Contents

Foreword ix

Introduction xi

Chapter 1. A First Encounter with Graphs 1

1.1. A few definitions 1
1.1.1. Directed graphs 1
1.1.2. Unoriented graphs 9
1.2. Paths and connected components 14
1.2.1. Connected components 16
1.2.2. Stronger notions of connectivity 18
1.3. Eulerian graphs 23
1.4. Defining Hamiltonian graphs 25
1.5. Distance and shortest path 27
1.6. A few applications 30
1.7. Comments 35
1.8. Exercises 37

Chapter 2. A Glimpse at Complexity Theory 43

2.1. Some complexity classes 43
2.2. Polynomial reductions 46
2.3. More hard problems in graph theory 49

Chapter 3. Hamiltonian Graphs 53

3.1. A necessary condition 53
3.2. A theorem of Dirac 55

3.3. A theorem of Ore and the closure of a graph 56
3.4. Chvátal's condition on degrees 59
3.5. Partition of K_n into Hamiltonian circuits 62
3.6. De Bruijn graphs and magic tricks 65
3.7. Exercises 68

Chapter 4. Topological Sort and Graph Traversals 69

4.1. Trees 69
4.2. Acyclic graphs 79
4.3. Exercises 82

Chapter 5. Building New Graphs from Old Ones 85

5.1. Some natural transformations 85
5.2. Products 90
5.3. Quotients 92
5.4. Counting spanning trees 93
5.5. Unraveling 94
5.6. Exercises 96

Chapter 6. Planar Graphs 99

6.1. Formal definitions 99
6.2. Euler's formula 104
6.3. Steinitz' theorem 109
6.4. About the four-color theorem 113
6.5. The five-color theorem 115
6.6. From Kuratowski's theorem to minors 120
6.7. Exercises 123

Chapter 7. Colorings 127

7.1. Homomorphisms of graphs 127
7.2. A digression: isomorphisms and labeled vertices 131
7.3. Link with colorings 134
7.4. Chromatic number and chromatic polynomial 136
7.5. Ramsey numbers 140
7.6. Exercises 147

Chapter 8. Algebraic Graph Theory 151

8.1. Prerequisites 151
8.2. Adjacency matrix 154
8.3. Playing with linear recurrences 160

8.4. Interpretation of the coefficients 168
8.5. A theorem of Hoffman 169
8.6. Counting directed spanning trees 172
8.7. Comments 177
8.8. Exercises 178

Chapter 9. Perron–Frobenius Theory 183

9.1. Primitive graphs and Perron's theorem 183
9.2. Irreducible graphs 188
9.3. Applications 190
9.4. Asymptotic properties 195
9.4.1. Canonical form 196
9.4.2. Graphs with primitive components 197
9.4.3. Structure of connected graphs 206
9.4.4. Period and the Perron–Frobenius theorem 214
9.4.5. Concluding examples 218
9.5. The case of polynomial growth 224
9.6. Exercises 231

Chapter 10. Google's Page Rank 233

10.1. Defining the Google matrix 238
10.2. Harvesting the primitivity of the Google matrix 241
10.3. Computation 246
10.4. Probabilistic interpretation 246
10.5. Dependence on the parameter α 247
10.6. Comments 248

Bibliography 249

Index 263