Table of Contents

Introduction xiii
Charles-Edmond BICHOT, Patrick SIARRY

Chapter 1. General Introduction to Graph Partitioning 1
Charles-Edmond BICHOT

1.1. Partitioning 1
1.2. Mathematical notions 2
1.3. Graphs 4
1.4. Formal description of the graph partitioning problem 8
1.5. Objective functions for graph partitioning 11
1.6. Constrained graph partitioning 13
1.7. Unconstrained graph partitioning 14
1.8. Differences between constrained and unconstrained partitioning 16
1.9. From bisection to k-partitioning: the recursive bisection method 17
1.9.1. Creating a partition with a number of parts a power of 2, from a graph bisection algorithm 17
1.9.2. Creating a k-partition from a graph bisection algorithm using the partitioning balance 17
1.10. NP-hardness of graph partitioning optimization problems 19
1.10.1. The case of constrained graph partitioning 19
1.10.2. The case of unconstrained graph partitioning 20
1.11. Conclusion 22
1.12. Bibliography 22

PART 1: GRAPH PARTITIONING FOR NUMERICAL ANALYSIS 27

Chapter 2. A Partitioning Requiring Rapidity and Quality: The Multilevel Method and Partitions Refinement Algorithms 29
Charles-Edmond BICHOT

2.1. Introduction 29
2.2. Principles of the multilevel method 30

2.3. Graph coarsening 33
2.3.1. Introduction 33
2.3.2. Graph matching 34
2.3.3. Hendrickson-Leland coarsening algorithm 34
2.3.4. The Heavy Edge Matching (HEM) algorithm 35
2.4. Partitioning of the coarsened graph 37
2.4.1. State-of-the-art partitioning methods 37
2.4.2. Region growing methods 38
2.5. Uncoarsening and partitions refinement 40
2.5.1. Presentation of the uncoarsening and refinement phase 40
2.5.2. The Kernighan-Lin algorithm 41
2.5.3. Fiduccia-Mattheyses implementation 46
2.5.4. Adaptation to direct k-partitioning 47
2.5.5. Global Kernighan-Lin Refinement 48
2.5.6. The Walshaw-Cross refinement algorithm 50
2.6. The spectral method 52
2.6.1. Presentation 52
2.6.2. Some results of numerical system 52
2.6.3. Finding the eigenvalues of the Laplacian matrix of a graph 55
2.6.4. Lower bound for constrained graph partitioning 56
2.6.5. Spectral methods for contrained partitioning 56
2.6.6. Spectral methods for unconstrained graph partitioning 57
2.6.7. Problems and improvements 58
2.7. Conclusion 59
2.8. Bibliography 60

Chapter 3. Hypergraph Partitioning 65
Cédric CHEVALIER

3.1. Definitions and metrics 65
3.1.1. Hypergraph and partitioning 65
3.1.2. Metrics for hypergraph partitioning 67
3.2. Connections between graphs, hypergraphs, and matrices 67
3.3. Algorithms for hypergraph partitioning 68
3.3.1. Coarsening 69
3.3.2. Initial partitioning and uncoarsening and refinement 71
3.3.3. Uncoarsening and refinement 71
3.4. Purpose 72
3.4.1. Hypergraph partitioning benefits 72
3.4.2. Matrix partitioning 73
3.4.3. Practical results 75

3.4.4. Repartitioning 76
3.4.5. Use of hypergraphs within a mesh partitioning context 76
3.4.6. Other applications 76
3.5. Conclusion 77
3.6. Software references 78
3.7. Bibliography 78

Chapter 4. Parallelization of Graph Partitioning 81
François PELLEGRINI

4.1. Introduction 81
4.1.1. Need for parallelism 81
4.1.2. Multilevel framework 82
4.2. Distributed data structures 84
4.3. Parallelization of the coarsening phase 87
4.3.1. Construction of the coarse graph 87
4.3.2. Parallel matching algorithms 87
4.3.3. Collision reduction at process level 88
4.3.4. Collision reduction at vertex level 89
4.4. Folding 93
4.5. Centralization 95
4.6. Parallelization of the refinement phase 96
4.6.1. Parallelization of the local refinement methods 96
4.6.2. Band graphs 99
4.6.3. Multi-centralization 101
4.6.4. Parallelization of the global refinement methods 102
4.7. Experimental results 107
4.8. Conclusion 111
4.9. Bibliography 111

Chapter 5. Static Mapping of Process Graphs 115
François PELLEGRINI

5.1. Introduction 115
5.2. Static mapping models 116
5.2.1. Cost functions 116
5.2.2. Heterogeneity of target architectures 119
5.3. Exact algorithms 121
5.4. Approximation algorithms 123
5.4.1. Global methods 123
5.4.2. Recursive methods 126
5.5. Conclusion 133
5.6. Bibliography 134

PART 2: OPTIMIZATION METHODS FOR GRAPH PARTITIONING 137

Chapter 6. Local Metaheuristics and Graph Partitioning 139
Charles-Edmond BICHOT

6.1. General introduction to metaheuristics 140
6.2. Simulated annealing 141
6.2.1. Description of the simulated annealing algorithm 142
6.2.2. Adaptation of simulated annealing to the graph bisection problem 144
6.2.3. Generalizing this adaptation to k-partitioning 147
6.2.4. Assessment of simulated annealing adaptation to graph partitioning 148
6.3. Iterated local search 149
6.3.1. Presentation of iterated local search 149
6.3.2. Simple adaptation of iterated local search to graph partitioning 152
6.3.3. Iterated local search and multilevel method 156
6.4. Other local search metaheuristics 158
6.4.1. Greedy algorithms 158
6.4.2. Tabu search 159
6.5. Conclusion 159
6.6. Bibliography 159

Chapter 7. Population-based Metaheuristics, Fusion-Fission and Graph Partitioning Optimization 163
Charles-Edmond BICHOT

7.1. Ant colony algorithms 163
7.2. Evolutionary algorithms 165
7.2.1. Genetic algorithms 165
7.2.2. Standard process of genetic algorithm adaptation to graph partitioning 169
7.2.3. The GA's adaptation to graph bisection optimization of Bui and Moon 172
7.2.4. Multilevel evolutionary algorithm of Soper-Walshaw-Cross 177
7.2.5. Other adaptations of evolutionary algorithms to graph partitioning optimization 180
7.3. The fusion-fission method 182
7.3.1. Introduction 182
7.3.2. Fusion-fission method principles 184
7.3.3. Algorithm 185
7.3.4. Selection of the multilevel algorithm 187
7.3.5. Creation of the sequence of number of parts 188
7.3.6. Selection of the refinement algorithm 189

7.3.7. Evaluation 191
7.4. Conclusion 195
7.5. Acknowledgments 196
7.6. Bibliography 196

Chapter 8. Partitioning Mobile Networks into Tariff Zones 201
Mustapha OUGHDI, Sid LAMROUS, Alexandre CAMINADA

8.1. Introduction 201
8.1.1. Scheduled rating model 201
8.1.2. Rating model for a network 204
8.2. Spatial division of the network 208
8.2.1. Definitions 208
8.2.2. Formalization of the space division problem 212
8.2.3. Resolution of space division by a genetic algorithm 216
8.3. Experimental results 220
8.4. Conclusion 222
8.5. Bibliography 223

Chapter 9. Air Traffic Control Graph Partitioning Application 225
Charles-Edmond BICHOT, Nicolas DURAND

9.1. Introduction 225
9.2. The problem of dividing up the airspace 227
9.2.1. Creation of functional airspace blocks in Europe 228
9.2.2. Creation of a functional block in central Europe 230
9.3. Modeling the problem 231
9.3.1. Control workload in a sector 231
9.3.2. Objective: minimizing the coordination workload 232
9.3.3. Two constraints, the size of the qualification areas and size control centers 232
9.3.4. Analysis and processing of European air traffic data 233
9.3.5. Graph of European air traffic and adaptation to partitioning 234
9.4. Airspace partitioning: towards a new optimization metaheuristic 237
9.5. Division of the central European airspace 240
9.6. Conclusion 246
9.7. Acknowledgments 247
9.8. Bibliography 247

PART 3: OTHER APPROACHES TO GRAPH PARTITIONING 249

Chapter 10. Application of Graph Partitioning to Image Segmentation 251
Amir NAKIB, Laurent NAJMAN, Hugues TALBOT, Patrick SIARRY

10.1. Introduction 251
10.2. The image viewed in graph form 251

10.3. Principle of image segmentation using graphs 254
10.3.1. Choice of arc weights for segmentation 255
10.4. Image segmentation via maximum flows 257
10.4.1. Maximum flows for energy minimization 257
10.4.2. Minimal geodesics and surfaces 259
10.4.3. Minimum geodesics and surfaces via maximum flows 263
10.4.4. Continuous maximum flows 265
10.5. Unification of segmentation methods via graph theory 265
10.6. Conclusions and perspectives 269
10.7. Bibliography 271

Chapter 11. Distances in Graph Partitioning 275
Alain GUÉNOCHE

11.1. Introduction 275
11.2. The Dice distance 276
11.2.1. Two extensions to weighted graphs 278
11.3. Pons-Latapy distance 281
11.4. A partitioning method for distance arrays 283
11.5. A simulation protocol 286
11.5.1. A random graph generator 286
11.5.2. Quality of the computed partition 286
11.5.3. Results 290
11.6. Conclusions 292
11.7. Acknowledgments 293
11.8. Bibliography 293

Chapter 12. Detection of Disjoint or Overlapping Communities in Networks 297
Jean-Baptiste ANGELELLI, Alain GUÉNOCHE, Laurence REBOUL

12.1. Introduction 297
12.2. Modularity of partitions and coverings 299
12.3. Partitioning method 301
12.3.1. Fusion and/or fission of clusters 302
12.3.2. Algorithm complexity 303
12.3.3. Simulations 303
12.4. Overlapping partitioning methods 307
12.4.1. Fusion of overlapping classes 308
12.4.2. Simulations 309
12.5. Conclusion 311
12.6. Acknowledgments 312
12.7. Bibliography 312

Chapter 13. Multilevel Local Optimization of Modularity 315
Thomas AYNAUD, Vincent D. BLONDEL, Jean-Loup GUILLAUME and Renaud LAMBIOTTE

13.1. Introduction 315
13.2. Basics of modularity 317
13.3. Modularity optimization 319
13.3.1. Existing methods 319
13.3.2. Known limitations 320
13.3.3. Louvain method 321
13.3.4. Modularity increase 324
13.3.5. Convergence of the algorithm 325
13.4. Validation on empirical and artificial graphs 327
13.4.1. Artificial graphs 328
13.4.2. Empirical graphs 331
13.5. Discussion 333
13.5.1. Influence of the processing order of vertices 333
13.5.2. Intermediate communities 334
13.5.3. Possible improvements 337
13.5.4. Known uses 340
13.6. Conclusion 341
13.7. Acknowledgments 342
13.8. Bibliography 342

Appendix. The Main Tools and Test Benches for Graph Partitioning 347
Charles-Edmond BICHOT

A.1. Tools for constrained graph partitioning optimization 348
A.1.1. Chaco 348
A.1.2. Metis 349
A.1.3. Scotch 349
A.1.4. Jostle 350
A.1.5. Party 350
A.2. Tools for unconstrained graph partitioning optimization 350
A.2.1. Graclus 351
A.3. Graph partitioning test benches 351
A.3.1. Graph partitioning archives of Walshaw 351
A.3.2. Other test benches 353
A.4. Bibliography 354

Glossary 357

List of Authors 361

Index 365