

Table of Contents

Introduction	17
Chapter 1. Basic Concepts	21
1.1 The origin of the graph concept	21
1.2 Definition of graphs	24
1.2.1 Notation	24
1.2.2 Representation	25
1.2.3 Terminology	25
1.2.4 Isomorphism and unlabeled graphs	26
1.2.5 Planar graphs	27
1.2.6 Complete graphs	28
1.3 Subgraphs	28
1.3.1 Customary notation	29
1.4 Paths and cycles	29
1.4.1 Paths	29
1.4.2 Cycles	31
1.4.3 Paths and cycles as graphs	33
1.5 Degrees	33
1.5.1 Regular graphs	34
1.6 Connectedness	35

8	Graph Theory and Applications	
1.7	Bipartite graphs	36
1.7.1	Characterization	37
1.8	Algorithmic aspects	37
1.8.1	Representations of graphs inside a machine	38
1.8.2	Weighted graphs	41
1.9	Exercises	41
	Chapter 2. Trees	45
2.1	Definitions and properties	45
2.1.1	First properties of trees	46
2.1.2	Forests	47
2.1.3	Bridges	47
2.1.4	Tree characterizations	48
2.2	Spanning trees	49
2.2.1	An interesting illustration of trees	52
2.2.2	Spanning trees in a weighted graph	53
2.3	Application: minimum spanning tree problem	54
2.3.1	The problem	54
2.3.2	Kruskal's algorithm	55
2.3.3	Justification	57
2.3.4	Implementation	58
2.3.5	Complexity	59
2.4	Connectivity	59
2.4.1	Block decomposition	60
2.4.2	k -connectivity	61
2.4.3	k -connected graphs	62
2.4.4	Menger's theorem	63
2.4.5	Edge connectivity	63

2.4.6	<i>k</i> -edge-connected graphs	64
2.4.7	Application to networks	65
2.4.8	Hypercube	65
2.5	Exercises	66
Chapter 3. Colorings		71
3.1	Coloring problems	71
3.2	Edge coloring	71
3.2.1	Basic results	72
3.3	Algorithmic aspects	73
3.4	The timetabling problem	75
3.4.1	Room constraints	76
3.4.2	An example	78
3.4.3	Conclusion	81
3.5	Exercises	81
Chapter 4. Directed Graphs		83
4.1	Definitions and basic concepts	83
4.1.1	Notation	83
4.1.2	Terminology	83
4.1.3	Representation	84
4.1.4	Underlying graph	85
4.1.5	“Directed” concepts	85
4.1.6	Indegrees and outdegrees	86
4.1.7	Strongly connected components	87
4.1.8	Representations of digraphs inside a machine	88
4.2	Acyclic digraphs	90
4.2.1	Acyclic numbering	90

10 Graph Theory and Applications

4.2.2	Characterization	91
4.2.3	Practical aspects	92
4.3	Arborescences	92
4.3.1	Drawings	92
4.3.2	Terminology	93
4.3.3	Characterization of arborescences	94
4.3.4	Subarborescences	95
4.3.5	Ordered arborescences	95
4.3.6	Directed forests	95
4.4	Exercises	95

Chapter 5. Search Algorithms **97**

5.1	Depth-first search of an arborescence	97
5.1.1	Iterative form	98
5.1.2	Visits to the vertices	100
5.1.3	Justification	102
5.1.4	Complexity	102
5.2	Optimization of a sequence of decisions	103
5.2.1	The eight queens problem	103
5.2.2	Application to game theory: finding a winning strategy	105
5.2.3	Associated arborescence	105
5.2.4	Example	106
5.2.5	The minimax algorithm	106
5.2.6	Implementation	107
5.2.7	In concrete terms	108
5.2.8	Pruning	108
5.3	Depth-first search of a digraph	109
5.3.1	Comments	110

Table of Contents 11

- 5.3.2 Justification 112
- 5.3.3 Complexity 113
- 5.3.4 Extended depth-first search 113
- 5.3.5 Justification 114
- 5.3.6 Complexity 115
- 5.3.7 Application to acyclic numbering 115
- 5.3.8 Acyclic numbering algorithms 116
- 5.3.9 Practical implementation 117
- 5.4 Exercises 117

Chapter 6. Optimal Paths 119

- 6.1 Distances and shortest paths problems 119
 - 6.1.1 A few definitions 119
 - 6.1.2 Types of problems 120
- 6.2 Case of non-weighted digraphs: breadth-first search 120
 - 6.2.1 Application to calculation of distances 122
 - 6.2.2 Justification and complexity 123
 - 6.2.3 Determining the shortest paths 124
- 6.3 Digraphs without circuits 125
 - 6.3.1 Shortest paths 127
 - 6.3.2 Longest paths 127
 - 6.3.3 Formulas 127
- 6.4 Application to scheduling 128
 - 6.4.1 Potential task graph 128
 - 6.4.2 Earliest starting times 129
 - 6.4.3 Latest starting times 130
 - 6.4.4 Total slacks and critical tasks 131
 - 6.4.5 Free slacks 131

12	Graph Theory and Applications	
6.4.6	More general constraints	133
6.4.7	Practical implementation	133
6.5	Positive lengths	134
6.5.1	Justification	135
6.5.2	Associated shortest paths	138
6.5.3	Implementation and complexity	140
6.5.4	Undirected graphs	140
6.6	Other cases	142
6.6.1	Floyd's algorithm	142
6.7	Exercises	143
Chapter 7.	Matchings	149
7.1	Matchings and alternating paths	149
7.1.1	A few definitions	149
7.1.2	Concept of alternating paths and Berge's theorem	151
7.2	Matchings in bipartite graphs	152
7.2.1	Matchings and transversals	154
7.3	Assignment problem	156
7.3.1	The Hungarian method	156
7.3.2	Justification	158
7.3.3	Concept of alternating trees	159
7.3.4	Complexity	159
7.3.5	Maximum matching algorithm	160
7.3.6	Justification	161
7.3.7	Complexity	161
7.4	Optimal assignment problem	164
7.4.1	Kuhn-Munkres algorithm	165
7.4.2	Justification	168

7.4.3	Complexity	169
7.5	Exercises	171
Chapter 8. Flows		173
8.1	Flows in transportation networks	173
8.1.1	Interpretation	175
8.1.2	Single-source single-sink networks	176
8.2	The max-flow min-cut theorem	177
8.2.1	Concept of unsaturated paths	178
8.3	Maximum flow algorithm	180
8.3.1	Justification	182
8.3.2	Complexity	187
8.4	Flow with stocks and demands	188
8.5	Revisiting theorems	191
8.5.1	Menger's theorem	191
8.5.2	Hall's theorem	193
8.5.3	König's theorem	193
8.6	Exercises	194
Chapter 9. Euler Tours		197
9.1	Euler trails and tours	197
9.1.1	Principal result	199
9.2	Algorithms	201
9.2.1	Example	202
9.2.2	Complexity	204
9.2.3	Elimination of recursion	204
9.2.4	The Rosenstiehl algorithm	204
9.3	The Chinese postman problem	207

14	Graph Theory and Applications	
9.3.1	The Edmonds-Johnson algorithm	209
9.3.2	Complexity	210
9.3.3	Example	210
9.4	Exercises	212
Chapter 10. Hamilton Cycles		215
10.1	Hamilton cycles	215
10.1.1	A few simple properties	216
10.2	The traveling salesman problem	218
10.2.1	Complexity of the problem	219
10.2.2	Applications	219
10.3	Approximation of a difficult problem	220
10.3.1	Concept of approximate algorithms	221
10.4	Approximation of the metric TSP	223
10.4.1	An approximate algorithm	223
10.4.2	Justification and evaluation	224
10.4.3	Amelioration	226
10.4.4	Christofides' algorithm	227
10.4.5	Justification and evaluation	227
10.4.6	Another approach	230
10.4.7	Upper and lower bounds for the optimal value	231
10.5	Exercises	234
Chapter 11. Planar Representations		237
11.1	Planar graphs	237
11.1.1	Euler's relation	238
11.1.2	Characterization of planar graphs	240
11.1.3	Algorithmic aspect	242

11.1.4	Other properties of planar graphs	242
11.2	Other graph representations	242
11.2.1	Minimum crossing number	243
11.2.2	Thickness	243
11.3	Exercises	244
Chapter 12. Problems with Comments		247
12.1	Problem 1: A proof of k -connectivity	247
12.1.1	Problem	247
12.1.2	Comments	248
12.2	Problem 2: An application to compiler theory	249
12.2.1	Problem	249
12.2.2	Comments	249
12.3	Problem 3: Kernel of a digraph	251
12.3.1	Problem	251
12.3.2	Comments	253
12.4	Problem 4: Perfect matching in a regular bipartite graph . . .	253
12.4.1	Problem	253
12.4.2	Comments	254
12.5	Problem 5: Birkhoff-Von Neumann's theorem	254
12.5.1	Problem	254
12.5.2	Comments	255
12.6	Problem 6: Matchings and tilings	256
12.6.1	Problem	256
12.6.2	Comments	257
12.7	Problem 7: Strip mining	258
12.7.1	Problem	258
12.7.2	Comments	259

Appendix A. Expression of Algorithms	261
A.1 Algorithm	262
A.2 Explanations and commentaries	262
A.3 Other algorithms	265
A.4 Comments	265
Appendix B. Bases of Complexity Theory	267
B.1 The concept of complexity	267
B.2 Class P	269
B.3 Class NP	272
B.4 NP-complete problems	273
B.5 Classification of problems	274
B.6 Other approaches to difficult problems	276
Bibliography	277
Index	279