
Contents

Preface	ix
Introduction	xi
Chapter 1. Gaussian Processes	1
1.1. Some preliminaries	1
1.2. Gaussian variables and vectors	2
1.2.1. Gaussian variables	2
1.2.2. Gaussian vectors	3
1.3. Gaussian processes	10
1.3.1. Some preliminaries concerning stochastic processes	10
1.3.2. Gaussian processes. Definition and main properties. Existence of a Gaussian process with prescribed covariance function	12
1.3.3. Continuity and Hölder property of Gaussian processes	14
1.3.4. Uniform bounds and maximal inequalities for Gaussian processes	15
1.3.5. Equivalence and singularity of measures related to Gaussian processes	18
1.4. Exercises	28
Chapter 2. Fractional and Sub-fractional Brownian Motions	31
2.1. Fractional Brownian motion	31
2.2. Sub-fractional Brownian motion	33
2.2.1. Existence of the sfBm	34

2.2.2. Main properties of the sfBm	38
2.3. Long- and short-range dependence of fBm and sfBm	43
2.4. “Moving average” representation of fBm and sfBm	45
2.5. Spectral representation of fBm and sfBm	46
2.6. Asymptotic growth of a Gaussian self-similar process with application to sub-fractional Brownian motion	51
2.7. Wiener integration with respect to sub-fractional Brownian motion	53
2.7.1. Wiener integration with respect to a Gaussian process	53
2.7.2. Wiener integration with respect to fractional and sub-fractional Brownian motion	55
2.7.3. Sub-fractional Ornstein–Uhlenbeck process	58
2.8. Compact interval representations of fractional processes	63
2.8.1. Fractional integrals and fractional derivatives	63
2.8.2. Compact interval representation of fBm and sfBm: fundamental martingale	64
2.9. Girsanov theorem for sub-fractional Brownian motion	66
2.10. Comparison of fractional and sub-fractional processes via Slepian’s lemma	67
2.11. Exercises	71
Chapter 3. Mixed Fractional and Mixed Sub-fractional Brownian Motions	75
3.1. The main properties of mixed fractional and mixed sub-fractional Brownian motions	76
3.2. The behavior of the increments of mfBm and msfBm	79
3.3. Invertibility of the covariance matrix of mfBm and msfBm	84
3.4. Some properties of the mfBm’s and msfBm’s sample paths	87
3.4.1. Hölder continuity and non-differentiability of the sample paths of mfBm and msfBm	87
3.4.2. Hausdorff dimensions of some sets related to the sample paths of mfBm and msfBm	89
3.5. A series expansion of mixed sub-fractional Brownian motion	101
3.5.1. Explicit series expansion of mixed sub-fractional Brownian motion	101
3.5.2. Rate of convergence	105
3.5.3. Computer simulation of msfBm sample paths	110

3.6. Study of the semi-martingale property of the mixed processes	111
3.6.1. General properties of semi-martingales	111
3.6.2. Semi- and non-semi-martingale property for the mixed sub-fractional Brownian motion	115
3.7. Mixed sub-fractional colored-white heat equation	134
3.7.1. Moving average representation of msfBm	134
3.7.2. The heat equation driven by mixed sub-fractional noise	136
3.7.3. Existence of the solution	139
3.7.4. Covariance function and self-similarity of mild solution of mixed sub-fractional heat equation	141
3.7.5. Regularity and fractal properties of mild solution	143
3.8. Exercises	153
Appendix	155
Bibliography	185
Index	193