# Contents

**LIST OF TABLES** . . . . . xi

**LIST OF FIGURES** . . . . . xiii

**LIST OF ALGORITHMS** . . . . . xvii

**INTRODUCTION** . . . . . xix

**CHAPTER 1. BASIC CONCEPTS IN OPTIMIZATION AND GRAPH THEORY** . . . . . 1

1.1. Introduction . . . . . 1
1.2. Notation . . . . . 2
1.3. Problem structure and variants . . . . . 2
1.4. Features of an optimization problem . . . . . 4
1.5. A didactic example . . . . . 5
1.6. Basic concepts in graph theory . . . . . 6
1.6.1. Degree of a graph . . . . . 7
1.6.2. Matrix representation of a graph . . . . . 7
1.6.3. Connected graphs . . . . . 8
1.6.4. Itineraries in a graph . . . . . 8
1.6.5. Tree graphs . . . . . 9
1.6.6. The bipartite graphs . . . . . 11
1.7. Conclusion . . . . . 12

**CHAPTER 2. KNAPSACK PROBLEMS** . . . . . 13

2.1. Introduction . . . . . 13
2.2. Graph modeling of the knapsack problem . . . . 14
2.3. Notation . . . . . 14
2.4. 0-1 knapsack problem . . . . . 15
2.5. An example . . . . . 16
2.6. Multiple knapsack problem . . . . . 17
2.6.1. Mathematical model . . . . . 17
2.6.2. An example . . . . . 18
2.7. Multidimensional knapsack problem . . . . . 19
2.7.1. Mathematical model . . . . . 19
2.7.2. An example . . . . . 20
2.8. Quadratic knapsack problem . . . . . 21
2.8.1. Mathematical model . . . . . 22
2.8.2. An example . . . . . 22
2.9. Quadratic multidimensional knapsack problem 23
2.9.1. Mathematical model . . . . . 24
2.9.2. An example . . . . . 24
2.10. Solution approaches for knapsack problems . . 25
2.10.1. The greedy algorithm . . . . . 25
2.10.2. A genetic algorithm for the KP . . . . . 26
2.11. Conclusion . . . . . 28

**CHAPTER 3. PACKING PROBLEMS** . . . . . 31

3.1. Introduction . . . . . 31
3.2. Graph modeling of the bin packing problem . . . 32
3.3. Notation . . . . . 33
3.4. Basic bin packing problem . . . . . 33
3.4.1. Mathematical modeling of the BPP . . . . . 34
3.4.2. An example . . . . . 35
3.5. Variable cost and size BPP . . . . . 36
3.5.1. Mathematical model . . . . . 36
3.5.2. An example . . . . . 37
3.6. Vector BPP . . . . . 37
3.6.1. Mathematical model . . . . . 38
3.6.2. An example . . . . . 39

3.7. BPP with conflicts . . . . . 40
3.7.1. Mathematical model . . . . . 40
3.7.2. An example . . . . . 41
3.8. Solution approaches for the BPP . . . . . 42
3.8.1. The next-fit strategy . . . . . 42
3.8.2. The first-fit strategy . . . . . 43
3.8.3. The best-fit strategy . . . . . 44
3.8.4. The minimum bin slack . . . . . 44
3.8.5. The minimum bin slack' . . . . . 46
3.8.6. The least loaded heuristic . . . . . 46
3.8.7. A genetic algorithm for the bin packing problem . . . . . 47
3.9. Conclusion . . . . . 48

**CHAPTER 4. ASSIGNMENT PROBLEM** . . . . . 51

4.1. Introduction . . . . . 51
4.2. Graph modeling of the assignment problem . . . 52
4.3. Notation . . . . . 52
4.4. Mathematical formulation of the basic AP . . . . 53
4.4.1. An example . . . . . 54
4.5. Generalized assignment problem . . . . . 55
4.5.1. An example . . . . . 56
4.6. The generalized multiassignment problem . . . . 57
4.6.1. An example . . . . . 58
4.7. Weighted assignment problem . . . . . 59
4.8. Generalized quadratic assignment problem . . . 60
4.9. The bottleneck GAP . . . . . 61
4.10. The multilevel GAP . . . . . 61
4.11. The elastic GAP . . . . . 62
4.12. The multiresource GAP . . . . . 63
4.13. Solution approaches for solving the AP . . . . . 64
4.13.1. A greedy algorithm for the AP . . . . . 64
4.13.2. A genetic algorithm for the AP . . . . . 65
4.14. Conclusion . . . . . 67

**CHAPTER 5. THE RESOURCE CONSTRAINED PROJECT SCHEDULING PROBLEM** . . . . . 69

5.1. Introduction . . . . . 69
5.2. Graph modeling of the RCPSP . . . . . 70
5.3. Notation . . . . . 71
5.4. Single-mode RCPSP . . . . . 72
5.4.1. Mathematical modeling of the SM-RCPSP . . 73
5.4.2. An example of an SM-RCPSP . . . . . 74
5.5. Multimode RCPSP . . . . . 75
5.6. RCPSP with time windows . . . . . 75
5.7. Solution approaches for solving the RCPSP . . . 76
5.7.1. A greedy algorithm for the RCPSP . . . . . 76
5.7.2. A genetic algorithm for the RCPSP . . . . . 77
5.8. Conclusion . . . . . 82

**CHAPTER 6. SPANNING TREE PROBLEMS** . . . . . 83

6.1. Introduction . . . . . 83
6.2. Minimum spanning tree problem . . . . . 84
6.2.1. Notation . . . . . 84
6.2.2. Mathematical formulation . . . . . 84
6.2.3. Algorithms for the MST problem . . . . . 85
6.3. Generalized minimum spanning tree problem . . 88
6.3.1. Notation . . . . . 90
6.3.2. Mathematical formulation . . . . . 90
6.3.3. Greedy approaches for the GMST problem . . 93
6.3.4. Genetic algorithm for the GMST problem . . 96
6.4. $k$-cardinality tree problem KCT . . . . . 100
6.4.1. Problem definition . . . . . 100
6.4.2. An example . . . . . 100
6.4.3. Notation . . . . . 102
6.4.4. Mathematical formulation . . . . . 103
6.4.5. Greedy approaches for the $k$-cardinality tree problem . . . . . 103
6.4.6. Minimum path approach . . . . . 104
6.4.7. A genetic approach for the $k$-cardinality problem . . . . . 105

6.5. The capacitated minimum spanning tree problem . . . . . 106
6.5.1. Problem definition . . . . . 106
6.5.2. Notation . . . . . 107
6.5.3. An example . . . . . 107
6.5.4. Solution approaches for the CMST problem . 108
6.6. Conclusion . . . . . 112

**CHAPTER 7. STEINER PROBLEMS** . . . . . 113

7.1. Introduction . . . . . 113
7.2. The Steiner tree problem . . . . . 114
7.2.1. Problem definition . . . . . 114
7.2.2. Problem formulation . . . . . 115
7.2.3. Constructive heuristics for the Steiner tree problem . . . . . 115
7.3. The price collecting Steiner tree problem . . . . . 118
7.3.1. Problem definition . . . . . 118
7.3.2. Example . . . . . 118
7.3.3. Mathematical formulation . . . . . 118
7.3.4. A greedy approach to solve the PCSTP . . . . 120
7.3.5. A genetic algorithm for the PCSTP . . . . . 121
7.4. Conclusion . . . . . 123

**CHAPTER 8. A DSS DESIGN FOR OPTIMIZATION PROBLEMS** . . . . . 125

8.1. Introduction . . . . . 125
8.2. Definition of a DSS . . . . . 126
8.3. Taxonomy of a DSS . . . . . 127
8.4. Architecture and design of a DSS . . . . . 128
8.4.1. Architecture of a DSS . . . . . 129
8.4.2. DSS design . . . . . 130
8.5. A DSS for the knapsack problem . . . . . 131
8.6. A DSS for the DCVRP . . . . . 133
8.6.1. Statement and modeling of the CVRP . . . . . 136
8.6.2. Notation . . . . . 137
8.6.3. Mathematical formulation of the DCVRP . . 138
8.6.4. DCVRP–DSS interfaces . . . . . 139

8.6.5. A real application: the case of Tunisia . . . . . 141
8.7. Conclusion . . . . . 143

**CONCLUSION** . . . . . 145

**GLOSSARY** . . . . . 147

**BIBLIOGRAPHY** . . . . . 149

**INDEX** . . . . . 155