Table of Contents

Preface ix

Chapter 1. Electrostatic Field in Vacuum 1

1.1. Electric charges 1
1.2. Electrostatic force 2
1.2.1. Distribution along a line (linear) or a curve (curvilinear) 4
1.2.2. Distribution over a surface (surface charge density) 8
1.2.3. Distribution over a volume (volume charge density) 9
1.3. Electric field 12
1.3.1. Distribution with spherical symmetry 16
1.3.2. Thomson's model 19
1.3.3. Symmetry in an electric field 20
1.3.4. Field lines and tubes 25
1.4. Electric potential 28
1.5. Equipotential surfaces 35
1.6. Gauss' law 39
1.6.1. Integral form 40
1.6.2. Differential form 44
1.6.3. The Helmholtz–Hodge theorem 45
1.6.4. Applications 48
1.6.5. Maximum and minimum potentials 57
1.7. Electric field across a charged sheet 57
1.8. Model of the hydrogen atom 63

Chapter 2. Conductors in Electrostatic Equilibrium 67

2.1. Conductors 67
2.2. Electrostatic equilibrium 68
2.3. Electrostatic pressure 74
2.4. Influence between conductors at electrostatic equilibrium 77
2.4.1. Partial influence 77
2.4.2. Theorem of corresponding elements 79
2.4.3. Earthing 80
2.4.4. Total influence 80
2.4.5. Electrostatic influence coefficients 83
2.5. Capacitors 88
2.5.1. Definition and capacitance of the capacitor 88
2.5.2. Capacitor with closely spaced armatures 92
2.5.3. Examples of capacitors 93
2.5.4. Association of capacitors 100
2.5.5. Capacitors with dielectrics 103
2.6. Energy stored in a capacitor 106
2.7. Forces of attraction between the armatures of a capacitor 109
2.7.1. Calculating force from pressure 109
2.7.2. Force calculation from the energy expression 112

Chapter 3. Electrokinetics 117

3.1. Conduction in solids 117
3.2. Current density and intensity 118
3.3. Conservation of charge 124
3.4. Kirchhoff's first law and approximation of quasi-stationary regimes 126
3.5. Conduction model for charged gas 127
3.5.1. The drift–diffusion equation 128
3.5.2. Local Ohm's law 133
3.5.3. Superconductors 135
3.6. Resistance 136
3.6.1. Resistance between two parallel electrodes 136
3.6.2. Association of resistors 140
3.6.3. Joule effect 142
3.7. Direct current circuits 143
3.7.1. Characteristics of a circuit element 146
3.7.2. Association of electrical elements 149
3.7.3. Solve DC circuits problems 150
3.8. Relaxation time 161
3.8.1. Discharge of a uniformly charged sphere 161

3.8.2. Charging and discharging at the interface between two materials . . . 163
3.8.3. Modeling of an ohmic electrode . . . 166
3.9. Earth atmosphere as a spherical capacitor . . . 170
3.10. Space charge . . . 172
3.10.1. Vacuum diode, non-ohmic conductor . . . 172
3.10.2. Space charges in materials . . . 175

Chapter 4. Magnetic Field in Vacuum . . . 179

4.1. Force on a mobile charge . . . 179
4.1.1. Theoretical origin of Lorentz force . . . 179
4.1.2. Lorentz force . . . 181
4.1.3. Charge motion in a uniform magnetic field . . . 182
4.1.4. Transformation of $\overrightarrow{\mathbf{E}}$ and $\overrightarrow{\mathbf{B}}$ in a change of reference frames . . . 185
4.1.5. Hall effect . . . 186
4.2. Magnetic field, Biot and Savart law . . . 187
4.2.1. Symmetry properties of the magnetic field . . . 189
4.2.2. Examples of magnetic field calculation . . . 191
4.3. Conservation of magnetic flux . . . 202
4.4. Laplace force . . . 204
4.4.1. Definitions . . . 204
4.4.2. Laplace force between two conductors . . . 205
4.4.3. Laplace force work . . . 208
4.4.4. Maxwell's theorem . . . 209
4.4.5. Expression for magnetic potential energy . . . 211
4.5. Ampère's theorem . . . 212
4.5.1. Maxwell–Ampère equation . . . 212
4.5.2. Integral form of Maxwell–Ampère equation . . . 213
4.5.3. A few applications of Ampère's theorem . . . 214
4.6. Magnetic field continuity at a sheet interface . . . 220
4.7. Potential vector $\overrightarrow{\mathbf{A}}$ . . . 222
4.7.1. Uniqueness of $\overrightarrow{\mathbf{A}}$ . . . 222
4.7.2. Potential vector of a current distribution . . . 224
4.7.3. Potential vector and magnetic flux . . . 232
4.8. Magnetic scalar potential . . . 235

Chapter 5. Electric and Magnetic Moments . . . 239

5.1. Electric dipole . . . 239
5.1.1. Field and potential created by an electric dipole . . . 240
5.1.2. Equipotential surfaces and field lines . . . 243
5.1.3. Dipole moment of a charge distribution . . . 243

5.2. Electric multipole moments 245
5.2.1. Third-order expansion in $\frac{1}{r}$ 245
5.2.2. Tensor form of multipole expansion 250
5.2.3. Multipole expansion and spherical functions 252
5.2.4. Expansion on spherical harmonics 253
5.2.5. Dipole in a uniform external electric field 256
5.2.6. Dipole in a nonuniform external field 258
5.2.7. Potential energy 260
5.3. Magnetic dipole moment 261
5.3.1. Expressions of magnetic moment 261
5.3.2. Magnetic field created by a magnetic moment 267
5.3.3. Multipole expansion of vector potential 268
5.3.4. Interaction of the magnetic moment with an external field 270
5.3.5. Torque acting on the circuit 271
5.3.6. Gyromagnetic ratio 272
5.3.7. Magnetic moment of the hydrogen atom 274

Chapter 6. Boundary Problems for Static Potentials in Vacuum 275

6.1. Introduction 275
6.1.1. Laplace equation 275
6.1.2. Uniqueness theorem 276
6.2. 1D Laplace equation 277
6.3. 2D Laplace equation 278
6.3.1. 2D Cartesian coordinates 278
6.3.2. Polar coordinates 289
6.3.3. Spherical coordinates with axial symmetry along Oz 303
6.3.4. Numerical 2D resolution 308
6.3.5. Solving in complex plane 312
6.4. 3D Laplace equation 351
6.4.1. 3D Cartesian coordinates 351
6.4.2. Cylindrical coordinates 352
6.4.3. Spherical coordinates 358
6.4.4. Parabolic cylindrical coordinates 365
6.4.5. Paraboloidal coordinates 366

References 371

Index 377