

## Contents

<b>Preface . . . . .</b>	vii
<b>Chapter 1. Introduction . . . . .</b>	
1.1. Dynamic response . . . . .	2
1.2. Dynamic loading . . . . .	2
1.2.1. Periodic loadings . . . . .	3
1.2.1.1. Harmonic loadings . . . . .	3
1.2.1.2. Arbitrary periodic loadings . . . . .	3
1.2.2. Non-periodic loadings . . . . .	4
1.2.2.1. Impulse loadings . . . . .	4
1.2.2.2. Arbitrary loadings . . . . .	4
1.3. Additional considerations . . . . .	4
1.4. Formulation of the equation of motion . . . . .	5
1.4.1. System with one mass particle . . . . .	6
1.4.1.1. Newton's second law of motion . . . . .	6
1.4.1.2. D'Alembert's principle . . . . .	6
1.4.1.3. Virtual work principle . . . . .	7
1.4.1.4. Constraints . . . . .	8
1.4.2. System with many mass particles . . . . .	9
1.4.3. System with deformable bodies . . . . .	10
1.5. Dynamic degrees of freedom . . . . .	10
1.6. Modeling a dynamic problem . . . . .	12
1.6.1. Mass concentration . . . . .	12
1.6.2. Rayleigh–Ritz method . . . . .	13
1.6.3. Finite element method . . . . .	15
1.7. Dynamic analysis of structures . . . . .	18
1.8. Dynamic testing . . . . .	19
1.9. Measuring vibration levels . . . . .	20
1.10. Suggested reading . . . . .	23

<b>PART 1. SINGLE DEGREE OF FREEDOM SYSTEMS . . . . .</b>	<b>25</b>
<b>Chapter 2. Equation of Motion . . . . .</b>	<b>27</b>
2.1. Response parameters . . . . .	27
2.2. Immobile support . . . . .	28
2.3. Effect of gravity forces . . . . .	30
2.4. Motion of the support . . . . .	31
<b>Chapter 3. Free Response . . . . .</b>	<b>37</b>
3.1. Characteristic equation . . . . .	37
3.2. Undamped free response . . . . .	38
3.3. Conservation of energy . . . . .	47
3.4. Damped free response . . . . .	48
3.4.1. Subcritical damping . . . . .	49
3.4.2. Critical damping . . . . .	52
3.4.3. Overcritical damping . . . . .	54
3.5. Dissipation of energy in a system with subcritical damping . . . . .	55
3.6. Coulomb damping . . . . .	58
3.7. Logarithmic decrement . . . . .	63
<b>Chapter 4. Forced Response to Harmonic Loading . . . . .</b>	<b>71</b>
4.1. Forced response of conservative systems . . . . .	72
4.1.1. Forced response to cosine force . . . . .	77
4.2. Beating . . . . .	78
4.3. Forced response of dissipative systems . . . . .	79
4.4. Steady-state response to cosine force . . . . .	86
4.5. Resonance . . . . .	87
4.6. Dynamic amplification factors . . . . .	89
4.7. Resonant angular frequency . . . . .	91
4.8. Power absorbed in steady-state forced vibration . . . . .	92
4.9. Complex frequency response . . . . .	97
4.10. Nyquist plot . . . . .	101
4.11. Vibration measurement instruments . . . . .	105
4.11.1. Displacement sensor or vibrometer . . . . .	106
4.11.2. Velocity transducer or velometer . . . . .	108
4.11.3. Acceleration transducer or accelerometer . . . . .	108
4.12. Vibration isolation . . . . .	110
4.12.1. Vertical oscillating force . . . . .	110
4.12.2. Harmonic motion of the base . . . . .	112
4.13. Mass eccentricity . . . . .	116

<b>Chapter 5. Measurement of Damping . . . . .</b>	123
5.1. Free-decay method . . . . .	123
5.2. Amplification method . . . . .	124
5.3. Half-power bandwidth method . . . . .	125
5.4. Nyquist plots . . . . .	129
5.5. Energy dissipated by damping . . . . .	129
5.5.1. Viscous damping . . . . .	129
5.5.2. Internal material damping . . . . .	132
<b>Chapter 6. Forced Response to Periodic Loading . . . . .</b>	141
6.1. Representation of a periodic function as a Fourier series . . . . .	142
6.1.1. Trigonometric form of the Fourier series . . . . .	142
6.1.2. Complex or exponential form of the Fourier series . . . . .	148
6.2. Fourier spectrum . . . . .	149
6.3. Response to periodic loading . . . . .	151
6.3.1. Trigonometric Fourier series decomposition of the load function .	151
6.3.2. Exponential Fourier series decomposition of the load function .	155
<b>Chapter 7. Response to Arbitrary Loading in the Time Domain . . . . .</b>	161
7.1. Response to an impulse loading . . . . .	161
7.2. Dirac impulse or delta function . . . . .	163
7.3. Response to a Dirac impulse . . . . .	165
7.4. Duhamel integral . . . . .	165
7.5. Convolution integral . . . . .	167
7.6. Numerical evaluation of the Duhamel integral . . . . .	170
7.6.1. Conservative system . . . . .	170
7.6.2. Dissipative system . . . . .	171
7.7. Response to a step load . . . . .	176
7.8. Response to a linearly increasing force . . . . .	178
7.9. Response to a constant force applied slowly . . . . .	179
7.10. Response to impulse loads . . . . .	181
7.10.1. Sinusoidal impulse . . . . .	182
7.10.2. Rectangular impulse . . . . .	184
7.10.3. Triangular impulse . . . . .	187
7.10.4. Symmetric triangular impulse . . . . .	189
7.10.5. Shock response spectra . . . . .	190
<b>Chapter 8. Forced Response to Arbitrary Loading in Frequency Domain . . . . .</b>	195
8.1. Fourier transform . . . . .	195
8.2. Relationship between the frequency response function and the impulse response function . . . . .	199
8.3. Discrete Fourier transform . . . . .	200

8.4. Nyquist frequency . . . . .	203
8.5. Fast Fourier transform: Cooley–Tukey algorithm . . . . .	205
8.6. Signal flow graph . . . . .	213
8.7. Calculation of double nodes . . . . .	215
8.8. Calculation of the inverse fast Fourier transform . . . . .	216
<b>Chapter 9. Direct Time Integration of Linear Systems . . . . .</b>	<b>223</b>
9.1. General . . . . .	224
9.2. Exact numerical integration for piecewise linear loading functions . . . . .	226
9.3. Central difference method . . . . .	229
9.4. Newmark method . . . . .	236
9.4.1. Average acceleration method . . . . .	238
9.4.2. Linear acceleration method . . . . .	241
9.4.3. Generalization of the Newmark’s methods . . . . .	243
<b>Chapter 10. Direct Time Integration of Nonlinear Systems . . . . .</b>	<b>249</b>
10.1. Incremental equation of dynamic equilibrium . . . . .	249
10.2. Newmark’s methods . . . . .	251
10.3. Error reduction with Newton method . . . . .	255
<b>Chapter 11. Generalized Elementary Systems . . . . .</b>	<b>271</b>
11.1. Rigid-body assemblies . . . . .	271
11.2. Flexible system . . . . .	277
11.3. Elementary generalized system . . . . .	281
11.4. Rayleigh method . . . . .	283
11.4.1. Elementary system . . . . .	283
11.4.2. Continuous system . . . . .	284
11.4.3. Selection of a displacement function . . . . .	287
11.4.4. Improved Rayleigh method . . . . .	294
11.4.5. Discrete system . . . . .	297
<b>Chapter 12. Response to Earthquake Excitation . . . . .</b>	<b>307</b>
12.1. Earthquake response in the time domain . . . . .	308
12.2. Response spectrum . . . . .	312
12.3. Design spectrum . . . . .	319
12.4. Use of design spectra . . . . .	320
12.5. Earthquake intensity . . . . .	323
12.6. Fourier spectrum, relative velocity spectrum and energy . . . . .	324
12.7. Response of a generalized SDOF system . . . . .	328
12.8. Nonlinear response . . . . .	333
12.9. Inelastic response spectrum . . . . .	340

<b>PART 2. MULTI-DEGREES OF FREEDOM SYSTEMS . . . . .</b>	347
<b>Chapter 13. Equations of Motion . . . . .</b>	349
13.1. Simplified model of a building . . . . .	350
13.2. Equation of dynamic equilibrium . . . . .	352
13.3. Stiffness influence coefficients . . . . .	354
13.4. Static condensation . . . . .	366
13.5. Support motions . . . . .	368
13.5.1. Synchronous support motion of a planar system . . . . .	369
13.5.2. Structure with multiple support motions . . . . .	374
13.5.3. Additional mass method . . . . .	379
<b>Chapter 14. Finite Element Method . . . . .</b>	385
14.1. Finite element method: overview . . . . .	385
14.2. Global formulation using the principle of virtual works . . . . .	388
14.3. Local formulation using the principle of virtual work . . . . .	399
14.4. Coordinate transformations . . . . .	403
14.5. Generalized displacements, strains and stresses . . . . .	406
14.6. Two-node truss element . . . . .	411
14.7. Beam finite element . . . . .	414
14.8. Beam-column element . . . . .	418
14.9. Geometric stiffness matrix . . . . .	422
14.9.1. Two-node truss element . . . . .	423
14.9.2. Two-node beam-column element . . . . .	426
14.10. Rules for assembling element matrices . . . . .	429
14.11. Properties of the stiffness matrix . . . . .	433
14.12. Numerical solution . . . . .	434
14.13. Post-processing . . . . .	439
14.14. Convergence and compatibility . . . . .	440
14.15. Isoparametric elements . . . . .	441
<b>Chapter 15. Free Response of Conservative Systems . . . . .</b>	445
15.1. Physical significance of eigenvalues and eigenvectors . . . . .	446
15.2. Evaluation of vibration frequencies . . . . .	448
15.3. Evaluation of mode shapes . . . . .	450
15.4. Flexibility matrix formulation . . . . .	454
15.5. Influence of axial forces . . . . .	456
15.6. Orthogonality of mode shapes . . . . .	457
15.6.1. Normalization of eigenvectors . . . . .	460
15.7. Comparing prediction and measured data . . . . .	461
15.8. Influence of the mass matrix . . . . .	464

<b>Chapter 16. Free Response of Non-conservative Systems . . . . .</b>	471
16.1. Proportional damping matrix . . . . .	471
16.2. Superposition of modal damping matrices . . . . .	476
16.3. Damping measurement by harmonic excitation . . . . .	478
16.4. Non-proportional damping matrix . . . . .	481
16.5. Construction of non-proportional damping matrices . . . . .	483
<b>Chapter 17. Response to Arbitrary Loading by Modal Superposition . . . . .</b>	489
17.1. Normal coordinates . . . . .	490
17.2. Uncoupled equations of motion . . . . .	491
17.3. Modal superposition method . . . . .	493
17.3.1. Calculation of the response . . . . .	493
17.3.1.1. Direct numerical integration . . . . .	493
17.3.1.2. Calculation of Duhamel integral . . . . .	493
17.3.1.3. Fourier transform . . . . .	494
17.3.2. Initial conditions . . . . .	494
17.3.3. Total response . . . . .	495
17.3.4. Calculation of elastic forces . . . . .	495
17.3.5. Error due to the use of a truncated eigenvector base . . . . .	498
17.3.6. Harmonic amplification . . . . .	502
17.3.7. Static correction . . . . .	504
17.3.8. Modal acceleration method . . . . .	506
17.3.9. Summary of the modal superposition method . . . . .	507
<b>Chapter 18. Modal Superposition Response to Earthquake Excitation . . . . .</b>	511
18.1. Modal superposition . . . . .	511
18.2. Effective modal mass . . . . .	516
18.3. Error due to the use of a truncated modal base . . . . .	517
18.4. Superposition of spectral responses . . . . .	522
18.5. Response of systems with multiple supports . . . . .	528
<b>Chapter 19. Properties of Eigenvalues and Eigenvectors . . . . .</b>	533
19.1. Standard symmetric eigenvalue problem . . . . .	533
19.2. Similarity transformations . . . . .	536
19.3. Some properties of the symmetric eigenvalue problem . . . . .	537
19.4. Generalized symmetric eigenvalue problem . . . . .	539
19.4.1. Fundamental properties . . . . .	540
19.4.2. Multiplicity of eigenvalues . . . . .	543
19.5. Standard eigenvalue problem for a real non-symmetric matrix . . . . .	545
19.6. Spectral shift . . . . .	548
19.7. Zero masses . . . . .	550
19.8. Transformation of generalized eigenvalue problems to standard form .	551

19.9. Rayleigh quotient . . . . .	554
19.9.1. Homogeneity property . . . . .	555
19.9.2. Stationarity property . . . . .	555
19.9.3. Bounding property . . . . .	556
19.10. Max-min and min-max characterization of the eigenvalues . . . . .	558
19.11. Cauchy's interlace theorem . . . . .	561
19.12. Properties of characteristic polynomials . . . . .	563
19.13. Sylvester's law of inertia . . . . .	566
<b>Chapter 20. Reduction of Coordinates . . . . .</b>	<b>571</b>
20.1. Kinematic constraints . . . . .	572
20.2. Static condensation . . . . .	576
20.3. Rayleigh analysis . . . . .	581
20.4. Rayleigh–Ritz analysis . . . . .	583
20.5. Load-dependent Ritz vectors . . . . .	590
20.6. Guyan–Irons reduction method . . . . .	602
<b>Chapter 21. Numerical Methods for Eigenproblems . . . . .</b>	<b>607</b>
21.1. Iterative methods . . . . .	608
21.1.1. Inverse iteration . . . . .	608
21.1.2. Direct iteration . . . . .	615
21.1.3. Inverse iteration with spectral shift . . . . .	620
21.1.4. Inverse iteration with orthogonal deflation . . . . .	622
21.2. Rotation and reflection . . . . .	623
21.3. Transformation methods . . . . .	625
21.3.1. Jacobi method . . . . .	626
21.3.2. Generalized Jacobi method . . . . .	631
21.3.3. QR iteration . . . . .	639
21.4. HQRI iterations . . . . .	651
21.5. Subspace iterations . . . . .	660
21.5.1. Algorithm . . . . .	660
21.5.2. Choice of the starting iteration vectors . . . . .	663
<b>Chapter 22. Direct Time Integration of Linear Systems . . . . .</b>	<b>673</b>
22.1. Multi-step methods . . . . .	674
22.1.1. Multi-step methods for first-order equations . . . . .	674
22.1.2. Multi-step methods to solve second-order equations . . . . .	675
22.2. The central difference method . . . . .	676
22.3. Houbolt method . . . . .	681
22.4. Newmark methods . . . . .	683
22.5. Wilson- $\theta$ method . . . . .	687
22.6. Collocation methods . . . . .	689

22.7. HHT- $\alpha$ method . . . . .	694
22.8. Estimation of the highest eigenvalue . . . . .	699
22.9. Stability analysis . . . . .	705
22.9.1. Exact solutions . . . . .	706
22.9.2. Discrete approximation . . . . .	707
22.9.3. Central difference method . . . . .	707
22.9.4. Houbolt method . . . . .	708
22.9.5. Newmark method . . . . .	709
22.9.6. Wilson- $\theta$ method . . . . .	710
22.9.7. HHT- $\alpha$ method . . . . .	711
22.10. Stability conditions . . . . .	713
22.10.1. Central difference method . . . . .	719
22.10.2. Newmark methods . . . . .	721
22.10.3. Wilson- $\theta$ method . . . . .	726
22.10.4. HHT- $\alpha$ method . . . . .	726
22.10.5. Comparison of the various methods . . . . .	727
22.11. Analysis of the consistency of a finite-difference scheme . . . . .	728
22.12. Analysis of the accuracy . . . . .	730
22.12.1. Accuracy of the Newmark method . . . . .	731
22.12.2. Measure of the accuracy of integration schemes . . . . .	732
22.13. Filtering of unwanted artificial modes and overestimation of the response . . . . .	735
22.14. Selection of a numerical direct integration method . . . . .	740
<b>Chapter 23. Direct Time Integration of Nonlinear Systems</b> . . . . .	743
23.1. Incremental equation of motion . . . . .	743
23.2. The central difference explicit method . . . . .	744
23.3. Implicit Newmark methods . . . . .	747
23.4. Error reduction with the Newton method . . . . .	748
23.5. Nonlinear analysis of a building under seismic loading . . . . .	753
<b>Appendix A: Complex Numbers</b> . . . . .	759
A.1. Algebraic representation . . . . .	759
A.2. Operations . . . . .	760
A.3. Geometric representation . . . . .	760
A.4. Trigonometric form . . . . .	761
A.5. Roots . . . . .	764
<b>Bibliography</b> . . . . .	767
<b>Index</b> . . . . .	775