

Series Editor
Guy Pujolle

Digital Communication Techniques

Christian Gontrand

Color section

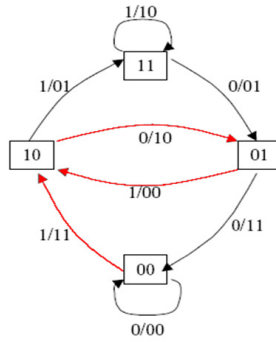


Figure I.7. Transition diagram

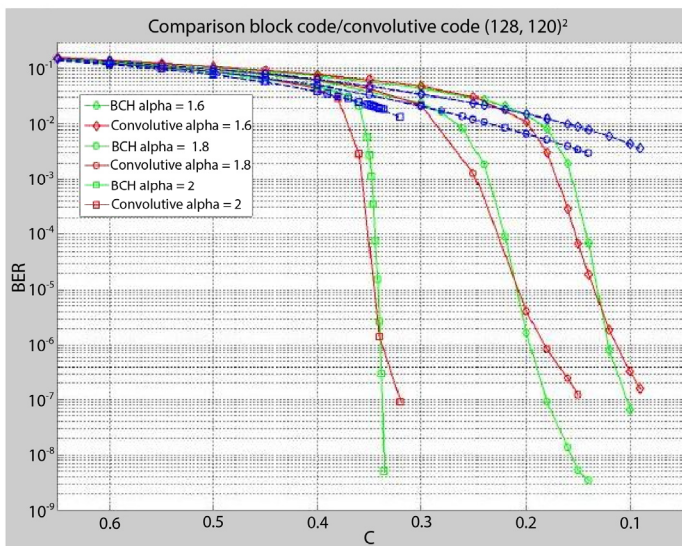


Figure I.14. Turbocodes

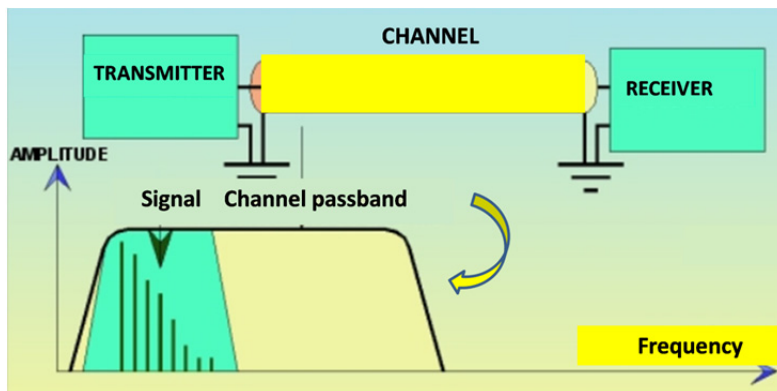


Figure 1.1. *Transmission of a signal in baseband*

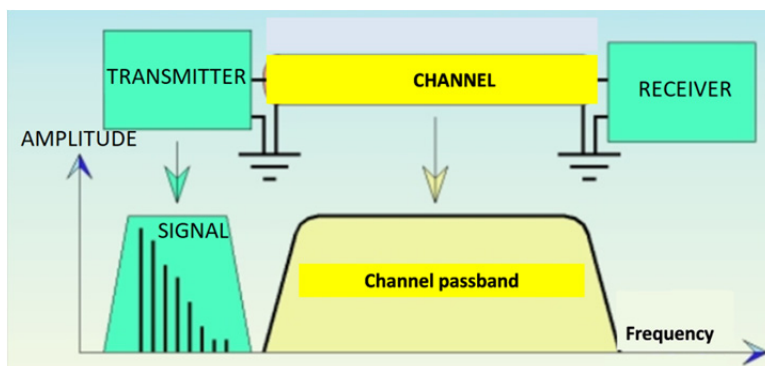


Figure 1.2. *Transmission of a signal via modulation*

Modulations- (heterodyne) frequency transpositions

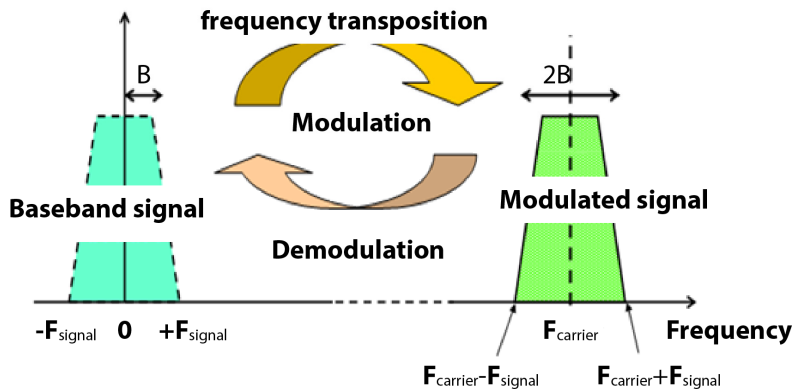


Figure 1.3. Heterodyne system

Probability of error

'Over the course of the transmission, the useful signal is 'attenuated' at the same time as a parasite signal is superposed on it.'

Suppose that the noise $n(t)$ has the following properties

- **Null** average value
- Average quadratic value (standard deviation) such as σ^2 is the variance, it is also the standardized noise power
- Gaussian process. The probability that $n(t)$ lies between s and $s+ds$ is $p(s).ds$ with

$$\sigma^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} n^2(t) dt$$

$$p(s) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{s^2}{2\sigma^2}}$$

- **Stationary** process (independent of time) and **ergodic** (statistical average = temporal average)
- Unilateral DSP is $N = cte$ (white noise), the standardized power is equal, in frequency band B (equivalent noise bandwidth) at $No. B$

Figure 1.10. Probability of error

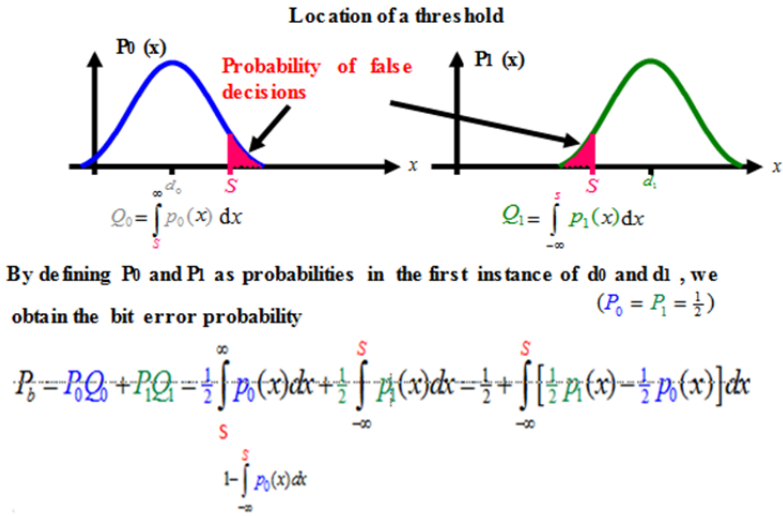


Figure 1.11. Probability of wrong decisions

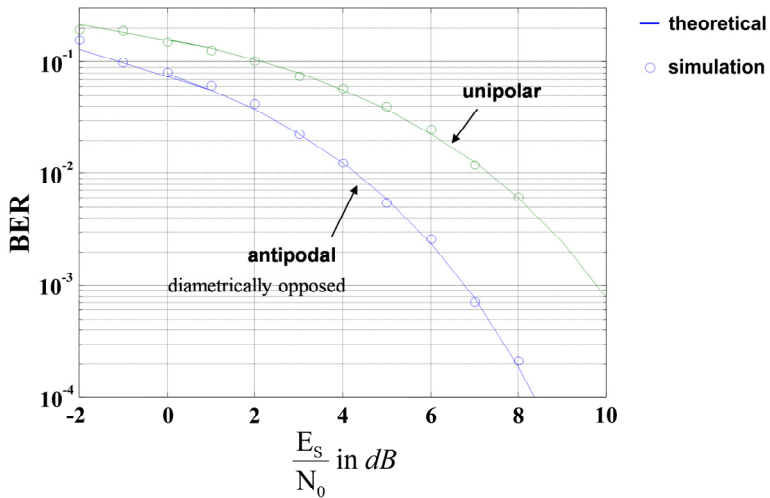


Figure 1.12. Error rate by bit, for a unipolar and antipodal transmission, according to the signal to noise ratio

Baseband signal

"0" ← Level u_1

"1" ← Level u_2

Probability of error, i.e. of deciding that 1 has been received while a 0 was transmitted (or vice versa), is given by:

$$p_e = \int_{\frac{u_1+u_2}{2}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-u_1)^2}{2\sigma^2}} dx$$

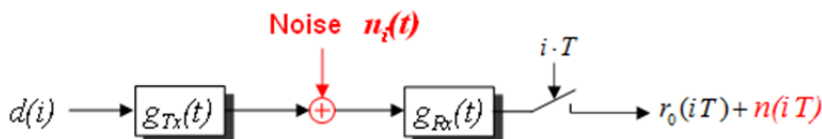
The complementary error function is generally involved :

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} du$$

Finally

$$p_e = \frac{1}{2} \operatorname{erfc}\left(\frac{u_2 - u_1}{2\sigma\sqrt{2}}\right)$$

Figure 1.13. Probability of error in erfc (erf complementary)



Hypotheses.

- Binary transmission, with: $d(i) \in [d_0, d_1]$
- Transmission system verifying the first Nyquist criteria
- Noise $n(iT)$, independent of the data source

Probability density
Average and variance

$n(iT)$

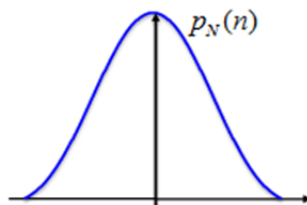


Figure 1.14. Probability of error by bit

Interferences Between Symbols (IBS)

Eye diagram

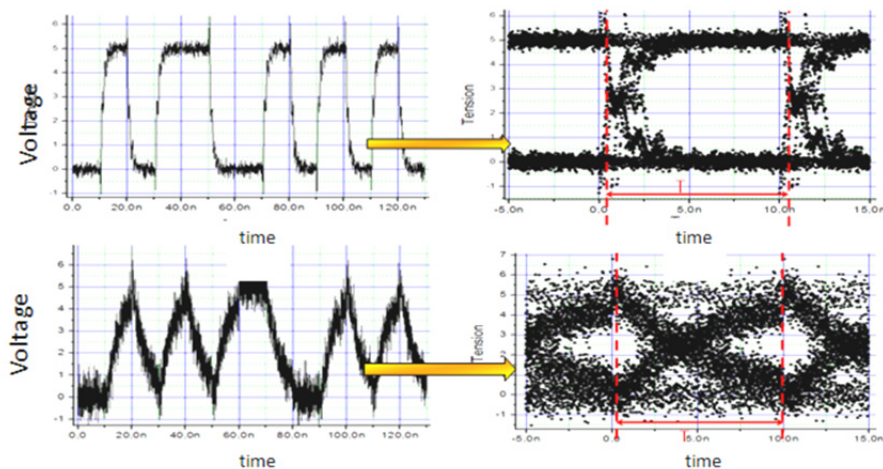


Figure 1.22. Intersymbol interferences and eye diagrams

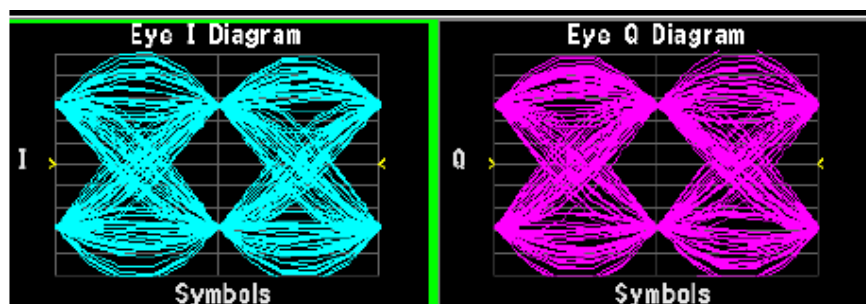


Figure 1.23. Eye diagrams (e.g. QPSK; Agilent)

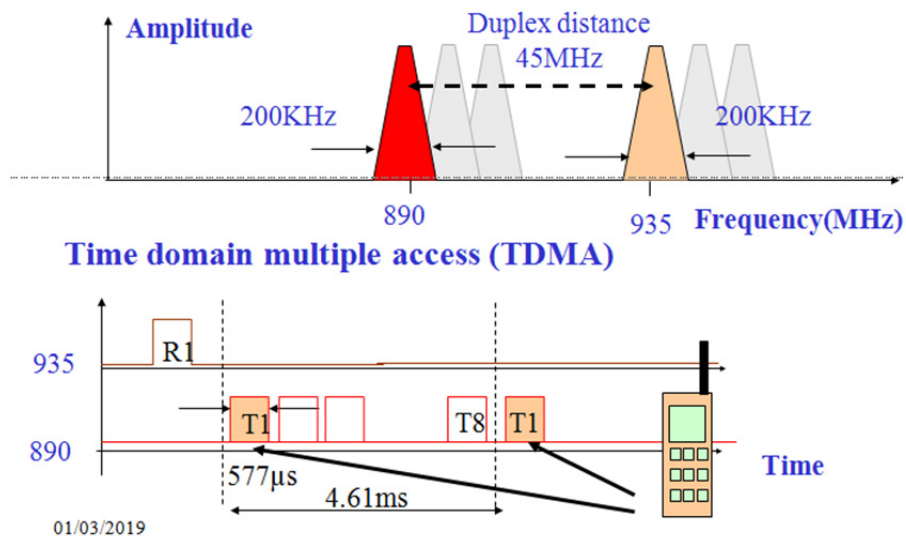


Figure 1.25. CDMA: all users on each frequency and users are separated by code

Example: BPSK-binary phase shift keying

BPSK-Binary Phase Shift Keying

Example: BPSK-binary phase shift keying

$n = 1, M = 2$ and $\varphi_k = 0$ or π

This is a binary modulation (a single bit transmitter/period):

$$m(t) = \pm A_p \cdot \cos(\omega_p t + \varphi_p)$$

The symbol $e^{j\varphi_k} = e^{j\varphi_k}$ therefore takes its value: $\{-1, 1\}$

➡ In the interval, we can write $[0, T]$, we can write:
BPSK Constellation

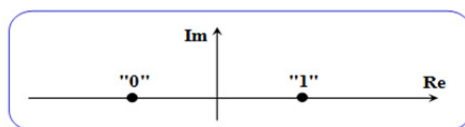


Figure 1.30. BPSK

Impact of noise on a modulated signal

Sj Spectral efficiency :

- ➤ BPSK example : $F_s = 100 \text{ KBd}$, $F_{\text{Bit}} = 100 \text{ Kbits/s}$, $F_p = 1 \text{ MHz}$
- ➤ calculation of the spectral occupation

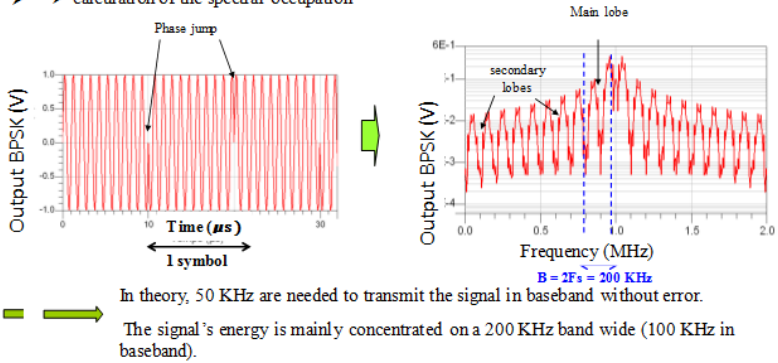


Figure 1.36. Spectral efficiency of a BPSK

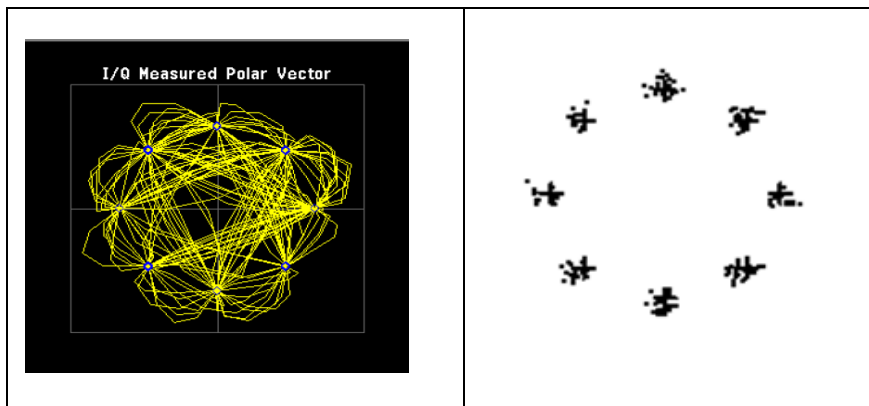


Figure 1.37. Constellation diagram (Agilent)

M-ary digital modulations – QPSK

➤ Quadrature phase-shift key (QPSK) demodulation (QPSK)

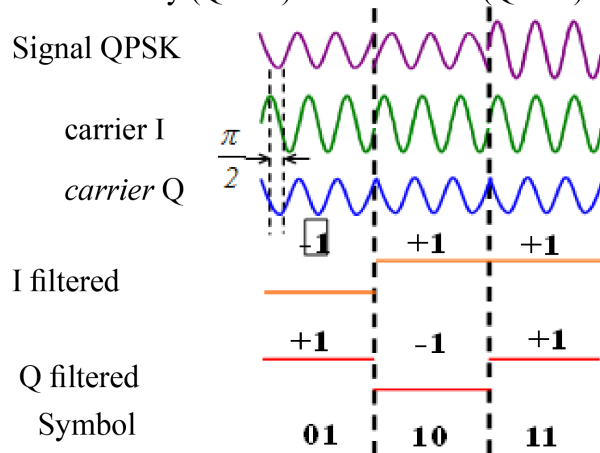


Figure 1.40. QPSK: I/Q

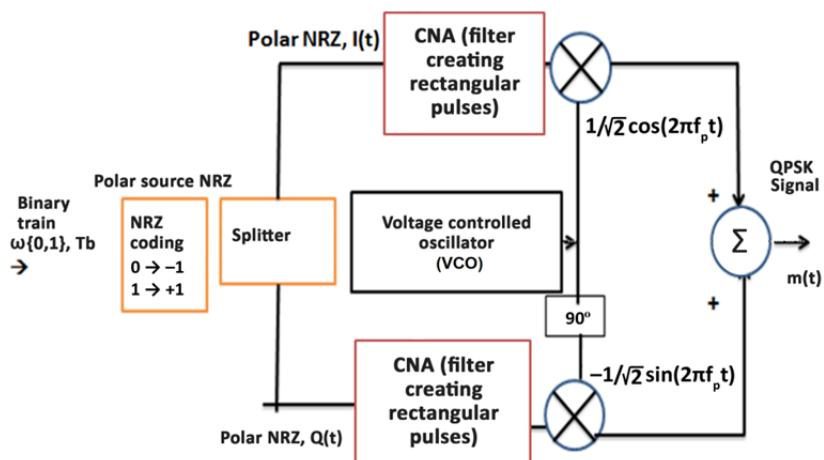


Figure 1.42. QPSK modulator

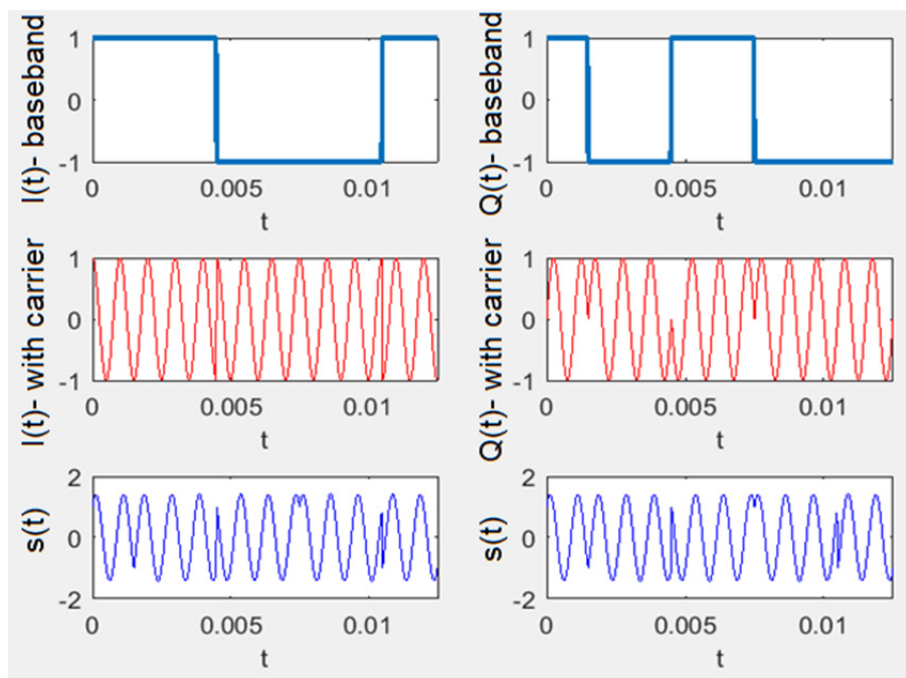


Figure 1.44. QPSK; at the transmitter: timing diagrams

Filtering – Pulse shaping

Exact middle between rectangular impulses and sinc impulses: **raised cosine impulse**

$$f(t) = \text{sinc}\left(\frac{t}{T_s}\right) \frac{\cos\left(\frac{\pi r t}{T_s}\right)}{1 - \left(\frac{2r t}{T_s}\right)^2}$$

T_s : sampling period

r : slip factor, or stiffening or excess band or ROLL-OFF FACTOR

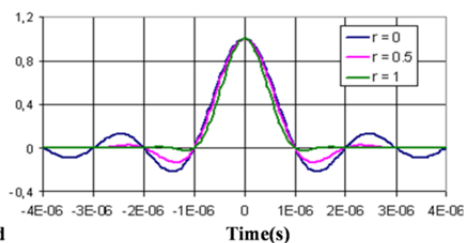


Figure 1.51. Filtering through various raised cosines

Filtering – pulse shaping

➤ Impulse spectrum in raised cosine

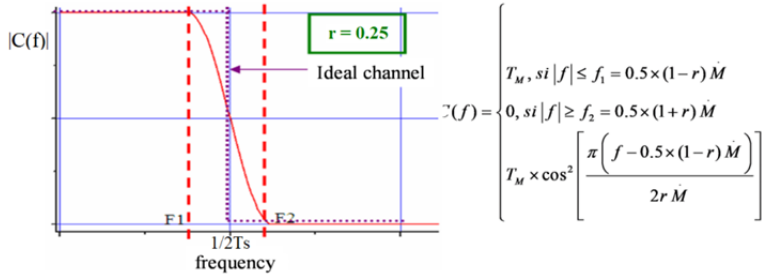


Figure 1.52. Filtering an impulse in raised cosine

Spectrum; frequency (and below)

➤ Example: signal modulated in QPSK

➤ Using a filter in raised cosine ($r = 0.7$; r (roll off))

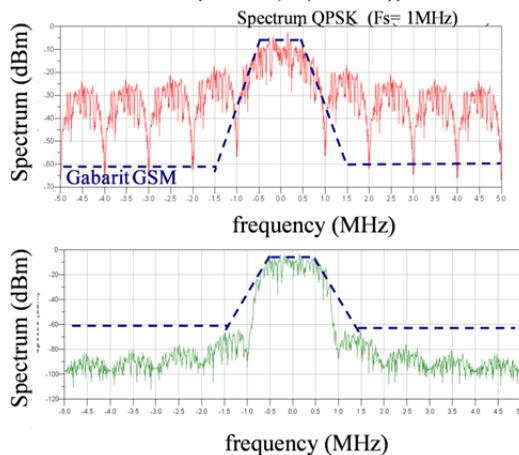


Figure 1.53. Filtering a QPSK signal using a raised cosine

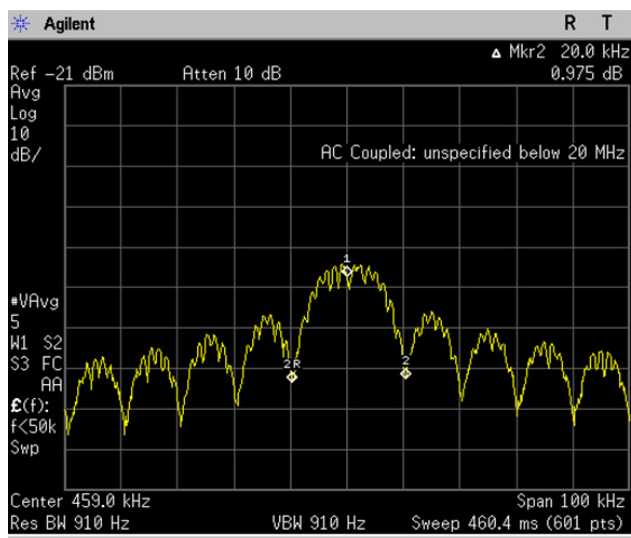


Figure 1.54. Spectrum of the modulated QPSK signal for a binary flow of 20 Kbits/s without filtering streams I and Q (Agilent)

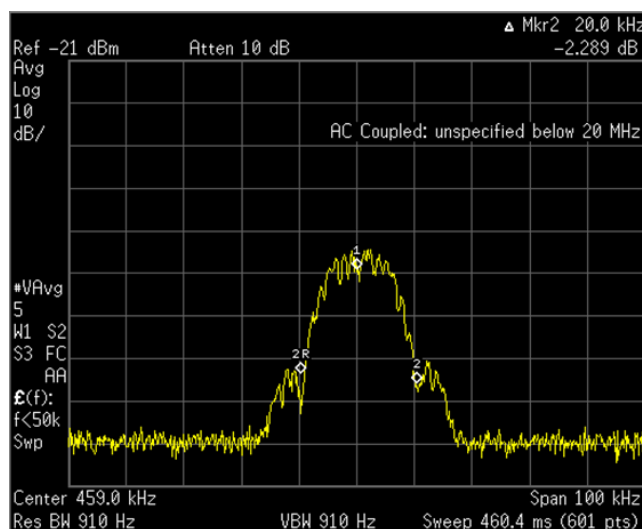
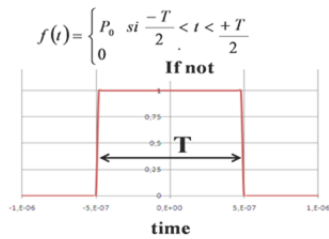


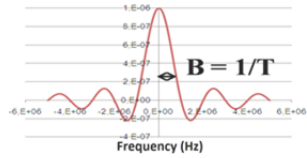
Figure 1.55. Spectrum of the modulated QPSK signal for a binary flow of 20 Kbits/s with filtering (Agilent)

Filtering – pulse shaping

➤ Limit of a rectangular impulse



$$F(f) = P_0 T \sin c(\pi f T)$$



➤ Impulse in limited time (low risk of ISI)...

➤ ... But the spectrum that extends infinitely.

Figure 1.56. Spectrum of a rectangular impulse

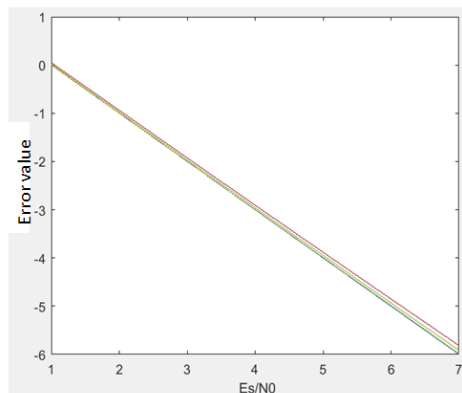


Figure 1.58. Amplitude of the vector error depending on the signal/noise ratio

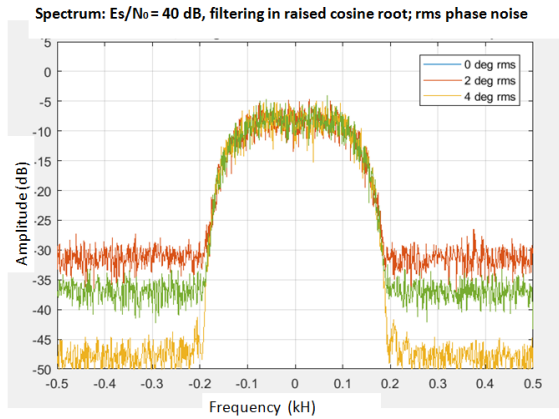


Figure 1.59. Calculation of a typical spectrum of a vector error

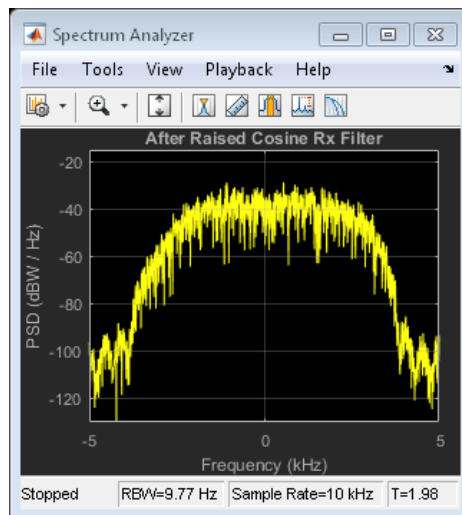


Figure 1.60. Typical spectrum of a vector error, filtered by a raised cosine (MATLAB)

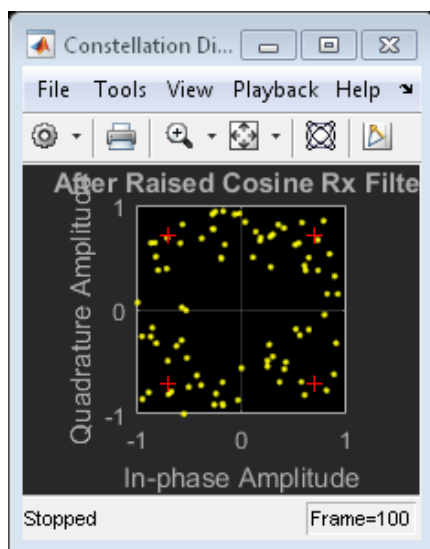


Figure 1.61. Constellation after filtering in raised cosine

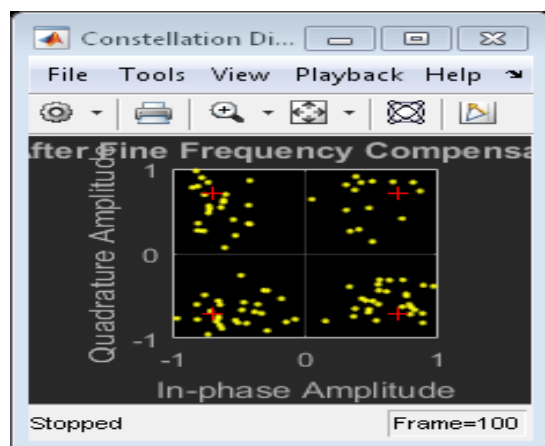


Figure 1.62. Constellation after fine frequency offsets

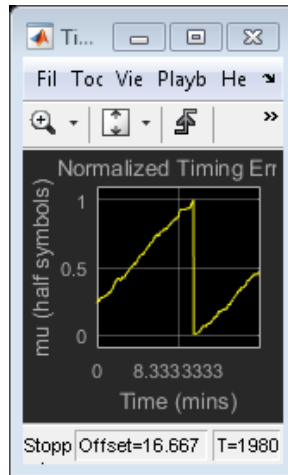


Figure 1.63. *Characteristic of a phase detector (the zig-zags are not ideal)*

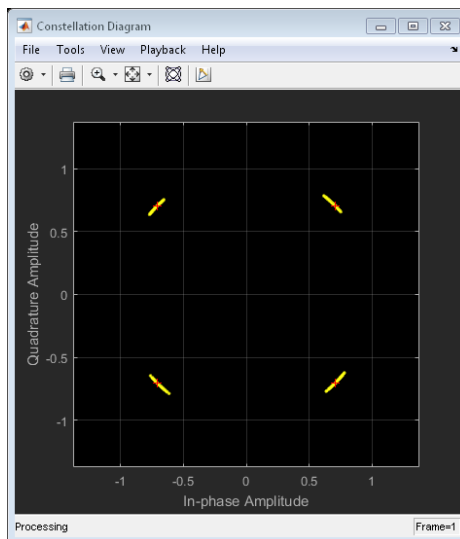


Figure 1.66. *Noisy in-phase constellation*

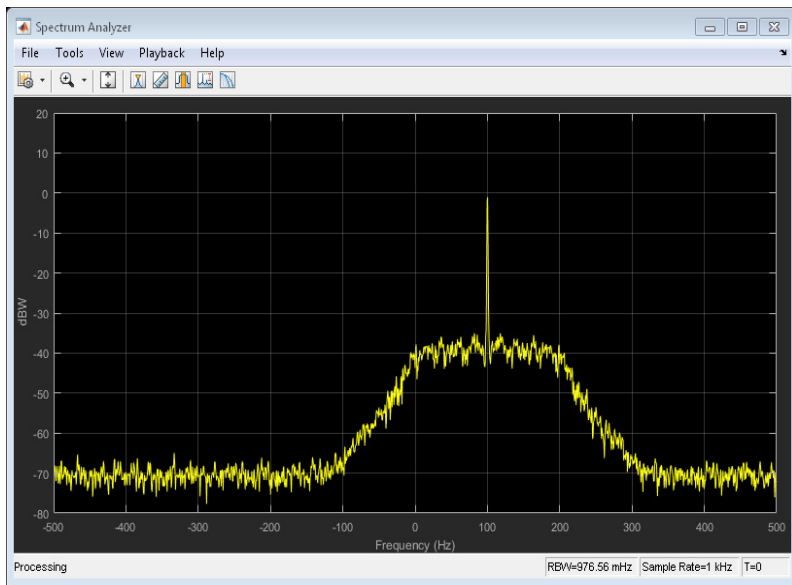


Figure 1.71. Carrier (spur: see line) and phase noise

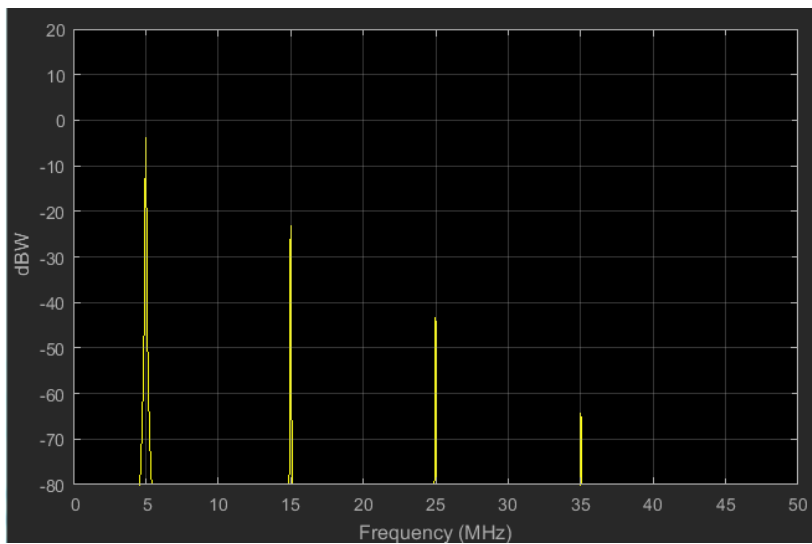


Figure 1.73. Simulation of a spectrum analyzer

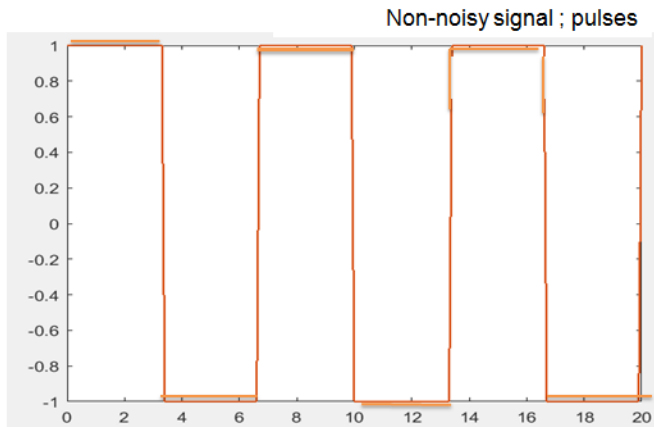


Figure 1.75. *At the transmitter: pulses: 101010*

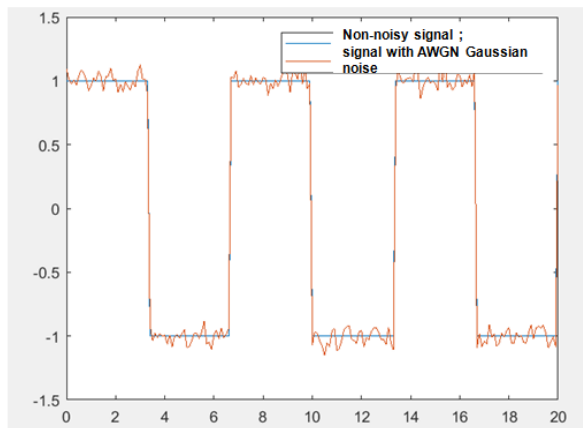


Figure 1.76. *In the channel: pulses: 101010*

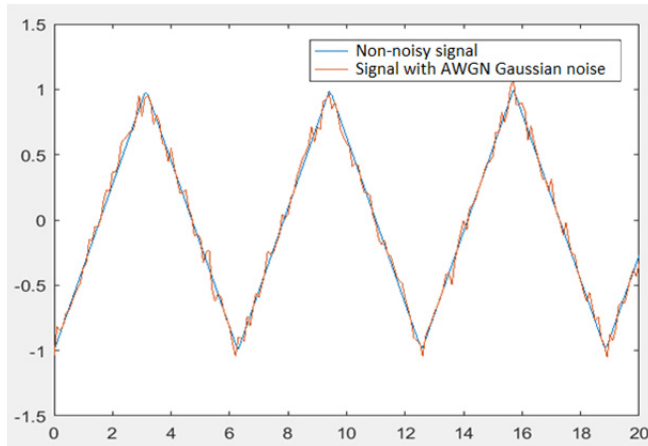


Figure 1.77. *Filtering (average, integration)*

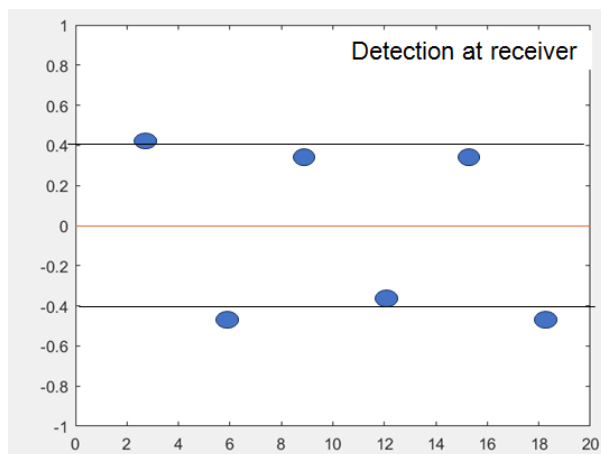


Figure 1.78. *Sampling/thresholding (without error) (101010)*

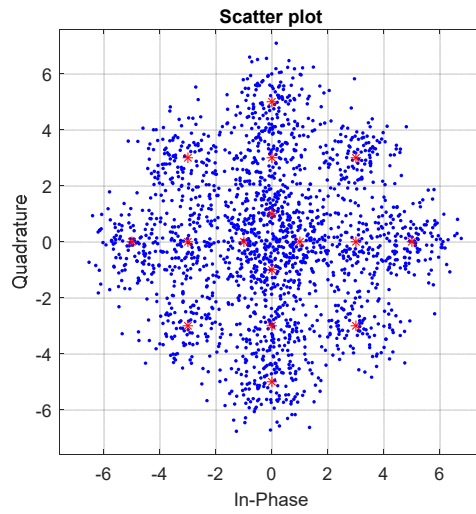


Figure 1.80. Trace of scatter from a 16 QAM

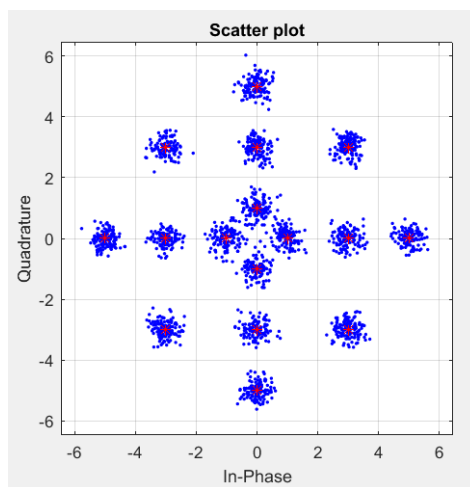


Figure 1.81. 16 constellations

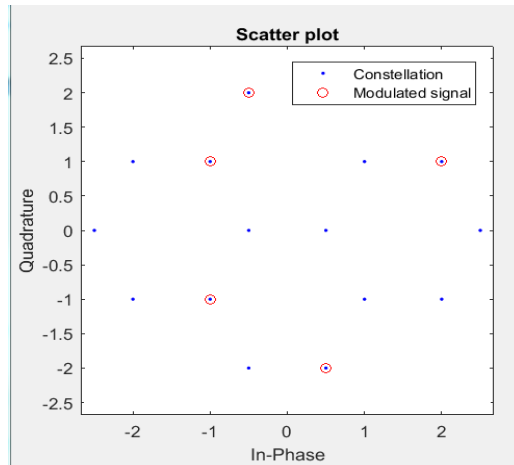
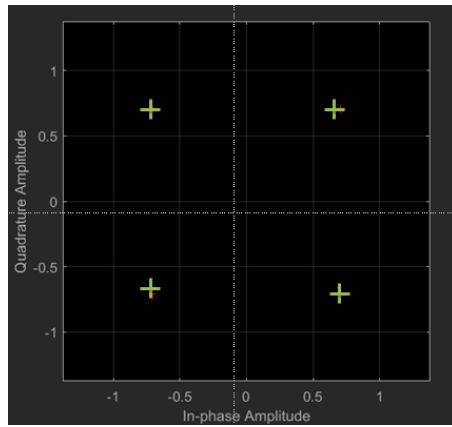
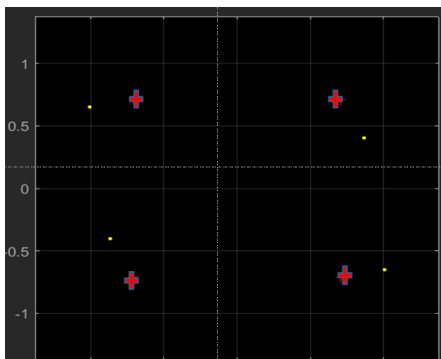


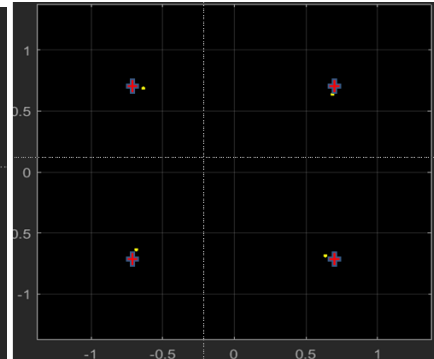
Figure 1.82. *Plot of a QAM constellation*



(a)



(b)



(c)

Figure 1.84. Removing I/Q imbalance

THE SPECTRUMS (continuous phase FSK)

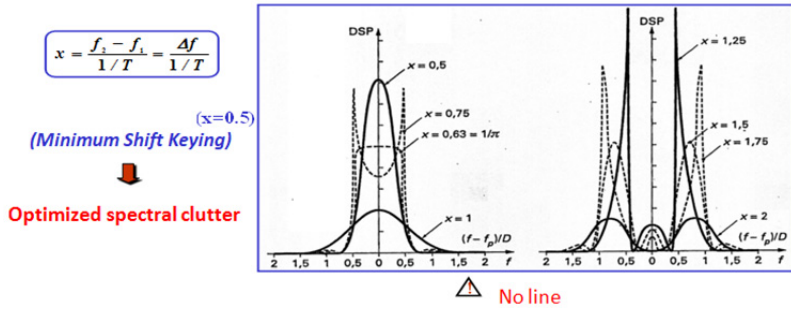


Figure 1.85. Phase shifting

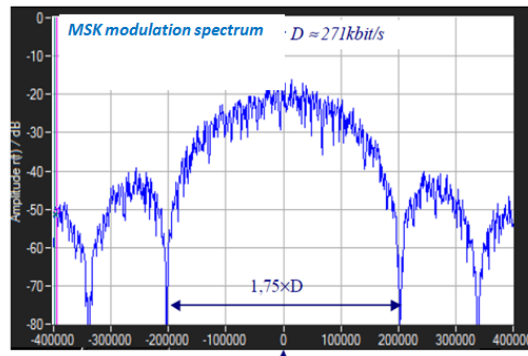


Figure 1.86. Minimum-shift keying (MSK) modulation spectrum:
continuous phase, modulation index: 0.5

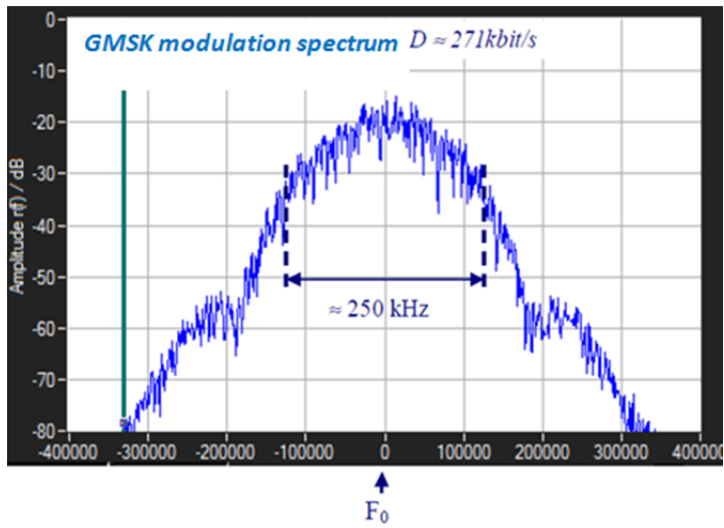


Figure 1.87. Gaussian MSK (GMSK); the data are, from the outset, processed using a Gaussian filter

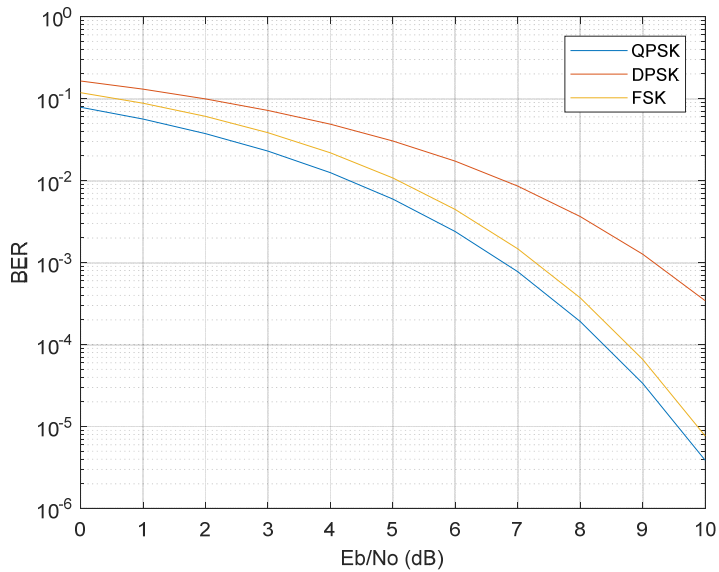


Figure 1.88. BER for different modulations

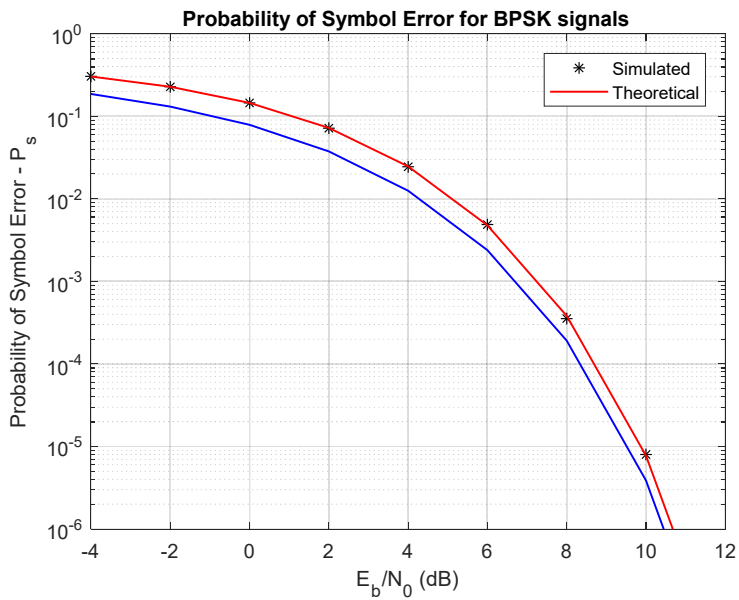


Figure 1.98. BPSK: probability of symbol error

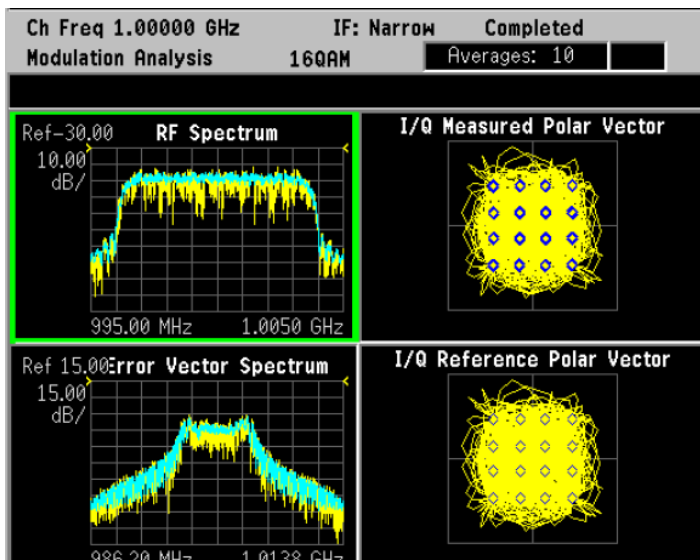


Figure 1.104. Occupied bandwidth (Agilent)

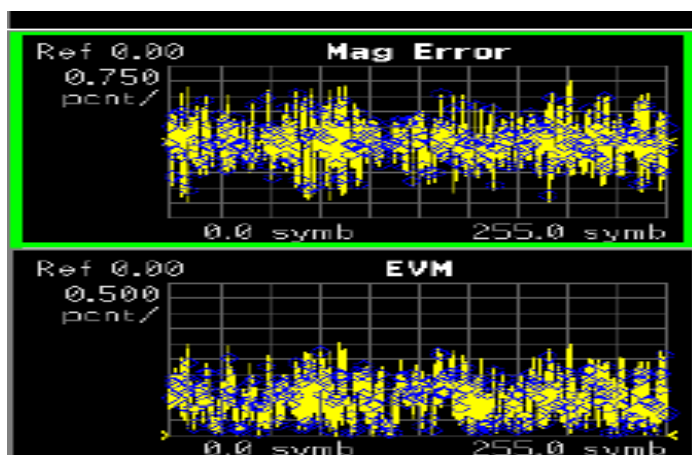


Figure 1.109. *EVM peaks (above) appear during the amplitude's passage to zero (below) (see phase noise – measures)*

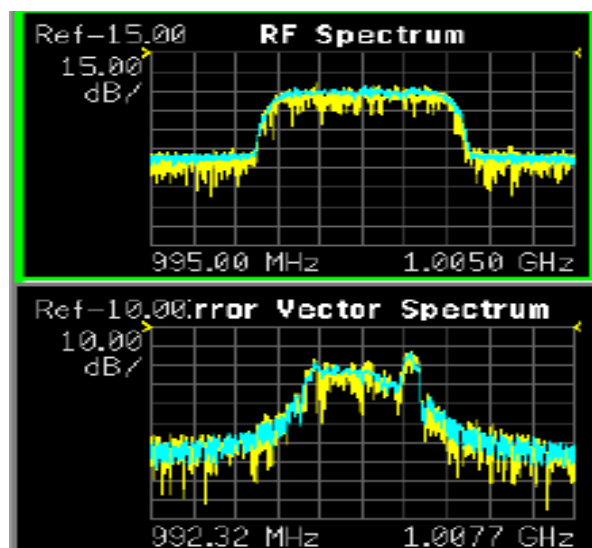


Figure 1.110. *RF spectrum (above) and error vector spectrum (below) (QPSK)*

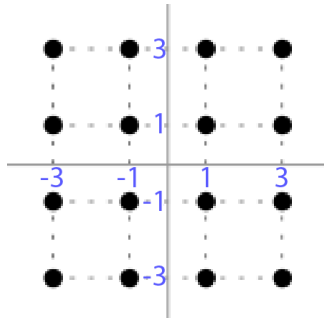


Figure 1.111. *Diagram of constellations for QAM at 16 states*

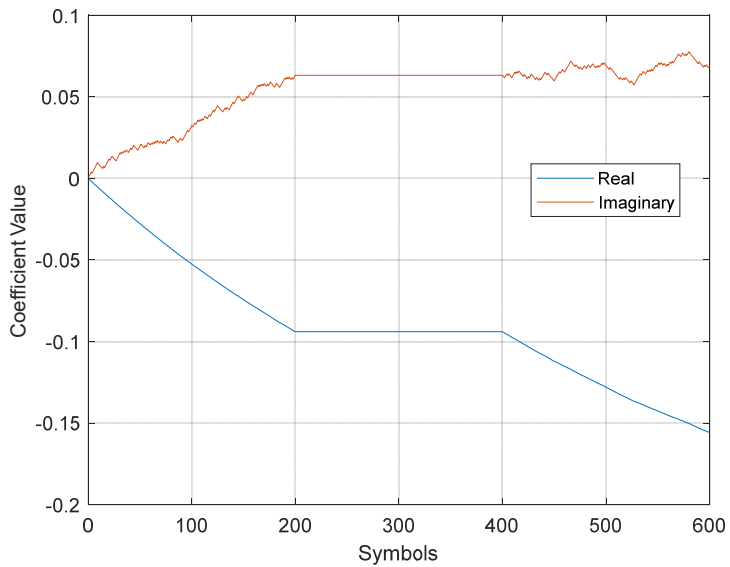


Figure 1.112. *I/Q imbalance measure – compensation coefficients*

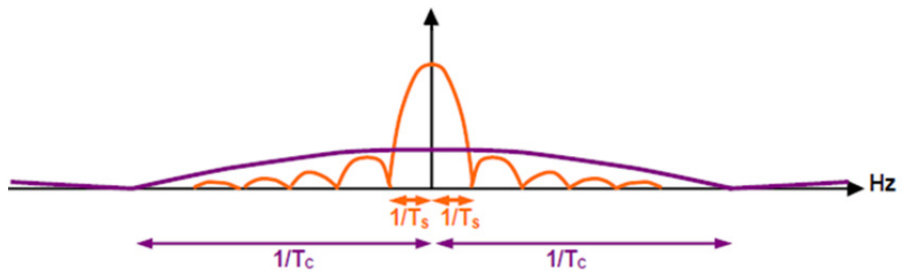


Figure 2.1. Spectrum spread

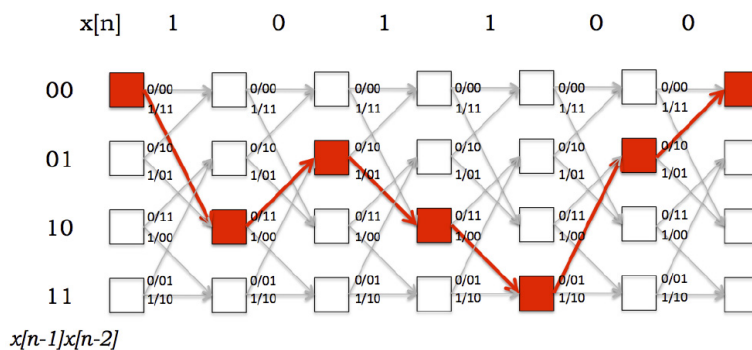
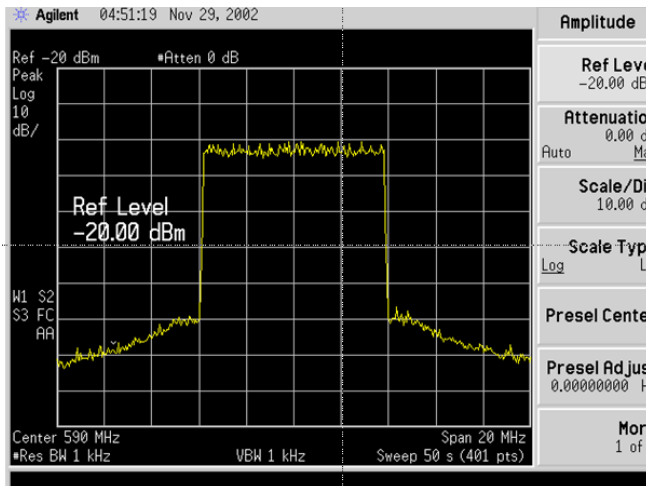
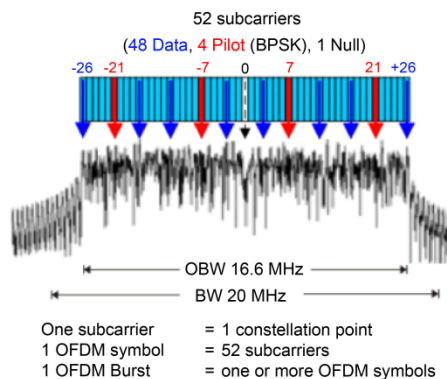


Figure 2.4. Viterbi algorithm. The lattice makes it possible to visualize the decoding and grasp the temporal evolution (from left to right) of a state machine



(a)

802.11a OFDM PHY Parameters	
BW	20 MHz
OBW	16.6 MHz
Subcarrier Spacing	312.5 KHz (20MHz/64 Pt FFT)
Information Rate	6/9/12/18/24/36/48/54 Mbits/s
Modulation	BPSK, QPSK, 16QAM, 64QAM
Coding Rate	1/2, 2/3, 3/4
Total Subcarriers	52 (Freq Index -26 to +26)
Data Subcarriers	48
Pilot Subcarriers*	4 (-21, -7, +7, +21)
DC Subcarrier	Null (0 subcarrier)



(b)

Figure 2.6. (a) A typical OFDM spectrum (measure). (b) Example: signal and parameters of the physical layer OFDM 802.11a

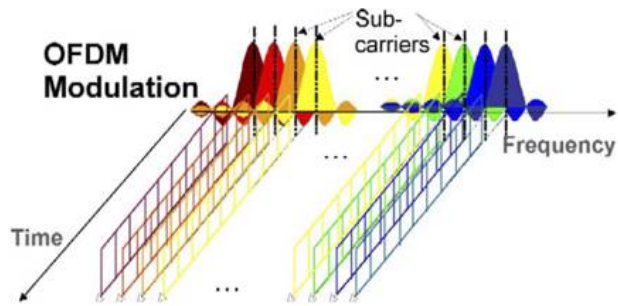


Figure 2.8. OFDM spectrum

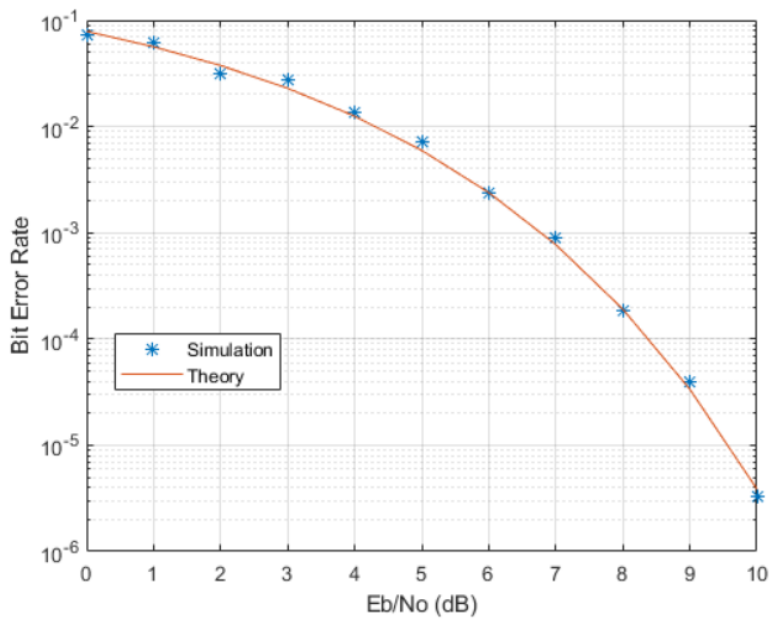


Figure 2.10. BER versus E_b/N_0

<pre> M = 4; % Modulation alphabet k = log2(M); % Bits/symbol numSC = 128; % Number of OFDM subcarriers cpLen = 32; % OFDM cyclic prefix length maxBitErrors = 100; % Maximum number of bit errors maxNumBits = 1e7; % Maximum number of bits transmitted </pre>
<p>Construct System objects needed for the simulation: QPSK modulator, QPSK demodulator, OFDM modulator, OFDM demodulator, AWGN channel, and an error rate calculator. Use name-value pairs to set the object properties.</p> <p>Set the QPSK modulator and demodulator so that they accept binary inputs.</p>
<pre> qpskMod = comm.QPSKModulator('BitInput',true); qpskDemod = comm.QPSKDemodulator('BitOutput',true); </pre>
<p>Set the OFDM modulator and demodulator pair according to the simulation parameters.</p>
<pre> ofdmMod = comm.OFDMModulator('FFTLength',numSC,'CyclicPrefixLength',cp Len); ofdmDemod = comm.OFDMDemodulator('FFTLength',numSC,'CyclicPrefixLength', cpLen); </pre>
<p>Set the NoiseMethod property of the AWGN channel object to Variance and define the VarianceSource property so that the noise power can be set from an input port.</p>
<pre> channel = comm.AWGNChannel('NoiseMethod','Variance',... 'VarianceSource','Input port'); </pre>
<p>Set the ResetInputPort property to true to enable the error rate calculator to be reset during the simulation.</p>
<pre> errorRate = comm.ErrorRate('ResetInputPort',true); </pre>
<p>Use the info function of the ofdmMod object to determine the input and output dimensions of the OFDM modulator (Matlab Inc).</p>
<pre> ofdmDims = info(ofdmMod) </pre>
<pre> ofdmDims = struct with fields: DataInputSize: [117 1] OutputSize: [160 1] </pre>
<p>Determine the number of data subcarriers from the ofdmDims structure variable.</p>
<pre> numDC = ofdmDims.DataInputSize(1) </pre>

numDC = 117

Determine the OFDM frame size (in bits) from the number of data subcarriers and the number of bits per symbol.

frameSize = [k*numDC 1];

Set the SNR vector based on the desired Eb/No range, the number of bits per symbol, and the ratio of the number of data subcarriers to the total number of subcarriers.

EbNoVec = (0:10)';

snrVec = EbNoVec + 10*log10(k) + 10*log10(numDC/numSC);

Initialize the BER and error statistics arrays.

berVec = zeros(length(EbNoVec),3);

errorStats = zeros(1,3);

Simulate the communication link over the range of Eb/No values. For each Eb/No value, the simulation runs until either maxBitErrors are recorded or the total number of transmitted bits exceeds maxNumBits.

for m = 1:length(EbNoVec)

snr = snrVec(m);

while errorStats(2) <= maxBitErrors && errorStats(3) <= maxNumBits

dataIn = randi([0,1],frameSize); % Generate random binary data

qpskTx = qpskMod(dataIn); % Apply QPSK modulation

txSig = ofdmMod(qpskTx); % Apply OFDM modulation

powerDB = 10*log10(var(txSig)); % Calculate Tx signal power

noiseVar = 10.^(0.1*(powerDB-snr)); % Calculate the noise variance

rxSig = channel(txSig,noiseVar); % Pass the signal through a noisy channel

qpskRx = ofdmDemod(rxSig); % Apply OFDM demodulation

dataOut = qpskDemod(qpskRx); % Apply QPSK demodulation

errorStats = errorRate(dataIn,dataOut,0); % Collect error statistics

end

berVec(m,:) = errorStats; % Save BER data

errorStats = errorRate(dataIn,dataOut,1); % Reset the error rate calculator

`end`

Use the `berawgn` function to determine the theoretical BER for a QPSK system.

```
berTheory = berawgn(EbNoVec,'psk',M,'nondiff');
```

Plot the theoretical and simulated data on the same graph to compare results

Figure

```
semilogy(EbNoVec,berVec(:,1),'*')
```

```
hold on
```

```
semilogy(EbNoVec,berTheory)
```

```
legend('Simulation','Theory','Location','Best')
```

```
xlabel('Eb/No (dB)')
```

```
ylabel('Bit Error Rate')
```

```
grid on
```

```
hold off
```

Box 2.1. *Bit error rate versus energy per bit*

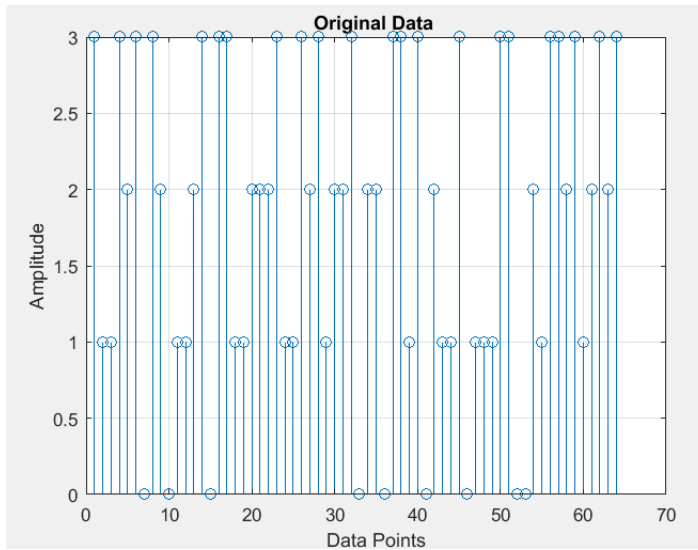


Figure 2.11. *Starting data*

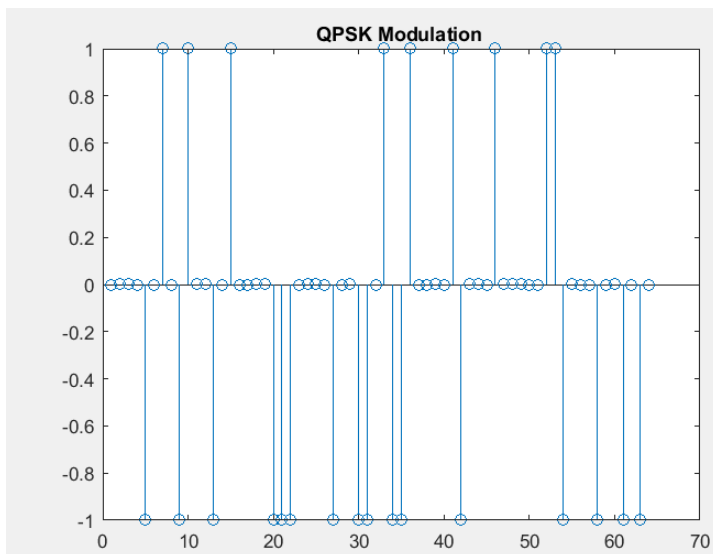


Figure 2.12. *QPSK modulations*

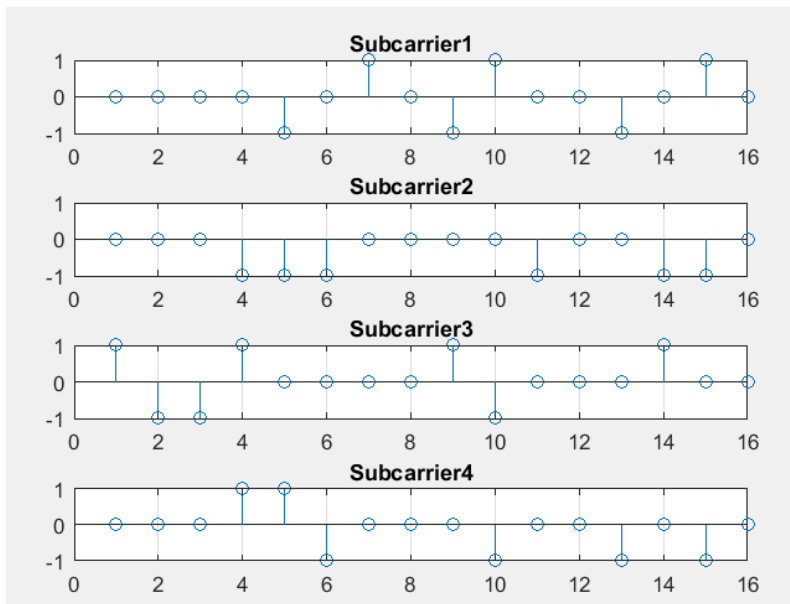


Figure 2.13. *Subcarriers*

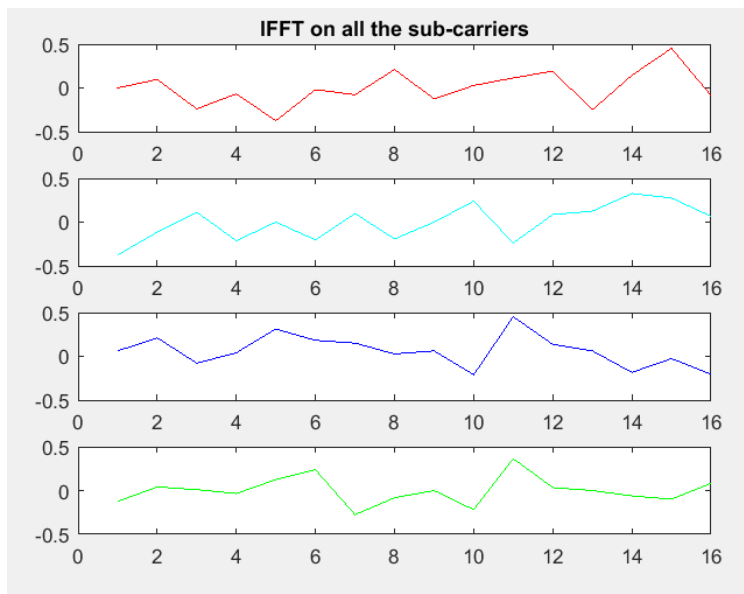


Figure 2.14. *Inverse Fourier transforms of the subcarriers*

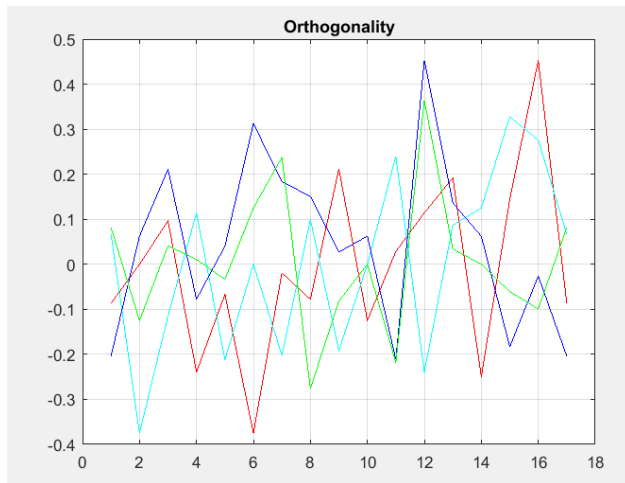


Figure 2.16. *Orthogonality*

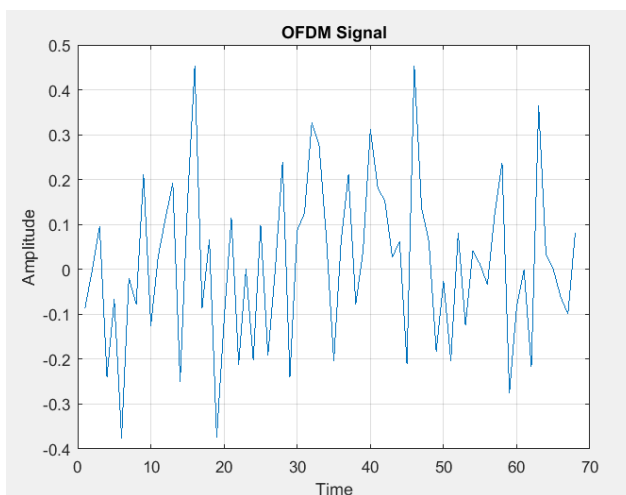


Figure 2.17. *The OFDM signal*

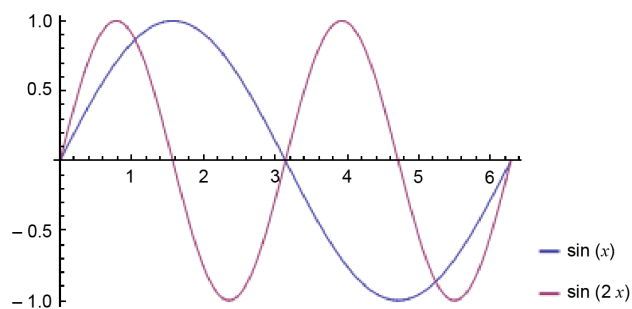


Figure 2.18. *Two sinusoidal curves of different periods; with a null algebraic sum*

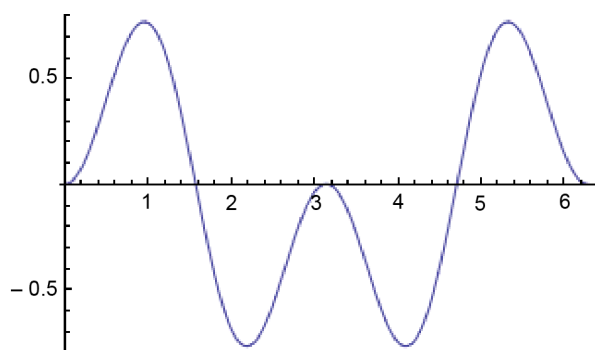


Figure 2.19. *Null mean value: the positive surface equals the negative one*

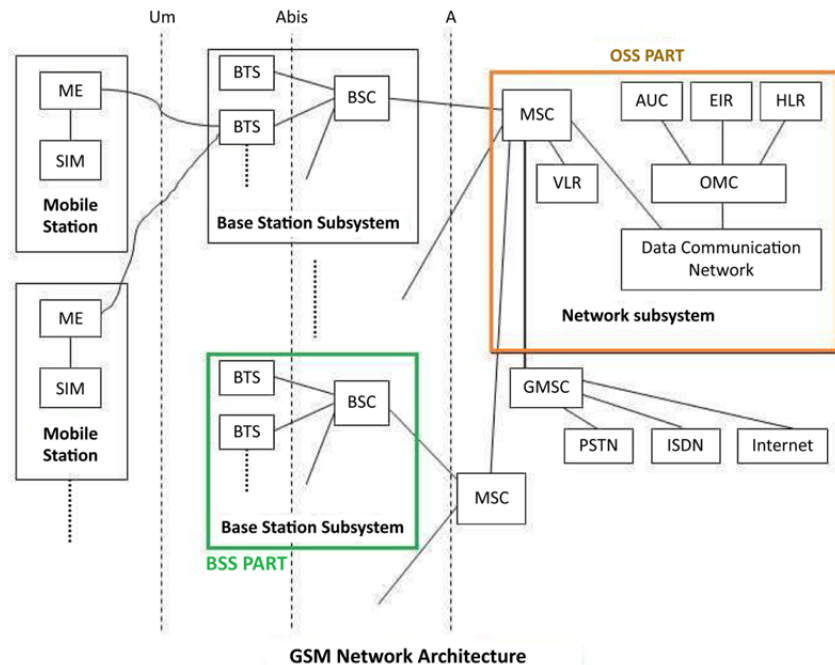


Figure 2.20. GSM network architecture

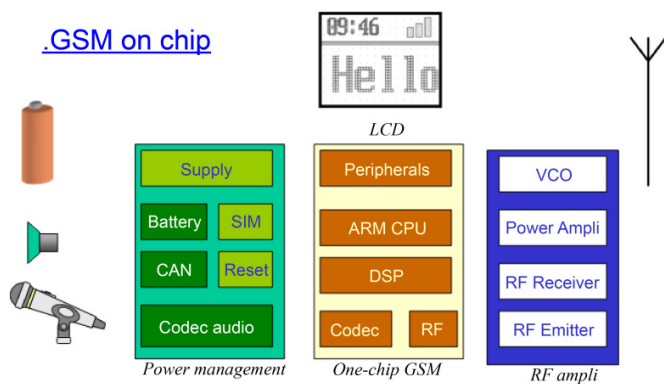


Figure 2.21. Composition of a GSM

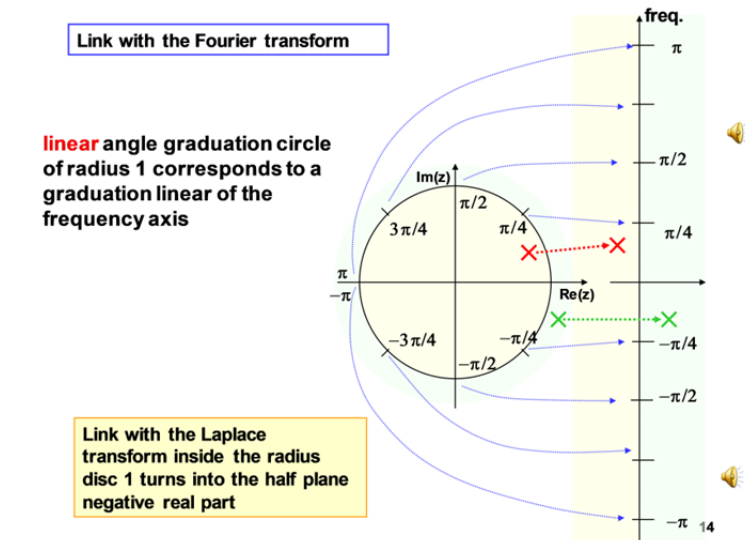


Figure 3.4. Links between z-transforms, Fourier z-transforms and Laplace z-transforms

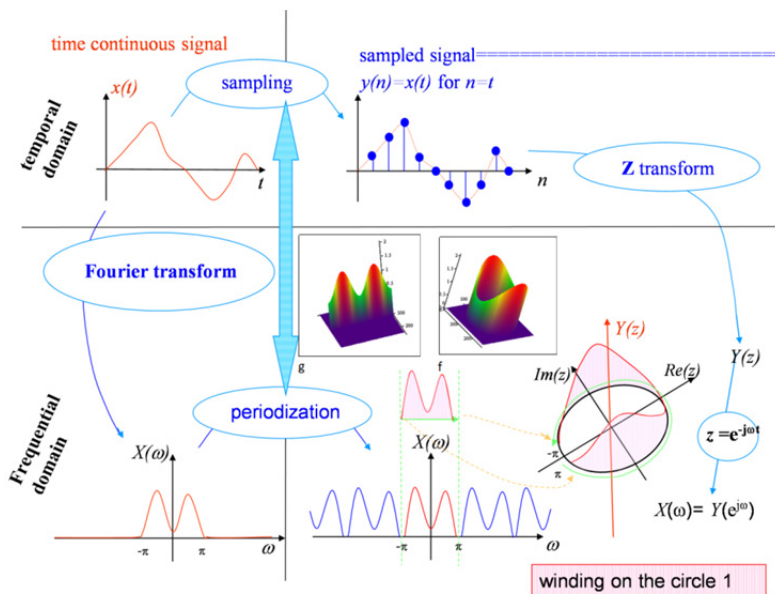


Figure 3.5. Continuous/discrete; temporal/frequency

OSCILLATOR, VCO

Voltage-controlled oscillator (VCO)



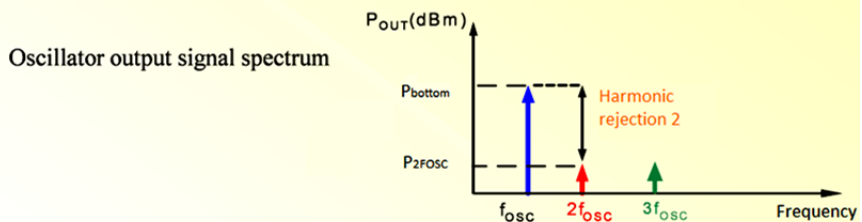
Main specifications

- ✓ Passband $[F_{OSC1}, F_{OSC2}]$
- ✓ Output power P_{OUT}
- ✓ Control voltage $[V_{T1}, V_{T2}]$
- ✓ Phase noise
- ✓ Harmonics rejection
- ✓ Consumption
- ✓ Pulling factor, pushing factor
- ✓ Oscillator linear tuning
- ✓ Temperature stability

Figure 4.1. Voltage-controlled oscillator

OSCILLATOR - VCO

- Output power, harmonic rejection



$$\text{Harmonic rejection } n(\text{dB}) = P_{BOTTOM}(\text{dBm}) - P_{nFOSC}(\text{dBm})$$

➔ The weaker the harmonic rejection, the less distorted the output signal

Figure 4.2. Power: harmonic rejection

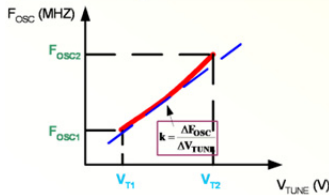
- Pulling factor, pushing factor

Pushing Measures oscillator sensitivity to variations in the **supply voltage V_{BIAS}** expressed in MHz/V

$$\Rightarrow k_P = \frac{\Delta f}{\Delta V_{BIAS}}$$

Pulling Measures the sensitivity of the oscillator to variations in the **output charge** expressed in MHz/ Ω

- Linearity of the oscillator tune



We seek a linear variation of F_{osc} with the control voltage V_{tune}

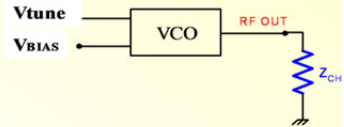
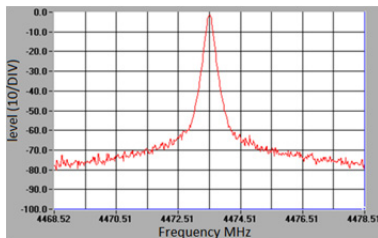
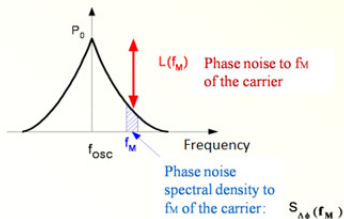


Figure 4.3. Pushing/pulling of a VCO



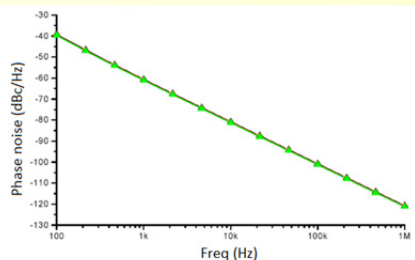
Main ray disrupted by fluctuations in frequency

Instability in the oscillation frequency



Expression of phase noise

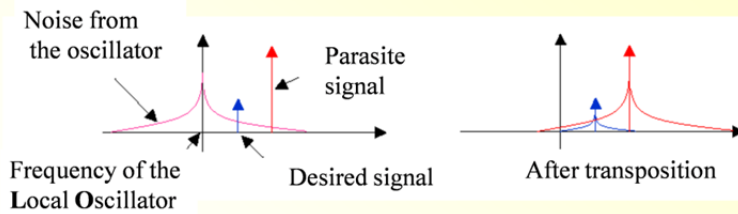
$$L(f_M) = \frac{S_{A\phi}(f_M)}{P_0}$$



Need to minimize this noise as much as possible

Figure 4.4. Phase noise

The phase noise around the frequency of the local oscillator affects radio reception as it mixes with a parasite signal (interferences)



The LO's phase noise, mixed with a parasite signal, can generate substantial noise in the reception channel

➡ Degradation of the radio link's binary error rate

Figure 4.5. *Phase noise caused by parasite signals.*
We note that LO stands for local oscillator

Study of an integrable 5 GHz VCO

Phase-locked loop (PLL):

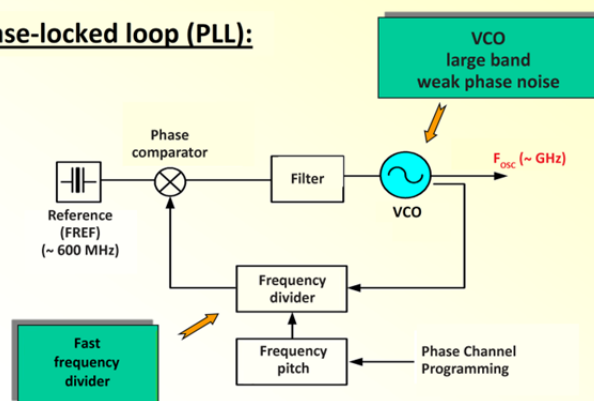


Figure 4.6. *Block schema of a phase-locked loop*

Study of an integrated 5 GHz VCO

Complete diagram of the VCO

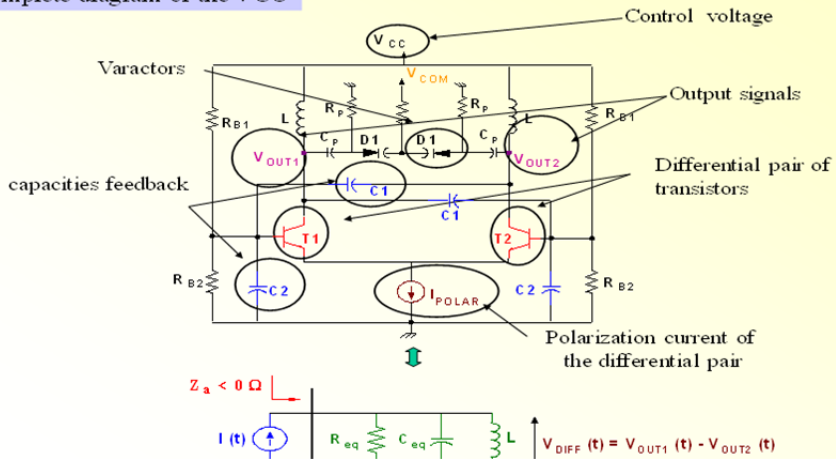


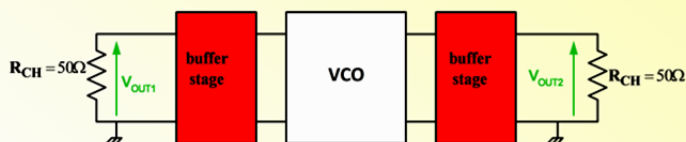
Figure 4.7. Diagram circuit of a VCO

Study of an integrable 5 GHz VCO

Need to use a buffer stage to isolate the VCO from its 50Ω charge



Reduction of the VCO's pulling factor



Classic buffer stage: **emitter follower** (common collector amplifier mounting; very low output resistance)

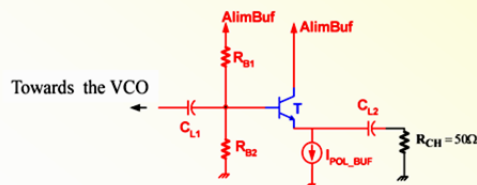
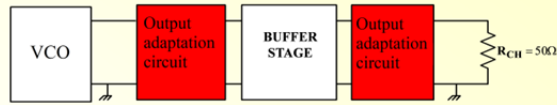


Figure 4.8. Adaptation at output

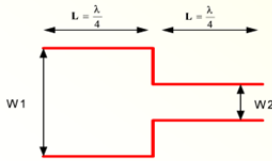
Study of an integrable 5 GHz VCO

- ✓ Addition of impedance matching circuits (LC networks, stubs, quarter-wave lines)



→ Limit: the chip's minimum footprint

Double quarter wave impedance transformer



On alumina substrate and at 5 GHz:
 $\lambda/4 \approx 4.5$ mm

Layout of a 0.9 nH coil

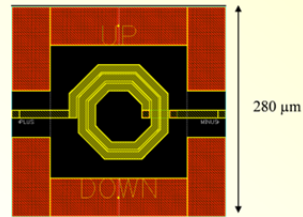


Figure 4.9. Adaptation and layout of an antenna

Study of an integrable 5 GHz VCO

Spectrum of the output power
phase noise and the oscillation
frequency following V_{COM}

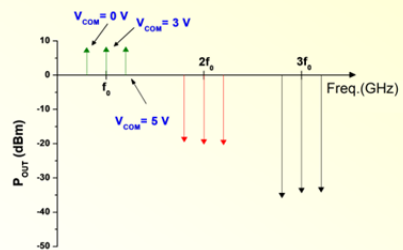
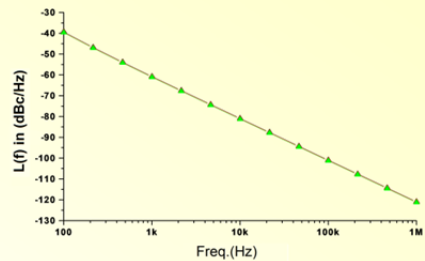
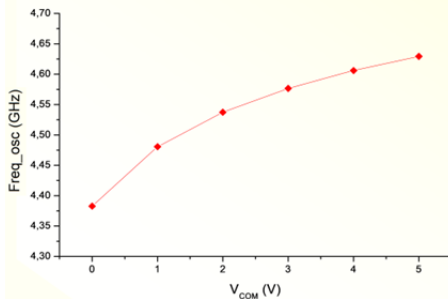


Figure 4.10. Important parameters of a VCO

Study of an integrable 5 GHz VCO

Making the layout

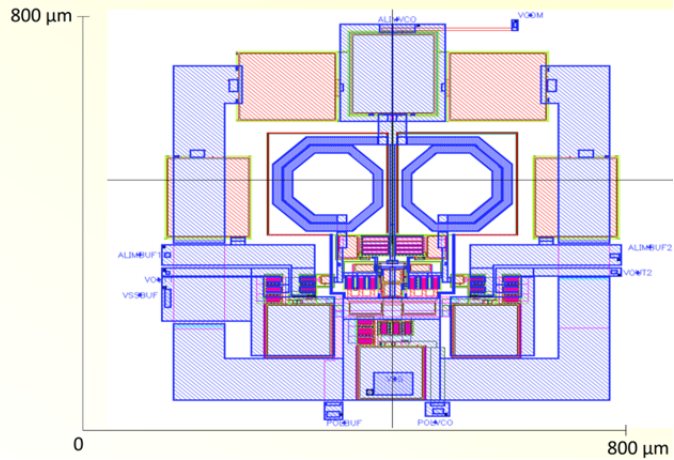


Figure 4.11. Sketch of layout

Colpitts oscillator

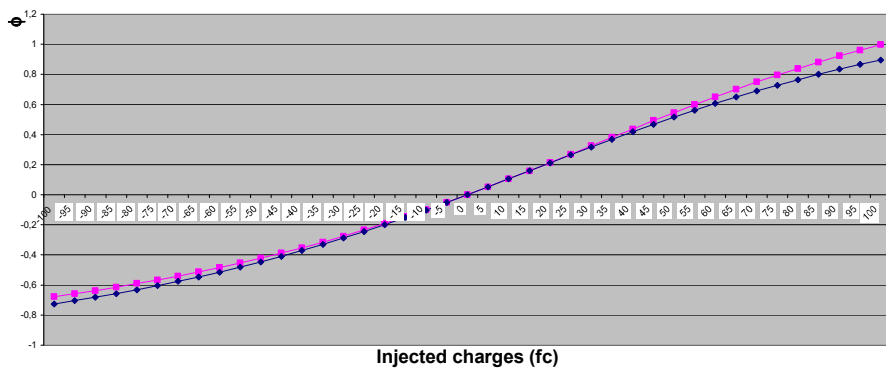


Figure 4.14. Phase shift versus injected charges – between collector (L) and ground (see Figure 4.12) – for oscillator in Figure 4.12. Mixed mode: squares; “arctan” fit: diamonds (lower curve)

Example of a phase detector(PD)

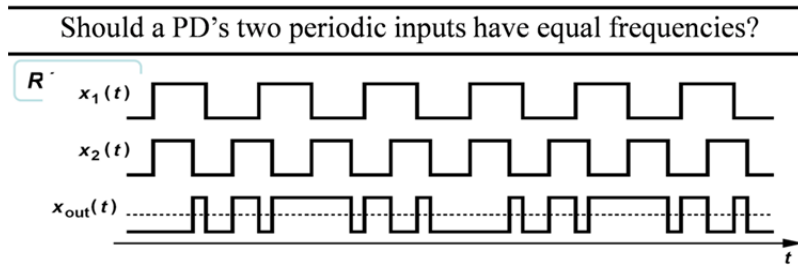
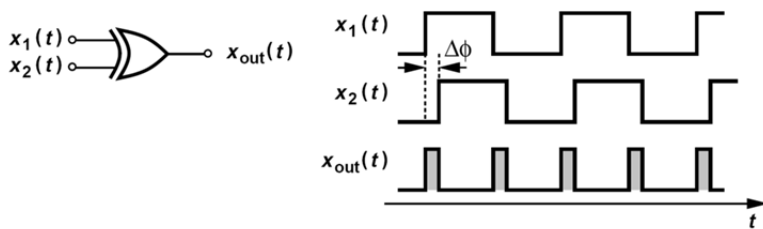


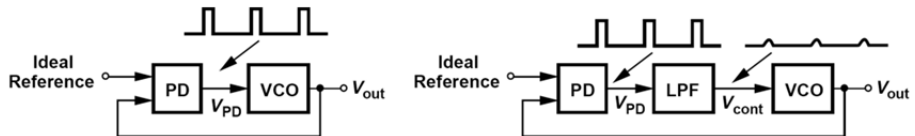
Figure 4.19. Phase difference versus frequency

How is the PD implemented?



- We seek a circuit whose average output is proportional to the input phase difference.
- An exclusive-or 'XOR' window can serve this purpose, generating impulses whose breadth is equal to $\Delta\phi$

Figure 4.20. Phase difference via an exclusive-or



- Negative feedback loop: if the 'loop gain' is high enough, the circuit minimizes the sampling error.
- Interpose a low-pass filter between the PD and the VCO to remove these impulses.

Figure 4.21. Simple PLL and filter loop. Note that the negative feedback loop should force the phase error to zero, in which case the PD generates no impulse and the VCO is not disrupted. Thus, the low-pass filter would not be needed. In fact, this feedback system suffers a finite loop gain presenting a finite phase error in stationary state. Even the PLL has an infinite loop gain containing nonlinearities that disturb V_{cont}

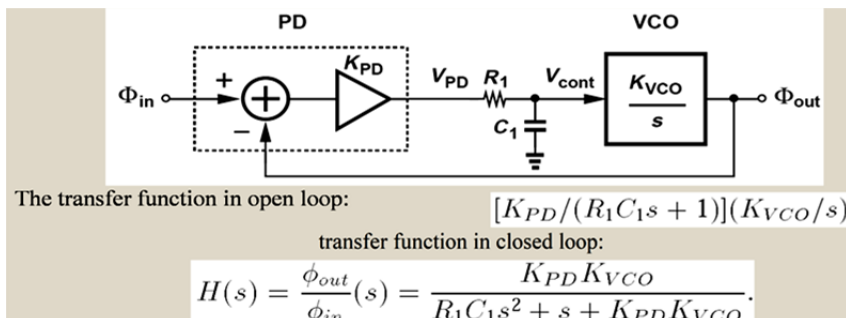
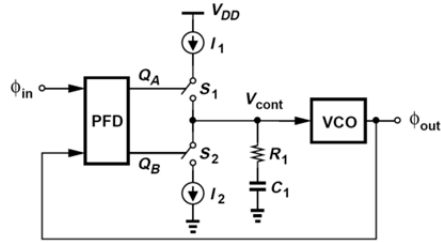
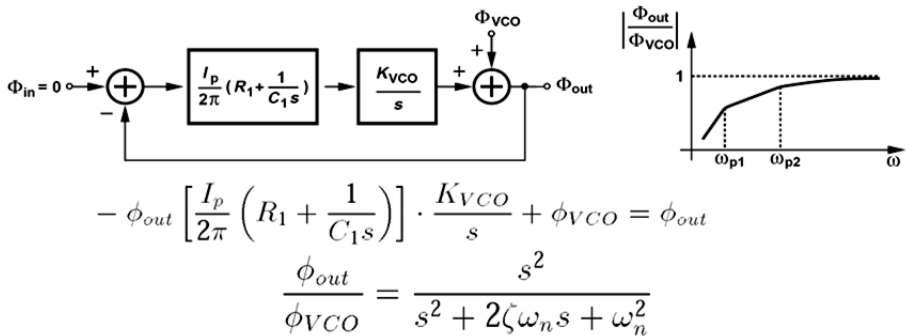


Figure 4.24. Loop dynamic: model of the phase domain



➤ If one of the integrators has losses, the system can be stabilized. This can be accomplished by inserting a resistance in series.

Figure 4.25. PLL: charge pump



➤ The PLL cancels only the slow phase variations of the VCO.

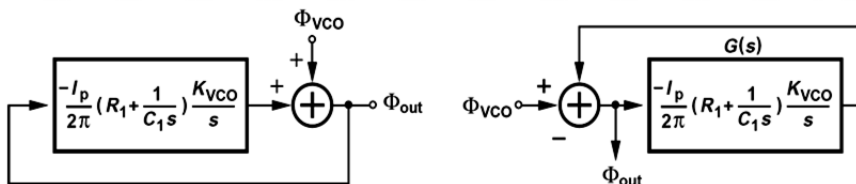


Figure 4.26. Phase noise in PLLs: phase noise

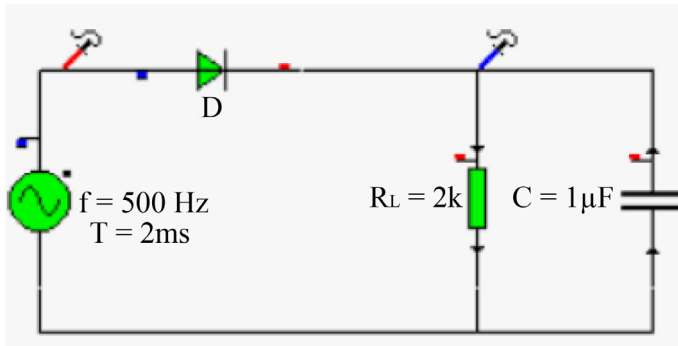


Figure A1.1. Schematic of a filtered mono-alternance rectifier

$$\begin{aligned}
 & (V_{DC} + m(t))\cos(\omega_c t) * \cos(\omega_c t) \\
 & (V_{DC} + m(t))\cos^2(\omega_c t) \\
 & (V_{DC} + m(t))\left(\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)\right) \\
 V_x = & \frac{V_{DC}}{2} + \frac{m(t)}{2} + \frac{V_{DC}}{2}\cos(2\omega_c t) + \frac{m(t)}{2}\cos(2\omega_c t)
 \end{aligned}$$

Figure A2.6. Coherent detection

$$s(t) = \begin{cases} A \cos(2\pi f_1 t) & \text{binary 1} \\ A \cos(2\pi f_2 t) & \text{binary 0} \end{cases}$$

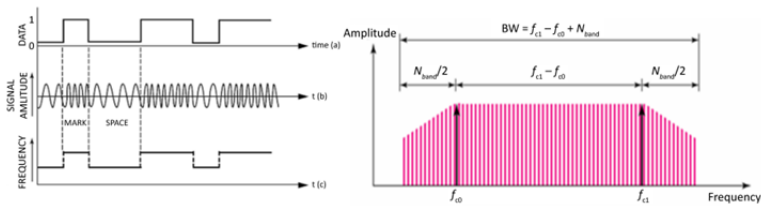


Figure A2.12. FSK spectrum

Carrier : $p(t) = A_p \cos 2\pi f_p t$ modulating: $m(t)$

$$\Rightarrow s(t) = (A_p + m(t)) \cos 2\pi f_p t$$

Or:

$$s(t) = A_p \left[1 + \frac{a}{A_p} m_0(t) \right] \cos 2\pi f_p t \quad \text{with } a = |\min(m(t))| \text{ and } m_0(t) = \frac{m(t)}{a}$$

EXPRESSION THAT BECOMES :

$$s(t) = A_p \left[1 + m \cdot m_0(t) \right] \cos 2\pi f_p t \quad \text{with } m = \frac{a}{A_p} \quad \text{index or modulation rate}$$

Figure A2.15. Linear amplitude modulations

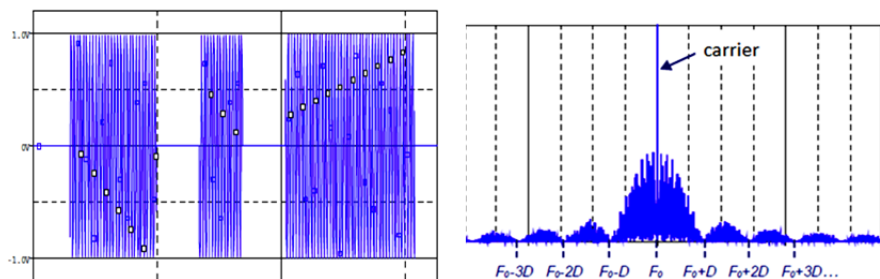


Figure A2.17. ASK modulation, with its Fourier transform

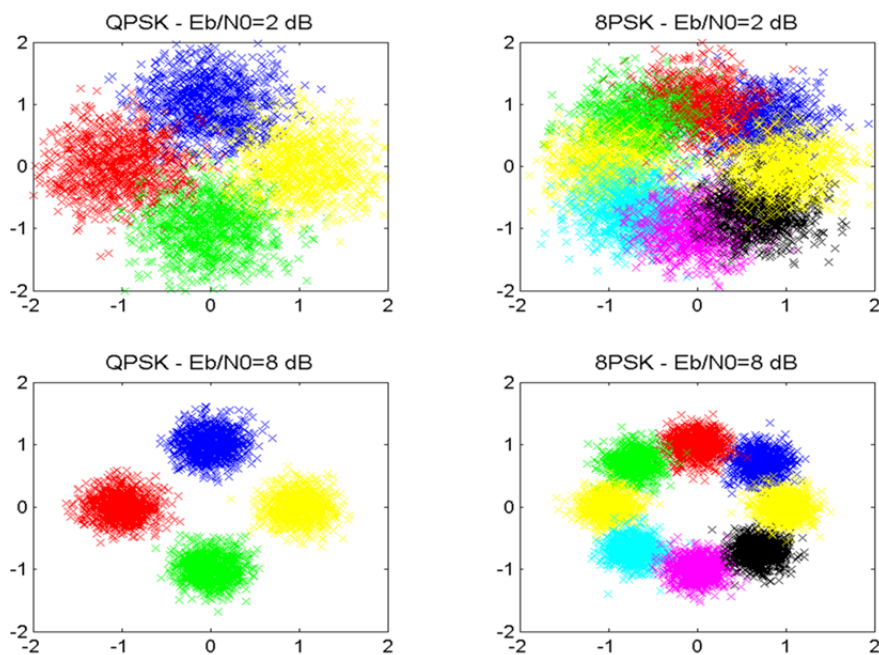


Figure A2.18. Example of samples of output from a filter adapted for some examples of passband QPSK/8PSK modulation

Signal in baseband:

$$x_{Tp}(t) = T \sum_{l=-\infty}^{\infty} (d'(l) + j d''(l)) g_{Tx}(t-lT)$$

Signal in the carrier's band:

$$\bar{E}_s = T^2 \overline{|D|^2} \int_{-\infty}^{\infty} g_{Tx}^2(t) dt$$

$$x_{Bp}(t) = \sqrt{2} \operatorname{Re} \{ x_{Tp}(t) e^{j2\pi f_0 t} \}$$

$$= \sqrt{2} T \left[\cos(2\pi f_0) \sum_{l=-\infty}^{\infty} d'(l) g_{Tx}(t-lT) - \sin(2\pi f_0) \sum_{l=-\infty}^{\infty} d''(l) g_{Tx}(t-lT) \right]$$

Average symbol energy:

$$\bar{E}_x = T^2 \cdot 2 \cdot \left[\frac{\overline{|D'|^2}}{2} + \frac{\overline{|D''|^2}}{2} \right] \cdot \int_{-\infty}^{\infty} g_{Tx}^2(t) dt = \bar{E}_s$$

Figure A2.19. Analysis of a baseband system

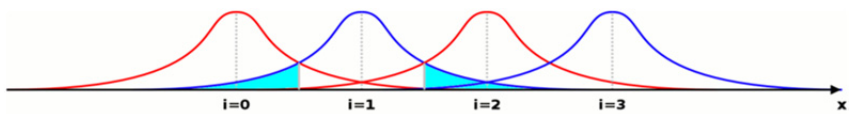


Figure A2.25. Error rate per bit

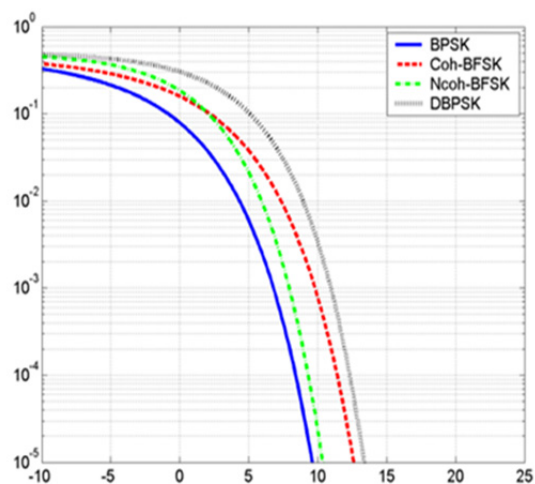


Figure A2.26. Example of BER – binary modulations

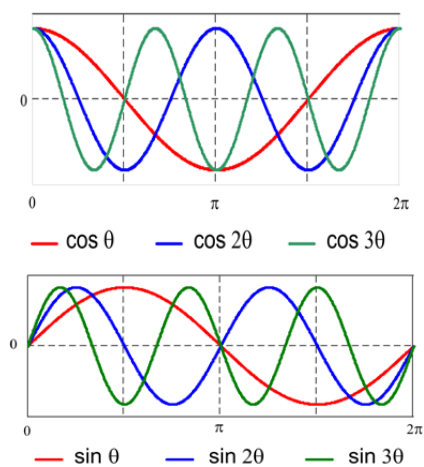


Figure A3.1. Some sines and cosines representations

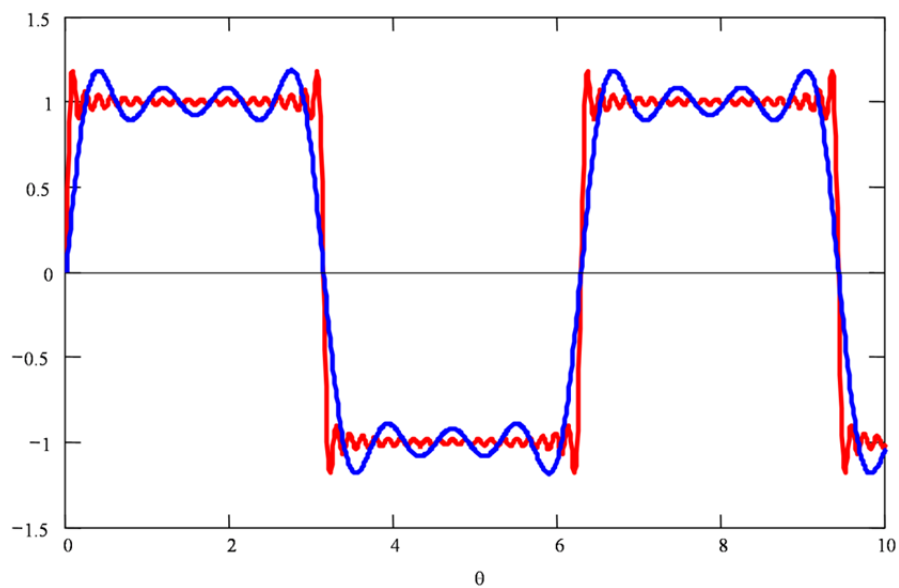


Figure A3.2. *Fourier analysis. The red curve is calculated with 20 terms and the blue with 4 terms*

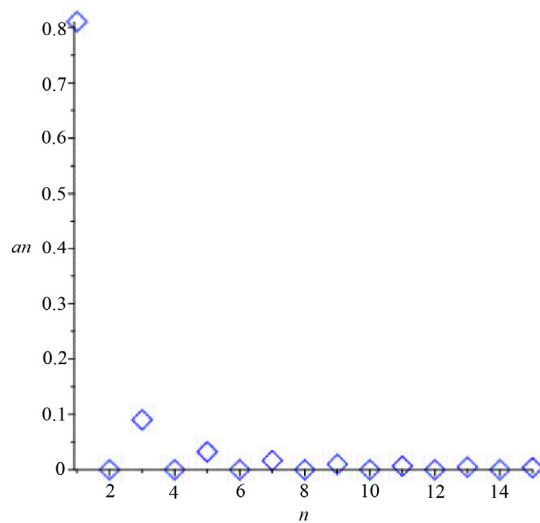


Figure A3.3. *a_n : function of n*

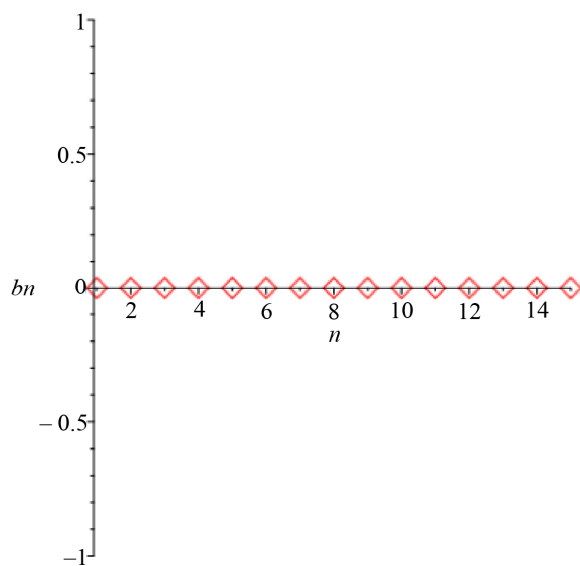


Figure A3.4. bn : function of n

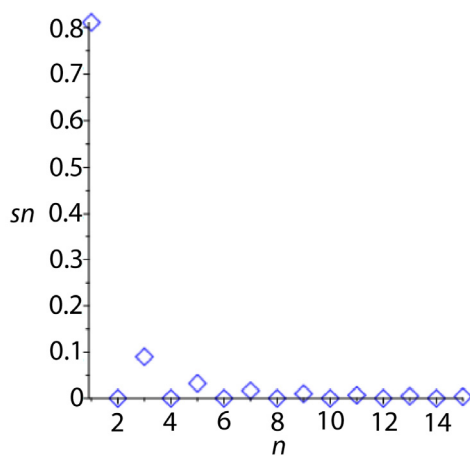


Figure A3.5. sn : function of n

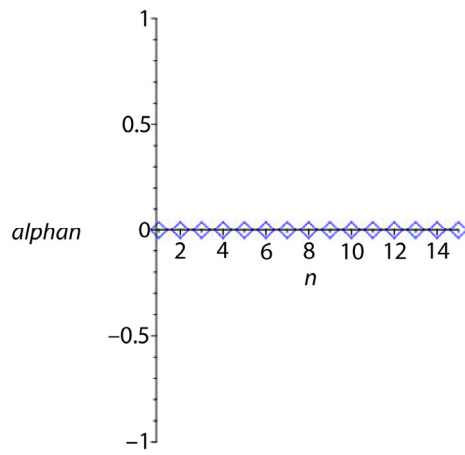


Figure A3.6. *alphan*: function of *n*

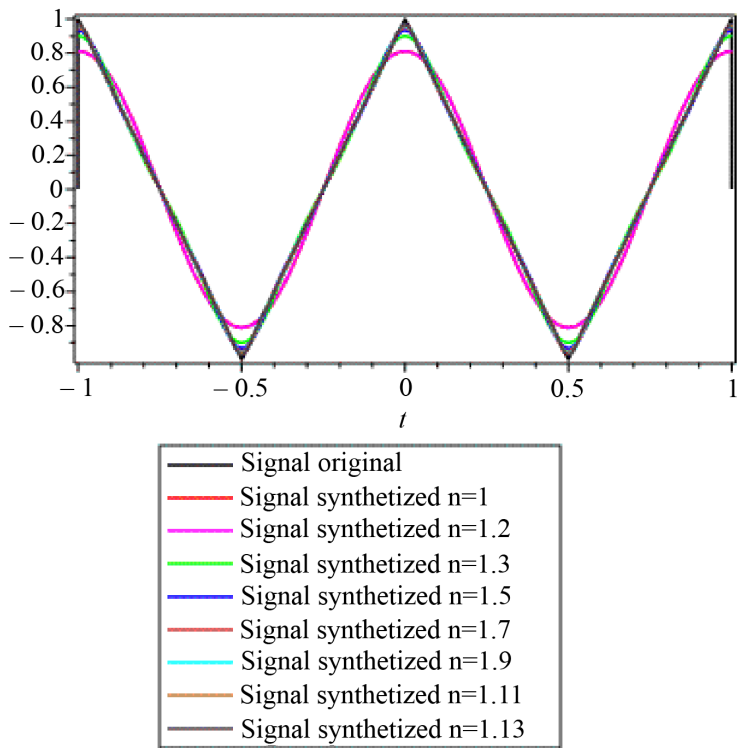


Figure A3.8. *Signal synthesized with different numbers of harmonics*