Contents

Preface ................................................................. xi

Part 1. Smoothness of the Survival
Probabilities with Applications ............................. 1

Chapter 1. Classical Results on the Ruin
Probabilities ....................................................... 3

1.1. Classical risk model ............................... 3
  1.1.1. Description of the model .................. 3
  1.1.2. Net profit condition and
         the behavior of the infinite-horizon ruin probability 5
  1.1.3. Integro-differential equations for
         the survival probabilities ...................... 6
  1.1.4. Integral equation for the infinite-horizon
         ruin probability ............................. 8
  1.1.5. Laplace transform of the infinite-horizon
         survival probability ......................... 10
  1.1.6. Analytic expressions for the infinite-horizon
         survival probability ......................... 11
  1.1.7. Cramér–Lundberg approximation
         for the infinite-horizon ruin probability .... 13
  1.1.8. Lundberg inequality for the infinite-horizon
         ruin probability .............................. 16
  1.1.9. Bibliographical notes ......................... 18

1.2. Risk model with stochastic premiums ................. 22
1.2.1. Description of the model .................................. 22
1.2.2. Basic results .............................................. 23
1.2.3. Bibliographical notes .................................... 25

Chapter 2. Classical Risk Model with Investments in a Risk-Free Asset ........................................... 27
2.1. Description of the model .................................. 27
2.2. Continuity and differentiability of the infinite-horizon survival probability .................................. 28
2.3. Continuity of the finite-horizon survival probability and existence of its partial derivatives ........... 32
2.3.1. Main theorem ............................................ 32
2.3.2. Examples ................................................. 40
2.4. Bibliographical notes .................................... 47

Chapter 3. Risk Model with Stochastic Premiums and Investments in a Risk-Free Asset ..................... 49
3.1. Description of the model .................................. 49
3.2. Continuity and differentiability of the infinite-horizon survival probability .................................. 50
3.2.1. Main results ............................................ 50
3.2.2. Zero interest rate ..................................... 56
3.3. Continuity of the finite-horizon survival probability and existence of its partial derivatives ........... 59

Chapter 4. Classical Risk Model with a Franchise and a Liability Limit ........................................... 65
4.1. Introduction ................................................ 65
4.2. Survival probability in the classical risk model with a franchise .............................................. 68
4.2.1. Analytic expression for the survival probability ................................................................. 68
4.2.2. Case of small and large enough initial surpluses ............................................................... 80
4.3. Survival probability in the classical risk model with a liability limit ......................................... 84
4.3.1. Analytic expression for the survival probability ................................................................. 85
4.3.2. Case of small enough and large enough initial surpluses ........................................ 91
4.4. Survival probability in the classical risk model with both a franchise and a liability limit .......... 93
  4.4.1. Analytic expression for the survival probability ...................................................... 94
  4.4.2. Case of small initial surpluses .......................................................... 102

Chapter 5. Optimal Control by the Franchise and Deductible Amounts in the Classical Risk Model ........ 105
  5.1. Introduction ................................................................. 105
  5.2. Optimal control by the franchise amount ......................................................... 106
    5.2.1. Problem statement .................................................. 106
    5.2.2. Hamilton–Jacobi–Bellman equation .................................... 108
    5.2.3. Existence theorem .................................................. 110
    5.2.4. Verification theorem .............................................. 113
    5.2.5. Exponentially distributed claim sizes .................................. 116
  5.3. Optimal control by the deductible amount ................................................... 119
    5.3.1. Problem statement .................................................. 119
    5.3.2. Hamilton–Jacobi–Bellman equation .................................... 121
    5.3.3. Existence and verification theorems .................................. 122
    5.3.4. Exponentially distributed claim sizes .................................. 123
  5.4. Bibliographical notes ........................................................ 124

Chapter 6. Risk Models with Investments in Risk-Free and Risky Assets ........................................ 127
  6.1. Description of the models ................................................... 127
  6.2. Classical risk model with investments in risk-free and risky assets ................................... 129
    6.2.1. Upper and lower bounds for the infinite-horizon survival probability .................. 129
    6.2.2. Continuity and differentiability of the infinite-horizon survival probability ............ 136
    6.2.3. Continuity of the finite-horizon survival probability and existence of its partial derivatives .......... 140
  6.3. Risk model with stochastic premiums and investments in risk-free and risky assets ............... 147
6.4. Accuracy and reliability of uniform approximations of the survival probabilities by their statistical estimates .......................... 150
6.4.1. Finite-horizon survival probability .................................. 150
6.4.2. Infinite-horizon survival probability .................................. 157
6.5. Bibliographical notes .................................................. 159

Part 2. Supermartingale Approach to the Estimation of Ruin Probabilities .............................................. 163

Chapter 7. Risk Model with Variable Premium Intensity and Investments in One Risky Asset .............................. 165

7.1. Description of the model ............................................ 165
7.2. Auxiliary results .................................................... 167
  7.2.1. Sufficient conditions for the solution to explode ............................. 167
  7.2.2. Finiteness of the first exit time ..................................... 172
7.3. Existence and uniqueness theorem ..................................... 175
7.4. Supermartingale property for the exponential process ............................. 176
7.5. Upper exponential bound for the ruin probability .......................... 181
7.6. Bibliographical notes .................................................. 184

Chapter 8. Risk Model with Variable Premium Intensity and Investments in One Risky Asset up to the Stopping Time of Investment Activity .......................... 187

8.1. Description of the model ............................................ 187
8.2. Existence and uniqueness theorem ..................................... 189
8.3. Redefinition of the ruin time ........................................... 190
8.4. Supermartingale property for the exponential process ............................. 192
8.5. Upper exponential bound for the ruin probability .......................... 196
8.6. Exponentially distributed claim sizes ..................................... 199
8.7. Modification of the model ............................................ 200
Chapter 9. Risk Model with Variable Premium
Intensity and Investments in One Risk-Free
and a Few Risky Assets .......................... 205
  9.1. Description of the model ..................... 205
  9.2. Existence and uniqueness theorem .............. 209
  9.3. Supermartingale property for the exponential
  process ........................................ 209
  9.4. Upper exponential bound for the ruin probability .... 214
  9.5. Case of one risky asset ..................... 223
  9.6. Examples .................................... 224

Appendix ............................................. 231

Bibliography ....................................... 239

Abbreviations and Notation ......................... 255

Index ................................................ 259