

Table of Contents

Preface	xvii
A. Ridha MAHJOUR	
Chapter 1. Partition Inequalities: Separation, Extensions, and Network Design	1
Mourad BAÏOU, Francisco BARAHONA and A. Ridha MAHJOUR	
1.1. Introduction	1
1.2. The case $b \leq -1$	3
1.2.1. The attack problem	3
1.2.2. Finding tight sets	6
1.3. The case $b > -1$	8
1.3.1. Queyranne's algorithm	9
1.4. Partition inequalities with terminals	10
1.5. Extensions	12
1.5.1. The strength of a network	12
1.5.2. Principal sequence of partitions of a graph	13
1.5.3. Network reinforcement	14
1.5.4. Packing spanning trees	16
1.5.5. Increasing the weight of minimum spanning trees	16
1.5.6. Potts' model in statistical physics	17
1.6. Survivable networks	18
1.6.1. Valid inequalities	19
1.6.2. Polyhedral consequences	24
1.7. Critical extreme points	25
1.8. Survivability with length constraints	29
1.8.1. Survivability with bounded rings	29
1.8.2. Hop-constrained paths	32
1.9. Bibliography	34

Chapter 2. Stable Sets in Claw-Free Graphs: A Journey Through Algorithms and Polytopes	41
Yuri FAENZA, Gianpaolo ORIOLO, Gautier STAUFFER and Paolo VENTURA	
2.1. Introduction	41
2.2. Stable sets in claw-free graphs: some classical results	43
2.2.1. Algorithms for the maximum weighted stable set problem	43
2.2.2. Stable sets in claw-free graphs: polyhedral issues	48
2.3. A breakthrough: decomposition of claw-free and quasi-line graphs	54
2.3.1. The composition of strips	55
2.3.2. Decomposition results for quasi-line and claw-free graphs	56
2.3.3. Algorithmic decomposition results for quasi-line and claw-free graphs	57
2.4. Following the breakthrough: stable sets in claw-free graphs revisited	60
2.4.1. Faster algorithm for computing the <i>MWSS</i>	60
2.4.2. The stable set polytope of quasi-line graphs	63
2.4.3. The stable set polytope of claw-free graphs with $\alpha \geq 4$ and no clique cutsets	65
2.4.4. Extended formulation for the stable set polytope of claw-free graphs	67
2.4.5. Separation routines for the stable set polytope of claw-free graphs	69
2.5. Open questions	71
2.5.1. Complete linear description of <i>STAB</i> (<i>G</i>) in the original space	71
2.5.2. Calvillo's theorem and the intersection property	72
2.5.3. Minimal linear description for the stable set polytope of quasi-line graphs	73
2.5.4. The Chvatal-Gomory rank of the stable set polytope of quasi-line graphs	73
2.5.5. Improving the complexity	73
2.5.6. A "short" proof of the Ben Rebea theorem	74
2.6. Acknowledgments	74
2.7. Bibliography	74
2.8. Appendix: Some details on Theorem 2.13.	78
Chapter 3. Algorithms for Submodular Totally Dual Integral Problems	81
S. Thomas MCCORMICK	
3.1. Introduction.	81
3.1.1. Total unimodularity	82
3.1.2. Total dual integrality	84

3.1.3. Submodularity	86
3.2. An algorithmic framework for some TDI systems	87
3.2.1. The primal–dual algorithm	87
3.2.2. The SSPs variant of primal–dual	88
3.2.3. A generic algorithm for some TDI systems	89
3.3. Hoffman’s models	91
3.3.1. Cut packing and blocking	91
3.3.2. The WAF model	93
3.3.3. The weighted abstract cut packing model	95
3.3.4. Blocking relation	96
3.3.5. Extending SSP for these abstract models	96
3.4. The SSP algorithm for WAF	98
3.4.1. Preliminaries	98
3.4.2. The SSP framework	100
3.4.3. Solving AMFMC	102
3.4.4. The WAF algorithm	121
3.4.5. A polynomial capacity-scaling WAF algorithm	129
3.5. The SSP algorithm for WACP	134
3.5.1. Preliminaries	135
3.5.2. Outer framework of the algorithm	135
3.5.3. Solving the restricted problems	138
3.5.4. The algorithm	142
3.5.5. A polynomial cost-scaling WACP algorithm	145
3.6. Extension to $0, \pm 1$ matrices	146
3.6.1. Signed submodularity is bisubmodularity	146
3.6.2. A signed version of WAF	147
3.6.3. A signed version of WACP	147
3.7. Future work	147
3.8. Acknowledgments	148
3.9. Bibliography	148
Chapter 4. Finding Descriptions of Polytopes via Extended Formulations and Liftings	151
Volker KAIBEL and Andreas LOOS	
4.1. Introduction	151
4.2. Lifting method	152
4.3. Path-set polytopes	155
4.4. Polytopes of small cliques	158
4.5. Orbisacks	162
4.6. Conclusion	168
4.7. Acknowledgments	169
4.8. Bibliography	169

Chapter 5. Relax-and-Cut as a Preprocessor and Warm Starter to Branch-and-Cut	171
Abilio LUCENA, Nelson MACULAN and Alexandre SALLES DA CUNHA	
5.1. Introduction	171
5.2. The degree-constrained minimum ST problem	173
5.2.1. A standard DCMSTP formulation and valid inequalities	175
5.3. An NDRC algorithm for DCMSTP	176
5.3.1. Separating BIs over Lagrangian subproblems	177
5.3.2. Using dual information to generate primal feasible solutions	179
5.3.3. Variable fixing tests and subgradient optimization cycles	181
5.4. NDRC as a warm starter to branch-and-cut	182
5.4.1. The crossover step	182
5.5. A branch-and-cut algorithm for DCMSTP	184
5.5.1. Valid inequalities and separation procedures	185
5.5.2. Cut pool rules	187
5.5.3. Some additional implementation details	187
5.6. Computational experiments	187
5.6.1. Results for NDRC as a stand-alone procedure	189
5.6.2. Results for the hybrid algorithm	191
5.7. Conclusions	196
5.8. Acknowledgments	197
5.9. Bibliography	197
Chapter 6. Weighted Transversals and Blockers for Some Optimization Problems in Graphs	203
Cédric BENTZ, Marie-Christine COSTA, Dominique DE WERRA, Christophe PICOULEAU and Bernard RIES	
6.1. Introduction	203
6.2. Preliminaries	204
6.3. Stable sets	208
6.3.1. Split graphs	209
6.3.2. Bipartite graphs	212
6.3.3. Trees	212
6.3.4. Cobipartite graphs	214
6.4. Cliques vs. stable sets	215
6.5. Matchings	216
6.5.1. Bipartite graphs	217
6.5.2. Chains and cycles	217
6.5.3. Complete graphs and regular bipartite graphs	218
6.5.4. Trees and grid graphs	218
6.6. Summary and conclusion	220

6.7. Acknowledgments	221
6.8. Bibliography	221
Chapter 7. On a Time-Dependent Formulation and an Updated Classification of ATSP Formulations	223
Maria Teresa GODINHO, Luis GOUVEIA and Pierre PESNEAU	
7.1. Introduction	223
7.2. Flow based formulations	225
7.2.1. A Generic Model	225
7.2.2. Path-based formulations	226
7.2.3. Circuit-based formulations	227
7.2.4. n -Circuit-based formulations	229
7.2.5. Enhancing the C-MCF model (using time-dependent variables)	232
7.2.6. Enhancing the C-MCF model (using precedence variables)	233
7.3. Comparing with the Picard and Queyranne formulation	235
7.4. Comparing with the Sherali and Driscoll Formulation	237
7.5. Comparing with the Sherali, Sarin and Tsai Formulation	245
7.6. Computational results and conclusions	250
7.7. Acknowledgments	252
7.8. Bibliography	252
Chapter 8. Cuts Over Extended Formulations by Flow Discretization	255
Eduardo UCHOA	
8.1. Introduction	255
8.2. The fixed charge network flow problem: natural and extended formulations	262
8.2.1. Natural formulation	262
8.2.2. The multicommodity extended reformulation	262
8.2.3. The discretized-flow extended reformulation	263
8.3. The CMST over the capacity-indexed formulation	265
8.3.1. Formulations and valid inequalities	266
8.3.2. Experimental results	270
8.4. Other applications	273
8.4.1. The VRP over the load-indexed formulation	273
8.4.2. The parallel machine scheduling problem over the arc-time-indexed formulation	277
8.5. Conclusions	279
8.6. Acknowledgments	279
8.7. Bibliography	279

Chapter 9. Model Equivalents and Cutting-Plane Decomposition Methods for Dominance-Constrained Two-stage Stochastic Programs . . .	283
Dimitri DRAPKIN, Oliver KLAAR and Rüdiger SCHULTZ	
9.1. Introduction	283
9.2. Mixed-integer recourse: model equivalents revisited and refined	286
9.3. Linear recourse: model equivalents based on duality	292
9.4. Cutting plane methods	297
9.5. Computations	304
9.5.1. First-order models	304
9.5.2. Second-order models	307
9.6. Bibliography	309
Chapter 10. Combinatorial Optimization Problems Arising from Interactive Congestion Situations	311
Laurent GOURVÈS and Stefano MORETTI	
10.1. Introduction	311
10.2. Preliminaries and notations	314
10.2.1. Notations for networks	314
10.2.2. Strategic games	315
10.2.3. Cooperative games	316
10.3. (Strategic) congestion games	317
10.3.1. A combinatorial model	317
10.3.2. Nash equilibria: existence, convergence, and computation	319
10.3.3. Inefficiency of equilibria	323
10.3.4. Strong equilibria	324
10.4. (Cooperative) congestion games	328
10.4.1. Cost allocation on connection situations	329
10.4.2. Congestion network situations	331
10.4.3. Extended models and applications	334
10.5. Conclusions	337
10.6. Acknowledgments	339
10.7. Bibliography	339
Chapter 11. Combinatorial Optimization Methods to Determine the Rank of a Matrix over a Communicative Ring, with Engineering Applications	343
András RECSKI	
11.1. A classical case	344
11.2. An “even more classical” case	345
11.3. Variations on a theme by Kirchhoff	346
11.4. Further variations	348

11.5. A hierarchy of the devices	350
11.6. Acknowledgments	351
11.7. Bibliography	351
Chapter 12. Robust Routing in Communication Networks	353
Walid BEN-AMEUR, Adam OUOROU and Mateusz ŻOTKIEWICZ	
12.1. Introduction	353
12.2. Notation	355
12.3. Modeling traffic uncertainty	356
12.3.1. Traffic matrix estimation	356
12.3.2. Models based on stochastic programming	357
12.3.3. Models based on robust optimization	358
12.4. Different robust routing strategies	361
12.4.1. Static routing	361
12.4.2. Dynamic routing	365
12.4.3. Domination-based approach	367
12.4.4. Multistatic routing	369
12.4.5. Affine routing	371
12.4.6. Multipolar routing	375
12.5. Local routing	379
12.6. Routing with additional requirements	380
12.7. Other robustness problems	382
12.8. Conclusion	384
12.9. Bibliography	384
Chapter 13. Single Machine Scheduling with a Common Due Date: Total Weighted Tardiness Problems	391
Imed KACEM, Hans KELLERER and Vitaly STRUSEVICH	
13.1. Introduction	391
13.2. Total weighted tardiness: complexity and dynamic programming algorithms	393
13.2.1. Dynamic programming algorithm based on fixing a straddling job	394
13.2.2. Dynamic programming algorithm without fixing a straddling job	398
13.3. Total weighted tardiness: approximation	399
13.3.1. A constant rate approximation algorithm	399
13.3.2. The FPTAS by Kellerer and Strusevich	400
13.3.3. The FPTAS by Kacem	401
13.3.4. Weighted tardiness minimization with a fixed number of due dates	403

13.4. Total weighted earliness and tardiness	404
13.4.1. The half-product problem	406
13.4.2. Unrestrictive case	408
13.4.3. The symmetric quadratic knapsack problem	410
13.4.4. Restrictive case	415
13.5. Conclusions	418
13.6. Bibliography	419
Chapter 14. Convergent Tabu Search for Optimal Partitioning	423
Fred GLOVER and Saïd HANAFI	
14.1. Introduction	423
14.2. Problem formulation.	424
14.2.1 0–1 IP problems	426
14.3. Convergent TS	427
14.3.1. Convergent TS (CTS)	428
14.3.2. Tabu tree search (TTS)	429
14.4. TS specialization for OP	430
14.4.1. Memory structure	431
14.4.2. Consequences of the specialized organization	432
14.4.3. Structure of the method	433
14.4.4. The specialized TS-OP method	433
14.5. A dynamic tabu list version	436
14.5.1. Dynamic TS-OP method.	438
14.6. Bibliography	439
14.7. Appendix: a bounded formulation.	440
Chapter 15. An Introduction to Exponential Time Exact	
Algorithms for Solving NP-hard Problems	443
Nicolas BOURGEOIS, Bruno ESCOFFIER and Vangelis Th. PASCHOS	
15.1. Introduction	443
15.2. Combinatorial methods	444
15.2.1. Brute force approaches	444
15.2.2. Dynamic programming	446
15.2.3. Inclusion-exclusion principle	447
15.3. Branching algorithms	451
15.3.1. Branch and reduce: basic principles	451
15.4. Other approaches	463
15.4.1. Sorting and searching	463
15.4.2. Iterative local search	464
15.5. Conclusion	465
15.6. Bibliography	466

Chapter 16. Moderately Exponential Approximation	469
Nicolas BOURGEOIS, Bruno ESCOFFIER, Vangelis Th. PASCHOS and Emeric TOURNIAIRE	
16.1. Introduction	469
16.2. What is moderately exponential approximation?	470
16.3. Generating a “small” number of candidate solutions	474
16.4. Divide-and-approximate	475
16.5. Approximately pruning the search tree	478
16.6. Randomization	482
16.7. Final remarks	484
16.8. Bibliography	485
Chapter 17. Progress in Semidefinite Optimization Techniques for Satisfiability	489
Miguel F. ANJOS	
17.1. Introduction	489
17.2. Background and notation	491
17.2.1. The satisfiability problem	492
17.2.2. Semidefinite optimization	493
17.3. The sphere relaxation and its application to MAX-SAT	493
17.4. Elliptic approximations and the gap relaxation of SAT	496
17.5. Relaxations of SAT and MAX-SAT via polynomial optimization	500
17.5.1. SOS relaxations	501
17.5.2. Higher lifting relaxations of SAT	503
17.5.3. Linearly sized higher lifting relaxations	505
17.6. Sufficient conditions for the exactness of the partial higher lifting relaxations	510
17.7. Exactness of SDP relaxations for structured classes of SAT	512
17.7.1. Pigeonhole instances	512
17.7.2. Tseitín instances	513
17.8. Semidefinite resolution and a direct proof of the exactness of Lasserre’s relaxation	514
17.9. Future research directions	517
17.10. Acknowledgments	517
17.11. Bibliography	517
Chapter 18. Disjunctive Cuts for Mixed Integer Nonlinear Programming Problems	521
Pierre BONAMI, Jeff LINDEROTH and Andrea LODI	
18.1. Introduction	521
18.2. Linear relaxations are enough for linear cuts	524

18.3. The concavity cut	526
18.4. Disjunctive programming basics	528
18.5. Disjunctive cuts for convex MINLPs	530
18.5.1. Disjunctive cuts via LP only	532
18.5.2. Disjunctive cuts via nonlinear programming	534
18.6. Disjunctive cuts for non-convex MINLPs	535
18.6.1. Indefinite quadratic constraints	537
18.6.2. Factorable MINLPs	539
18.7. Acknowledgments	540
18.8. Bibliography	541
Chapter 19. Using Extended MIP Formulations for a Production/Sequencing and a Production/Distribution Problem	545
Laurence A. WOLSEY	
19.1. Introduction	545
19.2. Extended formulations	546
19.2.1. When to use extended formulations?	546
19.3. What to do when the EFs become large?	547
19.3.1. Hybrid heuristics	547
19.3.2. Hybrid algorithms	548
19.4. Discrete lot-sizing with start-ups on identical parallel machines	549
19.4.1. The formulations	549
19.4.2. Computation	551
19.5. A two-level production/distribution problem	553
19.5.1. Formulations	553
19.5.2. Computation	555
19.6. Concluding remarks	556
19.7. Bibliography	557
Chapter 20. The Two-Phase Method for Multiobjective Combinatorial Optimization Problems	559
Anthony PRZYBYLSKI, Xavier GANDIBLEUX and Matthias EHRGOTT	
20.1. Introduction	559
20.2. The case of two objectives	562
20.2.1. Phase 1	562
20.2.2. Phase 2	564
20.3. Phase 1 with more than two objectives	566
20.3.1. Difficulties	566
20.3.2. Geometry, adjacency, and weight space decomposition	567
20.3.3. Foundations of the algorithm	571
20.3.4. The algorithm	575
20.4. Phase 2 with more than two objectives	578

20.4.1. Difficulties	579
20.4.2. Description of the search area	581
20.4.3. Exploration of the search area	588
20.4.4. Algorithm	591
20.5. Conclusion	591
20.6. Bibliography	592
List of Authors	597
Index	601