

Table of Contents

| | |
|--|-----------|
| Preface | xi |
| Chapter 1. Origins and Topicality of a Concept | 1 |
| 1.1. Historical milestones | 1 |
| 1.1.1. Dialogs concerning two new sciences | 1 |
| 1.1.2. Note on an application of the rules of maximum and minimum to some statical problems, relevant to architecture | 4 |
| 1.1.3. Compatibility between equilibrium and resistance | 6 |
| 1.2. Topicality of the yield design approach | 8 |
| 1.2.1. The Coulomb's Essay legacy | 8 |
| 1.2.2. Topicality | 9 |
| 1.3. Bibliography | 11 |
| Chapter 2. An Introductory Example of the Yield Design Approach | 19 |
| 2.1. Setting the problem | 19 |
| 2.1.1. The considered structure | 19 |
| 2.1.2. Loading mode of the structure | 20 |
| 2.1.3. Resistance of the elements of the structure | 20 |
| 2.1.4. The question | 21 |
| 2.2. Potential stability of the structure | 22 |
| 2.2.1. A necessary condition for the stability of the structure | 22 |
| 2.2.2. Instability and potential stability of the structure | 23 |
| 2.3. To what extent potential stability is a relevant concept? | 24 |
| 2.3.1. Linearly elastic and perfectly plastic rods | 25 |
| 2.3.2. Linearly elastic and perfectly plastic/brittle rods | 26 |

| | |
|---|-----------|
| 2.3.3. Conclusion | 27 |
| 2.4. Bibliography | 28 |
| Chapter 3. The Continuum Mechanics Framework | 29 |
| 3.1. Modeling the continuum | 29 |
| 3.1.1. Geometrical description | 29 |
| 3.1.2. Kinematics | 30 |
| 3.1.3. Conservation of mass | 33 |
| 3.2. Dynamics | 34 |
| 3.2.1. Quantity of acceleration | 34 |
| 3.2.2. External forces | 34 |
| 3.2.3. Internal forces: the Cauchy stress tensor | 35 |
| 3.2.4. Equation of motion | 36 |
| 3.2.5. Discontinuity of the Cauchy stress field | 38 |
| 3.2.6. Local analysis of stresses | 40 |
| 3.3. The theory of virtual work | 41 |
| 3.3.1. Virtual velocity fields | 41 |
| 3.3.2. Theorem/principle of virtual work | 42 |
| 3.4. Statically and kinematically admissible fields | 46 |
| 3.4.1. Volume and boundary data | 46 |
| 3.4.2. Statically admissible stress fields | 47 |
| 3.4.3. Kinematically admissible virtual velocity fields | 47 |
| 3.4.4. The virtual work equation | 47 |
| 3.5. Bibliography | 48 |
| Chapter 4. Primal Approach of the Theory of Yield Design | 51 |
| 4.1. Settlement of the problem | 51 |
| 4.1.1. Geometrical data | 51 |
| 4.1.2. Loading mode of the system | 52 |
| 4.1.3. Resistance of the constituent material | 55 |
| 4.1.4. The question | 57 |
| 4.2. Potentially safe loads | 57 |
| 4.2.1. Domain K | 57 |
| 4.2.2. Mathematical properties of the domain K | 58 |
| 4.2.3. Interior approach of the domain K | 58 |
| 4.3. Comments | 60 |
| 4.3.1. Permanent loads | 60 |
| 4.3.2. Convexity of $G(x)$ | 61 |
| 4.3.3. Constituent materials of the system | 62 |
| 4.3.4. The relevance of the concept of potentially safe loads | 63 |
| 4.4. Some usual isotropic strength criteria | 66 |

| | |
|---|------------|
| 4.4.1. 3D isotropic materials | 66 |
| 4.4.2. Isotropic interfaces | 69 |
| 4.5. Bibliography | 70 |
| Chapter 5. Dual Approach of the Theory of Yield Design | 73 |
| 5.1. A static exterior approach | 73 |
| 5.2. A kinematic necessary condition | 76 |
| 5.3. The π functions. | 78 |
| 5.3.1. Support function of $G(x)$ | 78 |
| 5.3.2. Maximum resisting (rate of) work | 79 |
| 5.3.3. Mathematical properties of the π function | 81 |
| 5.3.4. Dual definition of $G(\underline{x})$ | 82 |
| 5.3.5. Ω functions for interfaces. | 83 |
| 5.4. π functions for usual isotropic strength criteria | 84 |
| 5.4.1. Three dimensional isotropic materials. | 84 |
| 5.4.2. Isotropic interfaces | 87 |
| 5.5. Bibliography | 88 |
| Chapter 6. Kinematic Exterior Approach | 91 |
| 6.1. Equation of the kinematic exterior approach | 91 |
| 6.2. Relevant virtual velocity fields | 94 |
| 6.2.1. Definition | 94 |
| 6.2.2. Relevance conditions for usual isotropic strength criteria | 96 |
| 6.2.3. Implementation of the relevance condition | 99 |
| 6.3. One domain, two approaches | 100 |
| 6.3.1. Dual approach of the convex K | 100 |
| 6.3.2. Static interior approach combined with the kinematic exterior approach | 101 |
| 6.3.3. General comments | 104 |
| 6.4. Bibliography | 107 |
| Chapter 7. Ultimate Limit State Design from the Theory of Yield Design | 111 |
| 7.1. Basic principles of ultimate limit state design | 111 |
| 7.2. Revisiting the yield design theory in the context of ULSD | 113 |
| 7.2.1. Resistance parameters | 113 |
| 7.2.2. Potentially safe loads. | 113 |
| 7.3. The yield design theory applied to ULSD | 114 |
| 7.3.1. Static approach of ULSD | 114 |
| 7.3.2. Kinematic approach of ULSD | 115 |

| | |
|---|------------|
| 7.3.3. Partial factor for model uncertainties | 117 |
| 7.4. Conclusion | 117 |
| 7.5. Bibliography | 118 |
| Chapter 8. Optimality and Probability Approaches of Yield Design | 119 |
| 8.1. Optimal dimensioning and probabilistic approach | 119 |
| 8.2. Domain of potential stability | 120 |
| 8.2.1. Resistance parameters | 120 |
| 8.2.2. Potentially safe dimensionings | 122 |
| 8.2.3. Interior approach | 124 |
| 8.2.4. Kinematic exterior approach | 125 |
| 8.2.5. Potential stability under a set of loads | 126 |
| 8.2.6. Domain of potential stability of a system | 127 |
| 8.3. Optimal dimensioning | 130 |
| 8.3.1. Introductory remarks | 130 |
| 8.3.2. Optimal dimensioning based on potential stability | 130 |
| 8.3.3. Additional remarks | 132 |
| 8.4. Probabilistic approach of yield design | 133 |
| 8.4.1. Introductory remarks | 133 |
| 8.4.2. Settlement of the probabilistic yield design problem | 134 |
| 8.4.3. Probability of stability of a system | 136 |
| 8.4.4. Additional comments | 140 |
| 8.5. Bibliography | 141 |
| Chapter 9. Yield Design of Structures | 145 |
| 9.1. The curvilinear one-dimensional continuum | 145 |
| 9.1.1. Geometric description of the model | 146 |
| 9.1.2. Kinematics | 146 |
| 9.1.3. Dynamics | 150 |
| 9.1.4. Theorem/principle of virtual work | 155 |
| 9.2. Implementation of the yield design theory | 157 |
| 9.2.1. Settlement of the problem | 157 |
| 9.2.2. Interior approach | 160 |
| 9.2.3. Exterior approach | 161 |
| 9.3. Typical strength criteria | 164 |
| 9.3.1. Interaction formulas | 164 |
| 9.3.2. Assembly joints | 168 |
| 9.3.3. Structural supports | 170 |
| 9.4. Final comments | 172 |
| 9.5. Bibliography | 174 |

| | |
|---|-----|
| Chapter 10. Yield Design of Plates: the Model | 177 |
| 10.1. Modeling plates as two-dimensional continua. | 177 |
| 10.1.1. Geometric description of the model | 177 |
| 10.1.2. Kinematics | 179 |
| 10.2. Dynamics | 182 |
| 10.2.1. External forces. | 182 |
| 10.2.2. Internal forces | 184 |
| 10.2.3. Equilibrium equations | 184 |
| 10.3. Theorem/principle of virtual work | 191 |
| 10.3.1. Virtual motions | 191 |
| 10.3.2. The virtual work equation | 192 |
| 10.3.3. Tensorial wrench of internal forces | 196 |
| 10.4. Plate model derived from the three-dimensional continuum | 198 |
| 10.4.1. Internal forces | 198 |
| 10.4.2. Equilibrium equations and external forces | 199 |
| 10.4.3. Virtual work approach | 201 |
| 10.4.4. Final comments | 204 |
| 10.5. Bibliography | 204 |
| Chapter 11. Yield Design of Plates Subjected to Pure Bending | 205 |
| 11.1. The yield design problem. | 205 |
| 11.1.1. General outline | 205 |
| 11.1.2. Settlement of the problem | 206 |
| 11.2. Implementation of the yield design theory | 208 |
| 11.2.1. Interior approach | 208 |
| 11.2.2. Exterior approach | 209 |
| 11.3. Strength criteria and π functions | 213 |
| 11.3.1. Metal plates | 213 |
| 11.3.2. Reinforced concrete slabs | 216 |
| 11.3.3. Some typical support strength conditions | 224 |
| 11.4. Final comments. | 226 |
| 11.4.1. Hinge line virtual motions | 226 |
| 11.4.2. Circular plate subjected to a uniformly distributed load | 228 |
| 11.4.3. “Conical” virtual collapse mechanism | 233 |
| 11.5. Bibliography | 234 |
| Index | 237 |