Introduction

Since the early 1970s, graph partitioning has been a widely researched topic. After 40 years of development, it is now time to evaluate the situation of the work done in this area.

Graph partitioning is a problem transverse to several fields in engineering, as well as research. The diversity of backgrounds of people working on this subject may explain why no real international community exists, despite the fact that some workshops have been organized on this subject over different periods of time. However, the number of people who work or have worked on this topic is quite significant, as shown by the abundant literature cited throughout the chapters of this book. Confronted with this profusion and diversity, it is interesting to gather the knowledge accumulated over so many years in order to synthesize it and draw the common theoretical fundamentals of this field.

This book intends to present to the neophyte reader, as well as the expert in applied mathematics or the computer science researcher, tools and methods to solve graph partitioning optimization problems. For this purpose, we have collected several methodological chapters detailing different graph partitioning optimization approaches such as the multilevel method, metaheuristics, partitioning parallelization, and hypergraph partitioning. In order to complete this theoretical part, several graph partitioning applications have been described on subjects as diverse as mobile networks, image segmentation, air traffic control, social networks, etc.

Despite the large number of studies in the domain of graph partitioning, it is clear that a lot of work remains to be done to solve this problem more efficiently. Recent years have seen the sizes of graphs to be partitioned soaring from a few thousand vertices to several million, or even billions of vertices. We hope that by reading this book the reader will feel inspired not only to take an interest in this problem, but also try to solve it more efficiently, both in terms of quality of partitions found and computation time required.

This book has three parts: Part 1 is dedicated to the most common application of graph partitioning, numerical analysis; Part 2 describes and implements several combinatorial optimization methods for graph partitioning; Part 3 presents other uses of graph partitioning.

Chapter 1 provides a general introduction to this book and is therefore independent of any part. It analyzes graph partitioning in order to identify the different problems associated with it.

A large number of studies on graph partitioning have been undertaken within the domain of numerical analysis. Part 1 of this book is dedicated to their presentation. Thus, Chapter 2 describes the methods and algorithms commonly used in numerical analysis to solve graph partitioning problems: the multilevel method, refinement algorithms like the Kernighan-Lin or Fiduccia-Mattheyses algorithms, and the spectral method. Chapter 3 introduces the particular case of hypergraph partitioning, which often occurs in numerical analysis. Chapter 4 presents several parallel algorithms to partition a graph. Finally, Chapter 5 presents the problem of static mapping which occurs in parallel computing.

Graph partitioning is often studied through a combinatorial optimization point of view. This is the theme of Part 2 of this book, which is dedicated to the study of combinatorial optimization methods, and more particularly metaheuristics, for graph partitioning. This part consists of two theoretical chapters followed by two application chapters. Chapter 6 focuses on the use of several local metaheuristics, like simulated annealing or iterated local search. Chapter 7 provides details on the use of population-based metaheuristics to optimize partitioning. This chapter explains the work done on Ant Colony algorithms and describes several adaptations of Genetic Algorithms to graph partitioning. It also introduces a recent method for graph partitioning optimization, fusion-fission, which works as a meta-method that overlooks a multilevel algorithm. The last 2 chapters of this part provide application examples for this part. Chapter 8 applies a Genetic Algorithm to the problem of mobile network partitioning in tariff zones. Chapter 9 describes an Air Traffic Control problem and solves it using the fusion-fission method.

Part 3 of this book develops other approaches of graph partitioning. In this part, we have chosen to focus on graph partitioning optimization, and thus we limited the scope of this part, even if many other works on graph partitioning could have been added, like constraint programming for graph partitioning or graph decomposition. Chapter 10 outlines the image segmentation problem and offers several methods to solve this problem, based on graph partitioning. Chapter 11 compares several distances between vertices, in order to build communities, using classification methods. Chapter 12 proposes to partition networks into unconnected or overlapping communities. Finally, Chapter 13 concludes by describing how to partition very large networks into communities.

Further information that could not be included in this book is available at the book's Web address[1]. In particular, as this book is being printed in black and white, some figures will have lost their clarity when compared to their original color version. This is especially true for images in Chapters 9 and 10, respectively, about airspace partitioning and image segmentation. To overcome this problem, the original color versions of the figures concerned are also available at the book's Web address. You will also find at this Web address different graphs used in this book, as well as links to several softwares for graph partitioning.

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1 The book's Website, "Graph partitioning: optimization and applications", is available at: perso.ec-lyon.fr/charles-edmond.bichot/.