Contents

Preface xv
Claire GUILLE-BIEL WINDER and Teresa ASSUDE

Part 1. Articulations between Tangible Space, Graphical Space and Geometric Space 1

Chapter 1. The Geometry of Tracing, a Possible Link Between Geometric Drawing and Euclid's Geometry? 3
Anne-Cécile MATHÉ and Marie-Jeanne PERRIN-GLORIAN

1.1. Introduction 3
1.2. Geometry in middle school 5
1.2.1. What underlying axiomatics? 5
1.2.2. An example 6
1.2.3. The current lack of consistency 8
1.3. Geometry of tracing, a possible link between material geometry and Euclid's geometry? 8
1.3.1. Figure visualization and figure restoration 9
1.3.2. The geometrical use of tracing instruments, a first step to make sense to an axiomatic 10
1.3.3. Distinguishing between the hypothesis and the conclusion 12
1.3.4. Restoration, description, construction of figures and geometric language . 14
1.4. Dialectics of action, formulation and validation with regards to the reproduction of figures with instruments. 15
1.4.1. Formulation situations and possible variations 15
1.4.2. Validation situations 17
1.5. From tracing to the characterization of objects and geometric relationships . . 18
1.5.1. On the concepts of segments, lines and points 18
1.5.2. On the notion of perpendicular lines 21

1.6. Towards proof and validation situations in relation to figure restoration 27
1.6.1. Equivalence between two construction programs and the need for proof. 27
1.6.2. Validation situations involving programs for the construction of a square and introducing a proof process 29
1.7. Conclusion 31
1.8. References 32

Chapter 2. How to Operate the Didactic Variables of Figure Restoration Problems? 35
Karine VIÈQUE

2.1. Introduction 35
2.2. Theoretical framework 35
2.2.1. Studying a specific type of problem: figure restoration 35
2.2.2. Studying the concepts involved in figure restoration problems 37
2.3. Values of the didactic variables of the first problem family 39
2.3.1. Values of the didactic variables for the "figure" and the "beginning of the figure" 39
2.3.2. Value for the didactic variable "instruments made available" 40
2.3.3. Rules of action and theorems-in-action associated with development on the geometrical usage of the ruler 41
2.4. Conclusion 44
2.5. References 44

Chapter 3. Early Geometric Learning in Kindergarten: Some Results from Collaborative Research 47
Valentina CELI

3.1. The emergence of the first questions 47
3.2. Theoretical insights 48
3.2.1. Global understanding and visual perception of geometric shapes 48
3.2.2. Operative understanding and visual perception of geometric shapes 49
3.2.3. Topological understanding and visual perception of geometric shapes . . 50
3.2.4. Haptic perception 51
3.2.5. Association of visual and haptic perceptions: towards a sequential understanding of geometric shapes 52
3.3. The role of language in early geometric learning 53
3.3.1. But which lexicon? 54
3.3.2. Verbal and gestural language 58
3.4. Assembling shapes 60
3.4.1. Free assembly of shapes 60
3.4.2. Assembling triangles 62
3.5. Gestures to learn 68

3.6. Conclusion 69
3.7. References 71

Chapter 4. Using Coding to Introduce Geometric Properties in Primary School 73
Sylvia COUTAT

4.1. Coding in geometry 73
4.2. Two examples of communication activities requiring the use of coding 75
4.2.1. A co-constructed coding 75
4.2.2. Personal coding 77
4.3. Conclusion: perspectives on the introduction of coding in geometry 78
4.4. References 79

Chapter 5. Freehand Drawing for Geometric Learning in Primary School 81
Céline VENDEIRA-MARÉCHAL

5.1. Introduction 81
5.2. Drawings in geometry and their functions 82
5.3. Freehand drawing in research 83
5.4. Exploring the milieu around a freehand reproduction task of the Mitsubishi symbol on a blank white page 84
5.4.1. Freehand drawing reveals a reasoning between spatial knowledge and geometric knowledge 87
5.4.2. Freehand drawing as a dynamic process to build and transform knowledge 88
5.5. Conclusion 89
5.6. References 90

Part 2. Resources and Artifacts for Teaching 93

Chapter 6. Use of a Dynamic Geometry Environment to Work on the Relationships Between Three Spaces (Tangible, Graphical and Geometrical) 95
Teresa ASSUDE

6.1. Added value with a dynamic geometry environment: the ecological and economical point of view 95
6.2. Tangible space, graphical space and geometric space 100
6.3. Designing situations for first grade primary school 103
6.3.1. Our choices for designing situations 104
6.3.2. Presentation of situations 104
6.4. Analysis of the situations for the first-grade class 105
6.4.1. Instrumental dimension: perceptive–gestural level 105

6.4.2. Instrumental dimension: spatial–geometric relationships 106
6.4.3. Instrumental dimension: exploration and graphical space 107
6.4.4. Instrumental dimension: tool-geometric space symbiosis 108
6.4.5. Praxeological dimension 109
6.4.6. Praxeological dimension: observe and describe 111
6.5. Conclusion 113
6.6. References 115

Chapter 7. Robotics and Spatial Knowledge 119
Emilie MARI

7.1. Introduction 119
7.2. Theoretical framework and development for a categorization of spatial tasks. 120
7.2.1. Spatial knowledge 120
7.2.2. Types of spatial tasks 121
7.2.3. Types of tasks and techniques 121
7.3. Research methodology 122
7.4. Analysis: reproducing an assembly 123
7.4.1. Test item 123
7.4.2. Test results. 124
7.4.3. Analysis of the results 125
7.5. Conclusion 126
7.6. References 127

Chapter 8. Contribution of a Human Interaction Simulator to Teach Geometry to Dyspraxic Pupils 129
Fabien EMPRIN and Edith PETITFOUR

8.1. Introduction 129
8.2. General research framework 130
8.2.1. Teaching geometry 130
8.2.2. Dyspraxia and consequences for geometry 131
8.3. What alternatives are there for teaching geometry?. 132
8.3.1. Using tools in a digital environment 132
8.3.2. Dyadic work arrangement 135
8.4. Designing the human interaction simulator 138
8.4.1. General considerations 138
8.4.2. Choice of instrumented actions 139
8.4.3. Interaction choices 140
8.4.4. Ergonomic considerations 142
8.5. Initial experimental results 143
8.5.1. Data collected 144
8.5.2. Jim's diagnostic evaluation 144

8.5.3. Analysis of the first experimentation 146
8.5.4. Conclusion. 150
8.6. References 152

Chapter 9. Research and Production of a Resource for Geometric Learning in First and Second Grade 155
Jacques DOUAIRE, Fabien EMPRIN and Henri-Claude ARGAUD

9.1. Presentation of the ERMEL team's research on spatial and geometric learning from preschool to second grade 155
9.1.1. Origins of the research 156
9.1.2. Introduction to the chapter 156
9.2. Learning to trace straight lines 157
9.2.1. Significance of the straight line 157
9.2.2. Initial hypotheses 157
9.2.3. The RAYURE situation 159
9.2.4. Using straight lines 160
9.2.5. A few summary elements 161
9.3. Plane and solid figures 162
9.3.1. Findings and assumptions 162
9.3.2. The SQUARE AND QUASI-SQUARE situation 163
9.3.3. The emergence of criteria for comparing solids: the IDENTIFYING A SOLID situation 165
9.3.4. Identification of cube properties: the CUBE AND QUASI-CUBE situation 166
9.3.5. Progression on solids and plane figures. 167
9.4. The appropriation of research results by the resource 168
9.5. Conclusion 169
9.6. References 170

Chapter 10. Tool for Analyzing the Teaching of Geometry in Textbooks 171
Claire GUILLE-BIEL WINDER and Edith PETITFOUR

10.1. General framework and theoretical tools. 172
10.1.1. Didactic co-determination scale, mathematical and didactic organizations 172
10.1.2. Reference MO and theoretical tools for analysis 174
10.2. Analysis criteria: definition and methodology 181
10.2.1. Institutional conformity 181
10.2.2. Educational adequacy 182
10.2.3. Didactic quality 182
10.3. Introducing the analysis grid 183
10.3.1. Analysis of tasks and task types 183

10.3.2. Analysis of techniques 184
10.3.3. Analysis of knowledge 185
10.3.4. Analysis of ostensives 186
10.3.5. Analysis of organizational and planning elements 189
10.3.6. Summary 191
10.4. Conclusion 191
10.5. References 192

Part 3. Teaching Practices and Training Issues 197

Chapter 11. Study on Teacher Appropriation of a Geometry Education Resource 199
Christine MANGIANTE-ORSOLA

11.1. Introduction 199
11.2. Research background 200
11.2.1. Study on dissemination possibilities in ordinary education 200
11.2.2. Resource design approach 201
11.2.3. A working methodology based on assumptions 202
11.2.4. Designing a situation using the didactic engineering approach for development 205
11.3. Focus on the adaptability of this situation to ordinary education. 206
11.3.1. Details about the theoretical framework and the research question 206
11.3.2. Presentation on the follow-up of teachers, details of the research question and the methodology 207
11.3.3. Presentation of the analysis methodology 208
11.4. Elements of the analysis 209
11.4.1. Analysis a priori of the situation and anticipatory analysis of the teacher's activity 209
11.4.2. Analysis of practices 211
11.5. Conclusion 217
11.6. References 219

Chapter 12. Geometric Reasoning in Grades 4 to 6, the Teacher's Role: Methodological Overview and Results 221
Sylvie BLANQUART

12.1. Introduction 221
12.2. Theoretical choices and the problem statement 221
12.2.1. Geometrical paradigms 222
12.2.2. The different spaces 223
12.2.3. Study on reasoning 223
12.2.4. The role of the teacher 225
12.2.5. Problem statement 225

12.3. Methodology 225
12.3.1. General principle 225
12.3.2. The situations 226
12.3.3. Analysis methodology 226
12.4. Conclusion 227
12.5. References 229

Chapter 13. When the Teacher Uses Common Language Instead of Geometry Lexicon 231
Karine MILLON-FAURÉ, Catherine MENDONÇA DIAS, Céline BEAUGRAND and Christophe HACHE

13.1. Introduction 231
13.2. An attempt to categorize the uses of common vernacular terms in place of geometry lexicon terms within teacher discourse 232
13.2.1. The phenomenon of didactic reticence 232
13.2.2. The phenomenon of semantic analogy: comparison with common concepts to construct meaning for mathematical knowledge 233
13.2.3. The phenomenon of lexical competition: use of common vernacular terms to designate common concepts 234
13.2.4. The phenomena of repeating pupil formulations 235
13.2.5. The phenomenon of didactic repression 236
13.3. Conclusion 237
13.4. References 238

Chapter 14. The Development of Spatial Knowledge at School and in Teacher Training: A Case Study on *1, 2, 3… imagine!* 241
Patricia MARCHAND and Caroline BISSON

14.1. Introduction and research question 241
14.2. Conceptual framework 243
14.2.1. Components set to address SK in primary school 244
14.2.2. Levels of abstraction that value SK 245
14.2.3. Main variables in situations where SK is valued 246
14.3. Presentation of the activity 1, 2, 3 … imagine! 247
14.4. Experiments with this activity in primary school and in teacher training in Quebec 251
14.4.1. Teaching sequence experimented in primary school 251
14.4.2. Teaching sequence tested in teacher training 254
14.5. Experiment results 255
14.5.1. Experiment results of the teaching sequence in primary school 255
14.5.2. Experiment results of this teaching sequence in teacher training 257
14.6. Conclusion 259
14.7. References 260

Chapter 15. What Use of Analysis a priori by Pre-Service Teachers in Space Structuring Activities? 265
Ismaïl MILI

15.1. Introduction – an institutional challenge of transposing didactic knowledge 265
15.1.1. Choice of external transposition: institutional constraints 265
15.2. Theoretical framework 267
15.2.1. Choice of internal transposition: the moments of the study of the analysis a priori 268
15.3. Research questions 269
15.4. Methodology 269
15.4.1. Selection of activities and brief analysis 270
15.5. Results 272
15.6. Conclusion 273
15.7. References 273

Part 4. Conclusion and Implications 275

Chapter 16. Questions about the Graphic Space: What Objects? Which Operations? 277
Teresa ASSUDE

16.1. Semiotic tools of geometric work and graphic space 277
16.2. Graphic space: graphic expressions, denotation and meaning 280
16.2.1. How can we define the graphic space? 280
16.2.2. Which objects in the graphic space? 280
16.2.3. Graphic expressions: which operations? 282
16.3. References 285

Chapter 17. Towards New Questions in Geometry Didactics 289
Claire GUILLE-BIEL WINDER and Catherine HOUDEMENT

17.1. Current questions in geometry didactics 289
17.2. Continuities and breaks in the teaching of geometry 291
17.2.1. Institutional continuity? 291
17.2.2. Theoretical continuity from “geometry of tracing” to “abstract geometry”? 291
17.2.3. Praxis continuity from the “geometry of tracing“ to “abstract geometry” 294
17.3. Articulation between resources, practices and teacher training 297
17.4. References 299

Appendices 303

Appendix 1 305

Appendix 2 309

Appendix 3 311

Appendix 4 313

List of Authors 315

Index 317