{A} = \begin{pmatrix} \dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z} \\ \dfrac{\partial A_x}{\partial y} - \dfrac{\partial A_z}{\partial x} \\ \dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y} \end{pmatrix}$$

$$dC_x = \left(\overrightarrow{rot}\left(\vec{A}\right)\vec{u_x}\right).dS$$
$$dC_y = \left(\overrightarrow{rot}\left(\vec{A}\right)\vec{u_y}\right).dS$$
$$dC_z = \left(\overrightarrow{rot}\left(\vec{A}\right)\vec{u_z}\right).dS$$

If $dC_z = \left(\overrightarrow{rot}\left(\vec{A}\right)\vec{u_z}\right).dS \neq 0$, the vector is rotating around the point M, along the axis.

$\overrightarrow{rot}\left(\vec{A}(M)\right)$ represents, by its standard, the curvature of the vector field **around M**, and its direction (and orientation) gives the axis of this curvature **around M**.

If a vector field revolves around a given point, it does not mean that it revolves around all points in space

Figure A3.10. *Rotational*

ROTATIONAL IS A CIRCULATION PER UNIT AREA

$$\overrightarrow{rot}\,\vec{A} = \begin{pmatrix} \dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z} \\ \dfrac{\partial A_x}{\partial y} - \dfrac{\partial A_z}{\partial x} \\ \dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y} \end{pmatrix}$$

$$dC_x = \left(\overrightarrow{rot}\left(\vec{A}\right)\vec{u_x}\right).dS$$
$$dC_y = \left(\overrightarrow{rot}\left(\vec{A}\right)\vec{u_y}\right).dS$$
$$dC_z = \left(\overrightarrow{rot}\left(\vec{A}\right)\vec{u_z}\right).dS$$

Rotational is therefore the **circulation per unit area** dS when the area dS tends towards 0

This result can be generalized to any closed contour (not necessarily elementary) by cutting a closed contour into elementary contours.

Stokes' Theorem

$$\oint_C \vec{A}.\vec{dl} = \iint_S \overrightarrow{rot}\left(\vec{A}\right).\vec{dS}$$

The total **circulation** of a vector field along a **closed oriented** contour is equal to the **flow of the rotational** through the surface defined by the closed contour.

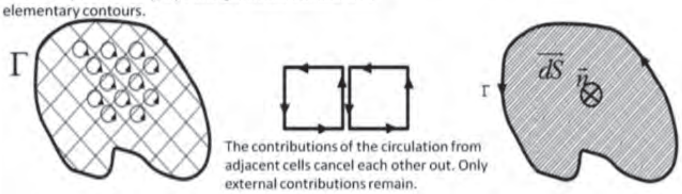


Figure A3.11. *Rotational: continuation and end*

PROPERTIES OF FIELD THEORY OPERATORS

Gradient, divergence, and rotational are **linear** operators

$$\overrightarrow{grad}(a+b) = \overrightarrow{grad}(a) + \overrightarrow{grad}(b)$$
$$div(\vec{A}+\vec{B}) = div(\vec{A}) + div(\vec{B})$$
$$\overrightarrow{rot}(\vec{A}+\vec{B}) = \overrightarrow{rot}(\vec{A}) + \overrightarrow{rot}(\vec{B})$$

$$\overrightarrow{grad}(\lambda a) = \lambda.\overrightarrow{grad}(a)$$
$$div(\lambda\vec{A}) = \lambda.div(\vec{A})$$
$$\overrightarrow{rot}(\lambda\vec{A}) = \lambda.\overrightarrow{rot}(\vec{A})$$

Combined operators always null

$$div(\overrightarrow{rot}(\vec{A})) = 0 \qquad \overrightarrow{rot}(\overrightarrow{\mathrm{grad}}(\mathrm{f})) = 0$$

Figure A3.13. *Form*

ORDER 2 OPERATOR: LAPLACIAN

Scalar Laplacian:
Applies to a scalar field to give a scalar

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\Delta f(M) = div\left(\overrightarrow{grad}(f(M))\right)$$

Vector Laplacian:
Applies to a vector field to give a vector

$$\vec{\Delta}\vec{E} = \begin{pmatrix} \Delta E_x \\ \Delta E_y \\ \Delta E_z \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} \\ \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} \\ \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} \end{pmatrix}$$

$$\vec{\Delta}\vec{A}(\mathrm{M}) = \overrightarrow{grad}\left(div\left(\vec{A}(M)\right)\right) - \overrightarrow{rot}\left(\overrightarrow{rot}\left(\vec{A}(M)\right)\right)$$

Figure A3.14. *Laplacian*

WRITING WITH THE "NABLA" OPERATOR

$$\vec{\nabla}=\begin{pmatrix}\dfrac{\partial}{\partial x}\\ \dfrac{\partial}{\partial y}\\ \dfrac{\partial}{\partial z}\end{pmatrix}$$

Scalar field

$$\overrightarrow{grad}(f)=\vec{\nabla}f$$

Vector field

Vector field

$$\overrightarrow{rot}\,\vec{A}=\vec{\nabla}\wedge\vec{A}$$

Pseudo - vector field

Vector field

$$div\,\vec{A}=\vec{\nabla}.\vec{A}$$

Scalar field

Figure A3.15. *Operational writing*

FORM:

$\vec{\nabla}.(\vec{\nabla}\wedge\vec{A}) = 0$	$\mathrm{div}(\overrightarrow{\mathrm{rot}}\vec{A}) = 0$
$\vec{\nabla}\wedge(\vec{\nabla}f) = 0$	$\overrightarrow{\mathrm{rot}}(\overrightarrow{\mathrm{grad}}f) = 0$
$\vec{\nabla}(f_1f_2)=f_1\vec{\nabla}(f_2)+f_2\vec{\nabla}(f_1)$	$\overrightarrow{\mathrm{grad}}(f_1f_2)=f_1\overrightarrow{\mathrm{grad}}f_2+f_2\overrightarrow{\mathrm{grad}}f_1$
$\vec{\nabla}.(f.\vec{A}) = \vec{A}.(\vec{\nabla}f)+f(\vec{\nabla}.\vec{A})$	$\mathrm{div}(f.\vec{A}) = \vec{A}.\overrightarrow{\mathrm{grad}}f+f\mathrm{div}\vec{A}$
$\vec{\nabla}\wedge(f.\vec{A}) = \vec{\nabla}f\wedge\vec{A}+f(\vec{\nabla}\wedge\vec{A})$	$\overrightarrow{\mathrm{rot}}(f.\vec{A}) = \overrightarrow{\mathrm{grad}}f\wedge\vec{A}+f\overrightarrow{\mathrm{rot}}\vec{A}$
$\vec{\nabla}.(\vec{A}_1\wedge\vec{A}_2) = \vec{A}_2.(\vec{\nabla}\wedge\vec{A}_1)-\vec{A}_1.(\vec{\nabla}\wedge\vec{A}_2)$	$\mathrm{div}(\vec{A}_1\wedge\vec{A}_2) = \vec{A}_2.\overrightarrow{\mathrm{rot}}\vec{A}_1-\vec{A}_1.\overrightarrow{\mathrm{rot}}\vec{A}_2$
$\Delta f = \vec{\nabla}.(\vec{\nabla}f)$	$\Delta f = \mathrm{div}(\overrightarrow{\mathrm{grad}}f)$
$\vec{\Delta}\vec{A}=\vec{\nabla}(\vec{\nabla}.\vec{A})-\vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{A})$	$\vec{\Delta}\vec{A}=\overrightarrow{grad}(div\vec{A})-\overrightarrow{rot}(\overrightarrow{rot}\vec{A})$

Figure A3.16. *Form*