

Table of Contents

Preface	xiii
Chapter 1. Surprising Instabilities of Simple Elastic Structures	
Davide BIGONI, Diego MISSERONI, Giovanni NOSELLI and Daniele ZACCARIA	
1.1. Introduction	1
1.2. Buckling in tension	2
1.3. The effect of constraint's curvature	4
1.4. The Ziegler pendulum made unstable by Coulomb friction	8
1.5. Conclusions	12
1.6. Acknowledgments	13
1.7. Bibliography	13
Chapter 2. WKB Solutions Near an Unstable Equilibrium	
and Applications	
Jean-François BONY, Setsuro FUJIIÉ, Thierry RAMOND and Maher ZERZERI	
2.1. Introduction	15
2.2. Connection of microlocal solutions near a hyperbolic fixed point	18
2.2.1. A model in one dimension	19
2.2.2. Classical mechanics	21
2.2.3. Review of semi-classical microlocal analysis	23
2.2.4. The microlocal Cauchy problem – uniqueness	24
2.2.5. The microlocal Cauchy problem – transition operator	26
2.3. Applications to semi-classical resonances	28
2.3.1. Spectral projection and Schrödinger group	30
2.3.2. Resonance-free zone for homoclinic trajectories	33
2.4. Acknowledgment	37
2.5. Bibliography	37

Chapter 3. The Sign Exchange Bifurcation in a Family of Linear Hamiltonian Systems	41
Richard CUSHMAN, Johnathan M. ROBBINS and Dimitrii SADOVSKII	
3.1. Statement of problem	41
3.2. Bifurcation values of γ	45
3.3. Versal normal forms near the bifurcation values	46
3.3.1. Normal forms	46
3.3.2. Linear Hamiltonian Hopf bifurcation γ_{\pm}	47
3.3.3. The Switch twist bifurcation at γ^+	50
3.3.4. Sign exchange bifurcation	53
3.4. Infinitesimally symplectic normal form	57
3.4.1. Normal form of X_γ at γ^\pm	57
3.4.2. Normal form of X_γ at γ_{\pm}	60
3.5. Global issues	62
3.5.1. Invariant Lagrange planes	62
3.5.2. Symplectic signs	64
3.6. Bibliography	65
Chapter 4. Dissipation Effect on Local and Global Fluid-Elastic Instabilities	67
Olivier DOARÉ	
4.1. Introduction	67
4.2. Local and global stability analyses	68
4.2.1. Local analysis	69
4.2.2. Global analysis	69
4.3. The fluid-conveying pipe: a model problem	70
4.4. Effect of damping on the local and global stability of the fluid-conveying pipe	72
4.4.1. Local stability	72
4.4.2. Global stability	74
4.5. Application to energy harvesting	79
4.6. Conclusion	81
4.7. Bibliography	82
Chapter 5. Tunneling, Librations and Normal Forms in a Quantum Double Well with a Magnetic Field	85
Sergey Y. DOBROKHOTOV and Anatoly Y. ANIKIN	
5.1. Introduction	85
5.2. 1D Landau–Lifshitz splitting formula and its analog for the ground states	87

5.3. The splitting formula in multi-dimensional case	92
5.4. Normal forms and complex Lagrangian manifolds	98
5.4.1. Normal form in the classically allowed and forbidden regions	98
5.4.2. Complex continuation of integrals	99
5.4.3. Almost invariant complex Lagrangian manifolds	99
5.5. Constructing the asymptotics for the eigenfunctions in tunnel problems	100
5.5.1. Complex WKB-method	100
5.5.2. WKB-methods with real and pure imaginary phases	101
5.5.3. Variational methods	102
5.6. Splitting of the eigenvalues in the presence of magnetic field	103
5.7. Proof of main theorem (a sketch)	104
5.7.1. Lifshitz–Herring formula	105
5.7.2. Instanton splitting formula	105
5.7.3. Asymptotic behavior of the libration action	106
5.7.4. Reduction to the 1D splitting problem	106
5.7.5. Asymptotic behavior of the Floquet exponents	107
5.7.6. Finishing the proof	107
5.8. Conclusion	107
5.9. Acknowledgments	108
5.10. Bibliography	108
Chapter 6. Stability of Dipole Gap Solitons in Two-Dimensional Lattice Potentials	111
Nir DROR and Boris A. MALOMED	
6.1. Introduction	111
6.2. The model	113
6.3. Solitons in the first bandgap: the SF nonlinearity	115
6.3.1. Solution families	115
6.3.2. Stability of solitons in the first finite bandgap	117
6.3.3. Bound states of solitons in the first bandgap	124
6.4. Stability GSs in the second bandgap	125
6.5. Conclusions	134
6.6. Bibliography	135
Chapter 7. Representation of Wave Energy of a Rotating Flow in Terms of the Dispersion Relation	139
Yasuhide FUKUMOTO, Makoto HIROTA and Youichi MIE	
7.1. Introduction	139
7.2. Lagrangian approach to wave energy	142
7.3. Kelvin waves	145

7.4. Wave energy in terms of the dispersion relation	148
7.5. Conclusion	150
7.6. Bibliography	151
Chapter 8. Determining the Stability Domain of Perturbed Four-Dimensional Systems in 1:1 Resonance	155
Igor HOVEIJN and Oleg N. KIRILLOV	
8.1. Introduction	155
8.1.1. Physical motivation	155
8.1.2. Setting	157
8.1.3. Main question and examples	158
8.2. Methods	159
8.2.1. Centralizer unfolding	159
8.2.2. Stability domain	160
8.2.3. Mapping into the centralizer unfolding	162
8.3. Examples	164
8.3.1. Modulation instability	164
8.3.2. Non-conservative gyroscopic system	169
8.4. Conclusions	172
8.5. Bibliography	172
Chapter 9. Index Theorems for Polynomial Pencils	177
Richard KOLLÁR and Radomír BOSÁK	
9.1. Introduction	177
9.2. Krein signature	179
9.3. Index theorems for linear pencils and linearized Hamiltonians	182
9.4. Graphical interpretation of index theorems	186
9.4.1. Algebraic calculation of Z^\downarrow and Z^\uparrow	191
9.5. Conclusions	197
9.6. Acknowledgments	197
9.7. Bibliography	197
Chapter 10. Investigating Stability and Finding New Solutions in Conservative Fluid Flows Through Bifurcation Approaches	203
Paolo LUZZATTO-FEGIZ and Charles H.K. WILLIAMSON	
10.1. Introduction	203
10.2. Counting positive-energy modes from IVI diagrams	204
10.3. An approximate prediction for the onset of resonance in 2D vortices . .	207
10.4. An example: three corotating vortices	209

10.4.1. Building a family of solutions from vorticity-preserving rearrangements	209
10.4.2. Computing signatures for one member of the family	209
10.4.3. The velocity-impulse diagram	212
10.4.4. Uncovering bifurcations by introducing imperfections	212
10.4.5. Counting positive-energy modes from turning points in impulse	213
10.4.6. Recovering the underlying bifurcation structure	214
10.4.7. An approximate prediction for resonance	215
10.5. Comparison with exact eigenvalues and discussion	216
10.6. Conclusions	218
10.7. Bibliography	219
Chapter 11. Evolution Equations for Finite Amplitude Waves in Parallel Shear Flows	223
Sherwin A. MASLOWE	
11.1. Introduction	223
11.2. Wave packets	226
11.2.1. Conservative systems	226
11.2.2. Applications to hydrodynamic stability	228
11.2.3. The Ginzburg–Landau equation	231
11.3. Critical layer theory	232
11.3.1. Asymptotic theory of the Orr–Sommerfeld equation	233
11.3.2. Nonlinear critical layers	234
11.3.3. The wave packet critical layer	237
11.4. Nonlinear instabilities governed by integro-differential equations	241
11.4.1. The zonal wave packet critical layer	241
11.5. Concluding remarks	244
11.6. Bibliography	244
Chapter 12. Continuum Hamiltonian Hopf Bifurcation I	247
Philip J. MORRISON and George I. HAGSTROM	
12.1. Introduction	247
12.2. Discrete Hamiltonian bifurcations	250
12.2.1. A class of 1 + 1 Hamiltonian multifluid theories	250
12.2.2. Examples	254
12.2.3. Comparison and commentary	261
12.3. Continuum Hamiltonian bifurcations	263
12.3.1. A class of 2 + 1 Hamiltonian mean field theories	263
12.3.2. Example of the CHH bifurcation	266
12.4. Summary and conclusions	278
12.5. Acknowledgments	279
12.6. Bibliography	279

Chapter 13. Continuum Hamiltonian Hopf Bifurcation II	283
George I. HAGSTROM and Philip J. MORRISON	
13.1. Introduction	284
13.2. Mathematical aspects of the continuum Hamiltonian Hopf bifurcation	285
13.2.1. Structural stability	285
13.2.2. Normal forms and signature	287
13.3. Application to Vlasov–Poisson	288
13.3.1. Structural stability in the space $C^n(\mathbb{R}) \cap L^1(\mathbb{R})$	292
13.3.2. Structural stability in $W^{1,1}$	294
13.3.3. Dynamical accessibility and structural stability	296
13.4. Canonical infinite-dimensional case	300
13.4.1. Negative energy oscillator coupled to a heat bath	301
13.5. Commentary: degeneracy and nonlinearity	303
13.6. Summary and conclusions	308
13.7. Acknowledgments	308
13.8. Bibliography	308
Chapter 14. Energy Stability Analysis for a Hybrid Fluid-Kinetic Plasma Model	311
Philip J. MORRISON, Emanuele TASSI and Cesare TRONCI	
14.1. Introduction	311
14.2. Stability and the energy-Casimir method	312
14.3. Planar Hamiltonian hybrid model	314
14.3.1. Planar hybrid model equations of motion	314
14.3.2. Hamiltonian structure	316
14.3.3. Casimir invariants	317
14.4. Energy-Casimir stability analysis	318
14.4.1. Equilibrium variational principle	319
14.4.2. Stability conditions	320
14.5. Conclusions	323
14.6. Acknowledgments	324
14.7. Appendix A: derivation of hybrid Hamiltonian structure	324
14.8. Appendix B: Casimir verification	326
14.9. Bibliography	327
Chapter 15. Accurate Estimates for the Exponential Decay of Semigroups with Non-Self-Adjoint Generators	331
Francis NIER	
15.1. Introduction	331
15.2. Relevant quantities for sectorial operators	334

15.3. Natural examples	336
15.3.1. An example related to linearized equations of fluid mechanics	336
15.3.2. Kramers–Fokker–Planck operators	338
15.4. Artificial examples	343
15.4.1. Adiabatic evolution of quantum resonances in the one-dimensional case	343
15.4.2. Optimizing the sampling of equilibrium distributions	345
15.5. Conclusion	347
15.6. Bibliography	348
Chapter 16. Stability Optimization for Polynomials and Matrices	351
Michael L. OVERTON	
16.1. Optimization of roots of polynomials	351
16.1.1. Root optimization over a polynomial family with a single affine constraint	352
16.1.2. The root radius	353
16.1.3. The root abscissa	355
16.1.4. Examples	357
16.1.5. Polynomial root optimization with several affine constraints	358
16.1.6. Variational analysis of the root radius and abscissa	360
16.1.7. Computing the root radius and abscissa	360
16.2. Optimization of eigenvalues of matrices	361
16.2.1. Static output feedback	362
16.2.2. Numerical methods for non-smooth optimization	363
16.2.3. Numerical results for some SOF problems	365
16.2.4. The Diaconis–Holmes–Neal Markov chain	369
16.2.5. Active derogatory eigenvalues	371
16.3. Concluding remarks	372
16.4. Acknowledgments	373
16.5. Bibliography	373
Chapter 17. Spectral Stability of Nonlinear Waves in KdV-Type Evolution Equations	377
Dmitry E. PELINOVSKY	
17.1. Introduction	377
17.2. Historical remarks and examples	379
17.3. Proof of theorem 17.1	382
17.4. Generalization of theorem 17.1 for a periodic nonlinear wave	393
17.5. Conclusion	397
17.6. Bibliography	398

Chapter 18. Unfreezing Casimir Invariants: Singular Perturbations	
Giving Rise to Forbidden Instabilities	401
Zensho YOSHIDA and Philip J. MORRISON	
18.1. Introduction	401
18.2. Preliminaries: noncanonical Hamiltonian systems and Casimir invariants	403
18.3. Foliation by adiabatic invariants	405
18.4. Canonization atop Casimir leaves	407
18.4.1. Extension of the phase space and canonization	407
18.4.2. “Minimum” canonization invoking Casimir invariants	408
18.5. Application to tearing-mode theory	409
18.5.1. Helicity and Beltrami equilibria	409
18.5.2. Tearing-mode instability	414
18.6. Conclusion	417
18.7. Acknowledgments	417
18.8. Bibliography	418
List of Authors	421
Index	425