Contents

Preface ix

Introduction xi

Chapter 1. The Model 1

1.1. About contact and friction conditions: nonsmooth nonlinearities 1
1.2. A class of discrete systems 4
1.2.1. The basic mass–spring systems 4
1.2.2. A special case without coupling 5
1.2.3. Larger size systems 6
1.3. Study of the restoring force 6
1.3.1. The geometrically linear case 7
1.3.2. The geometrical nonlinearity in 2D: adding smooth nonlinearities 8
1.4. The dynamical problem 10

Chapter 2. Mathematical Formulation 13

2.1. The complete mathematical statement 13
2.1.1. Existence results 16
2.1.2. Counterexamples to uniqueness 17
2.1.3. The functional framework for well-posedness 27
2.2. Computational methods 28
2.2.1. The event-driven methods 28
2.2.2. The time stepping algorithms 32
2.2.3. When the trajectories remain in contact 34

Chapter 3. The Equilibrium States 39

3.1. In the linearized case 39
3.1.1. The equilibrium states 39
3.1.2. Classification of the equilibria 41
3.2. Equilibria at large strains 44
3.2.1. Equilibrium solutions out of contact 44
3.2.2. Stationary solutions in contact 48

Chapter 4. Stability 57

4.1. Stability of the equilibrium states in the classical sense 57
4.1.1. Some qualitative phases of the dynamics 58
4.1.2. The stability results 62
4.2. Introducing a new notion of stability 73
4.3. Analysis of simple models 76
4.3.1. A chain of masses 76
4.3.2. The single mass model 78
4.4. A slightly more complicated mass–spring system 83
4.4.1. Equilibrium states 84
4.4.2. Stability 90
4.5. Numerical experiments on a finite element discretization of an elastic body 98
4.5.1. A reference state with imminent sliding contacts 100
4.5.2. The block perturbed by some horizontal shock 101

Chapter 5. Exploring the Case of the Linear Restoring Force 105

5.1. Equilibrium states in the $\{R_t, R_n\}$ plane under an oscillating loading 107
5.2. When no equilibrium solutions exist 111
5.2.1. Sliding periodic solutions 111
5.2.2. The transition for losing contact 124
5.2.3. Loss of contact 129
5.2.4. The global behavior 131
5.3. When a single equilibrium solution exists 131
5.3.1. The set of solutions when T is larger than T_α 134
5.3.2. When T is smaller than T_α 143
5.4. When infinitely many equilibria exist 150
5.5. Toward a more general excitation 154
5.5.1. Existence of equilibrium states 155
5.5.2. When infinitely many equilibrium solutions exist ($\varepsilon < (\mu A)/K_t$) 156
5.5.3. When no equilibrium solutions exist ($\varepsilon > (\mu A)/K_t$) 158

5.5.4. The transition where a single equilibrium state exists ($\varepsilon = (\mu A)/K_t$) 159

Chapter 6. The Case of the Nonlinear Restoring Force 161

6.1. Introduction 161
6.1.1. The equilibrium solutions considered 162
6.1.2. Recalling a few useful results 164
6.2. A particular case: $F_n > 0$ large enough and $|F_t|$ large enough 165
6.2.1. Linearization around an equilibrium $(u_t^0, 0)$ 165
6.2.2. Approximating the qualitative dynamics 167
6.2.3. Where the qualitative dynamics of the linearized problem differs from the nonlinear one 168
6.3. Qualitative study of the set of equilibria 169
6.3.1. Changes of the set of equilibria under an increasing load 169
6.3.2. Some preliminary calculations 170
6.3.3. Equilibria under tangential oscillations 173
6.4. Qualitative dynamics when ε is smaller than $\overline{\varepsilon}$ 174
6.4.1. Comments about the calculations 174
6.4.2. Preliminary: the case of constant loading 176
6.4.3. The qualitative dynamics 176
6.5. Periodic solutions when ε is greater than $\overline{\varepsilon}$ 183
6.5.1. For large periods 184
6.5.2. For small periods 187
6.5.3. In the interval $]T^*, T^{**}[$ 188

Chapter 7. Open Problems and Challenges 193

7.1. Complementary calculations 193
7.1.1. What has been done? 193
7.1.2. Complementary calculations for higher dimensional problems . . . 194
7.1.3. Investigating behavior under general excitations 195
7.1.4. Investigation in the presence of impacts 195
7.2. Toward more challenging problems 195
7.2.1. Enlarging the theory of stability of discrete systems 195
7.2.2. Improving the qualitative theory of dynamical systems 197
7.2.3. Refining the well-posedness criterion 197
7.2.4. And what about the continuous case... 198

Bibliography 201

Index 205