Contents

Foreword ix

Preface xi

Introduction xiii

Chapter 1. Stress 1

1.1. Notion of stress 1
1.1.1. External forces 1
1.1.2. Internal cohesive forces 2
1.1.3. Normal stress, shear stress 2
1.2. Properties of the stress vector 3
1.2.1. Boundary conditions 3
1.2.2. Torsor of internal forces 5
1.2.3. Reciprocal actions 8
1.2.4. Cauchy reciprocal theorem 9
1.3. Stress matrix 11
1.3.1. Notation 11
1.3.2. Invariants of the stress tensor 13
1.3.3. Relation between the stress matrix and the stress vector 15
1.3.4. Principal stresses and principal directions 18
1.4. Equilibrium equation 21
1.5. Mohr's circle 23

Chapter 2. Strain 27

2.1. Notion of strain 27
2.1.1. Displacement vector 27

2.1.2. Unit strain 28
2.1.3. Angular distortion. 30
2.2. Strain matrix 33
2.2.1. Definition of the strain matrix 33
2.2.2. Principal strains and principal directions 37
2.2.3. Volume expansion 39
2.2.4. Invariants of strain tensor 40
2.2.5. Compatibility condition 40
2.3. Strain measurement: strain gage 41

Chapter 3. Behavior Law 43

3.1. A few definitions 43
3.2. Tension test. 43
3.2.1. Brittle materials 44
3.2.2. Ductile materials 45
3.2.3. Particular cases 46
3.3. Shear test 46
3.3.1. Brittle materials 47
3.3.2. Ductile materials 48
3.4. General rule 48
3.4.1. Linear elasticity 48
3.5. Anisotropic materials: example of a composite 53
3.5.1. Elasticity 53
3.6. Thermoelasticity 54

Chapter 4. Resolution Methods 59

4.1. Assessment 59
4.2. Displacement method 61
4.3. Stress method 61
4.4. Finite element method 62

Chapter 5. Work-energy Theorem: Principle of Finite Element Method 63

5.1. Work-energy theorem 63
5.1.1. Hypotheses 63
5.1.2. Strain energy 64
5.1.3. Work of external forces 65
5.1.4. Strain energy 66
5.1.5. Energy minimization: Ritz method 68

5.2. Finite element method 69
5.2.1. General principle of finite element method 69
5.2.2. Example of the three-node triangular element 74
5.3. Application: triangle with plate finite element using Catia 80

Chapter 6. Sizing Criteria of an Aeronautical Structure 83

6.1. Introduction 83
6.2. Experimental determination of a sizing criterion 85
6.3. Normal stress or principal stress criterion: brittle material 87
6.4. Stress or maximum shear energy criterion: ductile material 91
6.4.1. Tresca criterion 91
6.4.2. Von Mises criterion 93
6.4.3. Rupture of a ductile material 96
6.5. Maximum shear criterion with friction:
compression of brittle materials 99
6.6. Anisotropic criterion: example of the composite 105

Chapter 7. Plasticity 109

7.1. Introduction 109
7.2. Plastic instability: necking, true stress and true strain 111
7.3. Plastic behavior law: Ramberg–Osgood law 116
7.4. Example of an elastic–plastic calculation:
plate with open hole in tension 118

Chapter 8. Physics of Aeronautical Structure Materials 127

8.1. Introduction 127
8.2. Aluminum 2024 130
8.3. Carbon/epoxy composite T300/914 135
8.4. Polymers 140

Chapter 9. Exercises 151

9.1. Rosette analysis 151
9.2. Pure shear 154
9.3. Compression of an elastic solid 154
9.4. Gravity dam 155
9.5. Shear modulus 156
9.6. Modulus of a composite 157
9.7. Torsional cylinder 158
9.8. Plastic compression 160
9.9. Bi-material beam tension 162

9.10. Beam thermal expansion . . . 164
9.11. Cube under shear stress . . . 165
9.12. Spherical reservoir under pressure . . . 166
9.13. Plastic bending . . . 169
9.14. Disc under radial tension . . . 171
9.15. Bending beam: resolution by the Ritz method . . . 173
9.16. Stress concentration in open hole . . . 174
9.17. Bending beam . . . 178

Chapter 10. Solutions to Exercises . . . 183

10.1. Rosette analysis . . . 183
10.2. Pure shear . . . 191
10.3. Compression of an elastic solid . . . 192
10.4. Gravity dam . . . 196
10.5. Shear modulus . . . 201
10.6. Modulus of a composite . . . 203
10.7. Torsional cylinder . . . 206
10.8. Plastic compression . . . 212
10.9. Bi-material beam tension . . . 215
10.10. Beam thermal expansion . . . 225
10.11. Cube under shear stress . . . 231
10.12. Spherical reservoir under pressure . . . 235
10.13. Plastic bending . . . 240
10.14. Disc under radial tension . . . 245
10.15. Bending beam: resolution by the Ritz method . . . 252
10.16. Stress concentration in open hole . . . 256
10.17. Bending beam . . . 259

Appendix . . . 273

Bibliography . . . 279

Index . . . 281