Table of Contents

Preface xi

Introduction xiii

List of Main Symbols xxiii

Chapter 1. A One-Dimensional Beam Metamodel 1

1.1. Models and metamodel 2
1.2. Internally unconstrained beams 3
1.2.1. Kinematics 3
1.2.2. Dynamics 5
1.2.3. The hyperelastic law 9
1.2.4. The Fundamental Problem 11
1.3. Internally constrained beams 12
1.3.1. The mixed formulation for the internally constrained beam . . . . kinematics and constraints 13
1.3.2. The displacement method for the internally constrained beam . . 18
1.4. Internally unconstrained prestressed beams 24
1.4.1. The nonlinear theory 25
1.4.2. The linearized theory 26
1.5. Internally constrained prestressed beams 29
1.5.1. The nonlinear mixed formulation 29
1.5.2. The linearized mixed formulation 30
1.5.3. The nonlinear displacement formulation 31
1.5.4. The linearized displacement formulation 32

1.6. The variational formulation 33
1.6.1. The total potential energy principle 34
1.6.2. Unconstrained beams 36
1.6.3. Constrained beams 37
1.6.4. Unconstrained prestressed beams 39
1.6.5. Constrained prestressed beams 41
1.7. Example: the linear Timoshenko beam 44
1.8. Summary 47

Chapter 2. Straight Beams 55

2.1. Kinematics 55
2.1.1. The displacement and rotation fields 55
2.1.2. Tackling the rotation tensor 60
2.1.3. The geometric boundary conditions 63
2.1.4. The strain vector 64
2.1.5. The curvature vector 68
2.1.6. The strain–displacement relationships 73
2.1.7. The velocity and spin fields 74
2.1.8. The velocity gradients and strain-rates 78
2.2. Dynamics 82
2.2.1. The balance of virtual powers 83
2.2.2. The inertial contributions 88
2.2.3. The balance of momentum 91
2.2.4. The scalar forms of the balance equations and boundary conditions 97
2.2.5. The Lagrangian balance equations 99
2.3. Constitutive law 102
2.3.1. The hyperelastic law 102
2.3.2. Identification of the elastic law from a 3D-model 104
2.3.3. Homogenization of beam-like structures 110
2.3.4. Linear viscoelastic laws 112
2.4. The Fundamental Problem 114
2.4.1. Exact equations 115
2.4.2. The linearized theory for elastic prestressed beams 118
2.5. The planar beam 122
2.5.1. Kinematics 122
2.5.2. Dynamics 124
2.5.3. The Virtual Power Principle 126
2.5.4. Constitutive laws 127
2.5.5. The Fundamental Problem 127
2.6. Summary 129

Chapter 3. Curved Beams 133

3.1. The reference configuration and the initial curvature 133
3.2. The beam model in the 3D-space 137
3.2.1. Kinematics 137
3.2.2. Dynamics 142
3.2.3. The elastic law 144
3.2.4. The Fundamental Problem 144
3.3. The planar curved beam 152
3.3.1. Kinematics 153
3.3.2. Dynamics 155
3.3.3. The Virtual Power Principle 157
3.3.4. Constitutive law 157
3.3.5. Fundamental Problem 158
3.4. Summary 160

Chapter 4. Internally Constrained Beams 163

4.1. Stiff beams and internal constraints 163
4.2. The general approach 166
4.3. The unshearable straight beam in 3D 168
4.3.1. The mixed formulation 169
4.3.2. The displacement formulation 172
4.4. The unshearable straight planar beam 177
4.5. The inextensible and unshearable straight beam in 3D 180
4.5.1. Hybrid formulation: Version I 181
4.5.2. Hybrid formulation: Version II 182
4.6. The inextensible and unshearable straight planar beam 183
4.6.1. Hybrid formulation: Version I 183
4.6.2. Hybrid formulation: Version II 185
4.6.3. The mixed formulation 186
4.6.4. The direct condensation of the elastica equilibrium equations . . . 188
4.7. The inextensible, unshearable and untwistable straight beam 190
4.8. The foil-beam 192
4.9. The shear–shear–torsional beam 193
4.10. The planar unshearable and inextensible curved beam 197
4.10.1. The hybrid formulation 197
4.10.2. The mixed formulation 200
4.11. Summary 201

Chapter 5. Flexible Cables 205

5.1. Flexible cables as a limit of slender beams 205
5.2. Unprestressed cables 207
5.2.1. Kinematics 207

5.2.2. Dynamics . . . 213
5.2.3. Constitutive law . . . 217
5.2.4. The Fundamental Problem . . . 218
5.3. Prestressed cables . . . 220
5.3.1. Quasi-exact models . . . 220
5.3.2. The linearized theory . . . 225
5.3.3. Taut strings . . . 227
5.4. Shallow cables . . . 230
5.4.1. An approximated nonlinear model . . . 231
5.4.2. An approximated linearized model . . . 234
5.5. Inextensible cables . . . 235
5.5.1. Inextensible unprestressed cables . . . 236
5.5.2. Inextensible prestressed cables . . . 237
5.6. Summary . . . 240

Chapter 6. Stiff Cables . . . 243

6.1. Motivations . . . 243
6.2. Unprestressed stiff cables . . . 246
6.2.1. Kinematics . . . 246
6.2.2. Dynamics . . . 249
6.2.3. The elastic law . . . 251
6.2.4. The Fundamental Problem . . . 251
6.3. Prestressed stiff cables . . . 252
6.3.1. Nonlinear model . . . 253
6.3.2. The linearized model . . . 256
6.3.3. Taut strings . . . 258
6.4. Reduced models . . . 261
6.4.1. Sagged cables . . . 261
6.4.2. Shallow cables . . . 263
6.5. Inextensible stiff cables . . . 264
6.5.1. Unprestressed cables . . . 265
6.5.2. Prestressed cables . . . 266
6.5.3. Reduced model . . . 268
6.6. Summary . . . 269

Chapter 7. Locally-Deformable Thin-Walled Beams . . . 271

7.1. Motivations . . . 271
7.2. A one-dimensional direct model for double-symmetric TWB . . . 273
7.3. A one-dimensional direct model for non-symmetric TWB . . . 277
7.4. Identification strategy from 3D-models of TWB . . . 284
7.5. A fiber-model of TWB . . . 285
7.6. Warpable, cross-undeformable TWB . . . 289
7.6.1. Kinematics . . . 289

7.6.2. Identification procedure 293
7.6.3. The Fundamental Problem 299
7.7. Unwarpable, cross-deformable, planar TWB 299
7.7.1. Kinematics 300
7.7.2. Identification procedure 303
7.7.3. The Fundamental Problem 307
7.8. Summary 308

Chapter 8. Distortion-Constrained Thin-Walled Beams 311

8.1. Introduction 311
8.2. Internal constraints 312
8.2.1. The Vlasov constraint for open TWB 312
8.2.2. The Bredt constraint for tubular TWB 314
8.2.3. The Brazier constraint for planar TWB 316
8.3. The non-uniform torsion problem for bi-symmetric cross-sections . . . 317
8.3.1. The unconstrained model 317
8.3.2. The mixed formulation for the constrained model 319
8.3.3. The displacement formulation for the constrained model 323
8.4. The general problem for warpable TWB 324
8.5. Cross-deformable planar TWB 328
8.6. Summary 332

Bibliography 335

Index 345