Preface

The mechanical testing of material is an important activity in research and industry. Scientists, engineers and technicians in a large range of domains (such as chemistry, metallurgy, mechanics, physics, polymer science, the rubber industry, aerospace and aeronautical industries, etc.) are interested in the technology used to investigate the mechanical properties of materials.

Static and dynamic tests are complementary and used concurrently. Static tests are often used in industry. Dynamic tests, however, are becoming more popular and, surprisingly, in many cases are easier to use than static ones, at least at lower frequencies. Let us take an example concerning the measurement of elastic Young's modulus or the shear modulus of a steel rod. In (nearly) static tests, glued strain gauges or special micro-displacement transducers are used to measure, the displacement of the sample in two or three directions at once, which enable us to evaluate the strains. With the measurement of applied force or torque, these two moduli are deduced from the basic definitions relating to stress and strain. There are a succession of measurements and calculations from the stress *versus* strain curves.

To obtain such elastic moduli using dynamic tests, evaluation of resonance frequencies only is required; dimensions and geometry of the sample and its weight, as well as boundary conditions, being known.

The main interest in dynamic testing, however, resides in characterization of the viscoelastic properties of materials, i.e. the dependency of technical moduli (or relaxation, creep functions) *versus* the frequency (or time).

Growing interest in dynamic tests

In industrially advanced countries, societies for material testing regularly publish recommendations concerning mechanical tests with indications on methods and test procedures. Over the last five decades, the methods of investigating dynamic mechanical properties have made significant progress. In the scientific and technical literature devoted to this problem, various viewpoints have been adopted. For example, the dynamic tests that interpret materials at a molecular level, i.e. structural factors, molecular weight, cross-linking, crystallinity, etc., constitute tools in the chemistry and physics of polymers. The science of rheology is being more frequently adopted in order to obtain technical moduli (or time functions) which serve in viscoelastic constitutive equations relating stress components to strain components.

Composite materials cover a large domain including laminated plastics and panels for the building industry. Special composite materials were initially designed and fabricated for advanced applications in the aeronautical and aerospace industries in the 1970s. The anisotropic properties of such materials are obtained by the appropriate arrangement of high-strength fibers in metallic or non-metallic matrices of the layers or by the orientation of the layers in the structural composite. The mechanical characterization of such anisotropic materials consequently requires special testing procedures which are more elaborate than the ones devoted to isotropic materials.

Characterization of metallic and non-metallic material damping capacities is of interest to specialists in chemistry and physics as well as in mechanics.

Measuring damping

The measurement of damping coefficients of mechanical structures gives rise to a large variety of methods in structural dynamics that deserve the attention of specialists in material testing. The transposition of those methods into rheology, however, requires some caution and adaptations. The damping of a mechanical structure depends on damping of the materials used in the structure and the geometry of the structure itself. Consequently it is necessary to have this distinction in mind. Material damping can be deduced from structural damping on the condition that the relationship between these two kinds of damping is known.

Damping capacities of materials cover a much larger range than structural damping. Globally, damping capacities of materials (defined as the quotient of the imaginary to real part of a complex modulus) can be divided into three classes:

a) low damping – tan $\delta < 10^{-3}$;

b) medium damping – $10^{-3} < \tan\delta < 5.10^{-2}$;

c) high damping – $5.10^{-2} < \tan\delta < 10$.

Mechanical structural damping, in the majority of cases, concerns class (b) and no caution concerning measurement technique is needed. Class (a) concerns steel or special metals and requires special caution when taking measurements in order to eliminate the predominant influence of air damping on the sample. Class (c) concerns some rubbers or special blends of polymers and metal powders. The usual methods adopted in structural dynamics require special adaptations.

Size and shape of the sample

In many circumstances, analysts have to deal with samples with special or unusual shapes and sizes. The sample cut off from a hollow cylinder is curved and necessitates a special sample holder. The sample can be very small and therefore commercially available apparatus cannot be used. This is the case in biomechanics, for example, where the sample is a cut-off from a small bone. Analysts then have to come up with and devise a special set up.

Appropriate knowledge on the elastodynamics of bounded media

Let us begin with some remarks about currently available instruments. In some apparatuses, the mechanical part and adopted loading system are designed in such a way that vibrations imposed on the sample are far from simple. It eventually gives rise to different kinds of vibrations (extensional, bending, torsion) which are coupled in the sample itself. Coupling of such vibrations is often neglected in proposed formulae giving the moduli. The last formulae are deduced from the elementary theory of vibration using localized mechanical parameters that are not necessarily valid for short and thick samples. Mechanical effects (such as shear and inertia effects) are not taken into account. When experiments are conducted in higher frequency ranges, wave dispersion phenomenon (which describes the variation of the wave velocity in the sample *versus* the frequency) is rarely taken into account.

Attachment of the sample by clamping, gluing or screw tightening creates zones where there is a three-dimensional state of stress that can be localized in the sample submitted to compression forces and also beyond the contact zone between the sample and holder system. This effect is particularly pronounced for a short sample.

These remarks, among others, show that confidence granted to an apparatus must not exclude critical thought and a mechanical background.

The book

I will now present and comment on the chapters in this book.

The authors have intentionally situated dynamic testing of materials in the context of bounded medium elastodynamics. The measurements of dynamic responses of the sample in a large range of frequencies are interesting for analysts who want to obtain viscoelastic complex moduli. Rheologists[1] are interested in the relationship between various resonance peaks of dynamic responses *versus* frequency and micromechanisms of the polymer being tested. Mechanical engineers wish to obtain the curves of complex moduli at low and high frequencies so as to include them in calculation of the dynamic responses of the mechanical structure.

The motion equations of the sample must be carefully chosen, taking into account the frequency range. The upper frequency guides the choice of degree of approximation, which is related to the set of motion equations in view of *wave dispersion* characterization at the chosen upper frequency range. Wave dispersion is not the only effect we need to account for. There is another dispersion phenomenon: the *viscoelastic dispersion*, which is also frequency dependent.

These two effects sometimes act in the same sense with respect to frequency, and *vice versa* regarding sample responses, depending on the type of stationary wave in the sample and the working frequency. This is the reason main wave dispersion should be taken into account and raises the delicate problem of reasoned choice of appropriate equations of motion, compatible with tractable numerical exploitation of experimental results.

Recently, specialists in structural computing science have focused on the *continuous element method* which permits structure calculation even in the ultrasonic frequency range. This method presents advantages and constitutes a serious competitor for *classical finite element method*. The elastodynamics of bounded media precisely furnishes theoretical foundations, particularly in the domain of wave dispersion. Consequently, this last topic is treated in detail for various wave types adopted in samples.

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1 Rheology designates the science which studies the flow (Greek radical *Rheos*) of solid or liquid materials.

One of the new methods of treating viscoelastic material characterization is to use continuous elements as a tool to numerically solve an inverse problem without recourse to closed-form eigenvalue solutions of boundary equations.

We try to bridge the gap between theoretical academic works on wave dispersion and practical applications that do not yet sufficiently exploit the literature. Many significant theoretical contributions concerning wave dispersion in bounded media during the last three decades merit being brought together, classified and examined in view of applications.

Part A is devoted to continuum mechanics (constitutive equations of materials including *anisotropic materials*). Chapter 1 covers linear and applied viscoelasticity. Chapter 2 looks at the principle of correspondence that permits the conversion of elastic equations of motion into viscoelastic ones, with the condition that boundary conditions and sample geometry remain the same.

Chapter 3 is devoted to Williams-Landel-Ferry's (WLF) method, which is very popular in the field of polymer chemistry and deserves the attention of mechanical engineers. It permits artificial enlargement of the modulus (or compliance) curve in an unusually large frequency range (often more than eight decades) on the condition that *the superposition principle temperature-frequency is applicable.*

Serious limitations of WLF's method must be taken into account when dealing with anisotropic artificial composite materials. The superposition principle may not be valid for such materials. The remaining possibility is to directly evaluate complex modulus (or compliance) over a large frequency range. This is the main reason to resort to appropriate wave dispersion theories for these materials.

The closed-form expression of viscoelastic modulus (or compliance) is often necessary in computer codes to evaluate the damping responses of structures. Examining this problem from a practical point of view, we notice that analog models, usually proposed in textbooks and publications, with a reduced number of springs and dashpots cleverly arranged in series and/or in parallel, indeed help the reader "visualize" the material.

For a given experimental dynamic curve, however, we do not know in advance how many associated mechanical elements (springs and dashpots) will be adopted, particularly when the frequency range is large. The appropriate model is often more complicated than the simple academic models indicated above. This unknown model belongs to the "*black box*" constituted by the material in the usual mechanical inverse problem to be solved![2]

Some methods are then proposed to obtain a closed-form expression of modulus (or compliance) *versus* frequency by quotient of polynomials of the same degree (without *a priori* assumption of its degree) or by fractional derivatives whose interest resides in the condensed mathematical expression.

In Chapter 4, various formulations of equations of motion are presented. As we have to deal with bounded medium and finite sample length, no exact equations are available: approximate equations of motion are to be found. The main question is: *what is the degree of approximation we must adopt*? This question raises a subsidiary question: how many generalized displacement components and generalized force components are to be adopted to fully cover the mechanical behavior of the sample?

All the methods presented in dynamics textbooks can be utilized. D'Alembert's principle and Lagrange's equations constitute the first group of methods. The second group includes Hamilton's variational principle using simple displacement field. Love's variational principle can be considered as derived from Hamilton's one.[3] Mixed field Reissner's principle is, in some cases, useful for correctly portraying the dispersion curve of the sample. This variational method is referred to in an accurate analysis of vibrations of an anisotropic rod.

Part B concerns various types of rod vibration: extension, bending and torsion. Vibration modes are a source of vocabulary confusion for analysts. Let us clarify some different definitions.

Vibration modes might concern the *nature of the vibration* as mentioned above. The nature of the vibration is related to the predominant strain in the sample, i.e. the extensional strain along the rod axis in longitudinal motion, shear strain in rod torsion, and axial strain in bending test.

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2 Recently in electrical engineering, as well as in mechanical engineering, attention has focused on distributed models in a ladder using linear elementary models (springs and dashpots) or fractional derivatives which constitute an elegant method to characterize materials in a large frequency range with minimum parameters.

3 Using variational principles and integrating by parts, we directly obtain equations of motion and natural boundary conditions as well.

In structural dynamics, vibration mode is related to *eigenfrequencies* and *eigenvectors,* which are portrayed by nodal lines on the sample surface whose density increases with frequency.

Elastodynamic vocabulary: attention is focused not only on the representation of nodal lines on the lateral surface of the sample, but also on the sample thickness itself. Let us take an example: the bending test on a rod with a rectangular cross-section. There is a neutral line in the thickness whose motion is representative of bending motion in the first elastodynamic mode. Higher elastodynamic modes correspond to discontinuous or undulating neutral lines in the cross-section. To create such modes, a special array of small piezoelectric exciters can be used. For the usual characterization of material, higher elastodynamic modes are rarely used, although they might constitute a good tool in fracture mechanics. To avoid confusion on the signification of vibration mode, additional indications between brackets will be used: (nature), (eigenfrequency or eigenvalue), (elastodynamics).

In some chapters, theoretical works are presented with proofs so as to facilitate the reader's consultation. Intentionally, Part B is presented with details in the theoretical formulation so as to facilitate the reader's work and reduce his/her burden in the search of scientific papers sometime published some centuries ago! Appendixes presented at the end of each chapter might help researchers to find the demonstration of formulae. For each kind of wave a collection of theories from elementary to sophisticated may present difficulties and a profusion of theories to a reader who approaches the problem for the first time. We have presented a set of theories as a toolbox: practitioners and researchers have to choose the appropriate tool for special applications themselves.

Some readers might be surprised by the unusual length of the chapters in Part B compared to a classical book devoted to the same topics. The authors' intention is to gather together all the possible groups of theories with various degrees of approximations, so the reader does not need to search elsewhere. The contributions of our research team are naturally presented with the intention of completing existing literature on the vibration of rods with finite and infinite lengths.

Coupled vibrations highlight the effect of non-diagonal elastic coefficients in the equations of motion. Coupled vibrations are intentionally used with an *off-axis anisotropic* rod. Matricial diagonal coefficients being known, such coupled vibrations permit us to evaluate *non-diagonal terms*.

Coupled vibrations exist even in an isotropic rod submitted to various vibration types, even for a closed section. In elementary theories these vibrations are neglected at lower frequencies. Shear effect in longitudinal and bending vibrations, however, occurs in equations of motion with higher degrees of approximation. Torsional rod vibration gives rise to axial strain and extensional vibration occurs. Consequently, two or more elastic (or viscoelastic) moduli are present in equations of motion.

The extensive utilization of rods in this book, instead of plates, necessitates explanation. In some technical and scientific publications, plate is indeed used to evaluate elastic (and/or viscoelastic) moduli. The objective of such works is to determine the whole set of elastic moduli. Elastic vibrations of plates necessitate measurements of vibration amplitude at many points and eventually for a certain number of (eigenfrequency) modes. On grounds of numerical calculation, optimization algorithms are referred to. The degree of complexity is considerably increased with respect to that concerning a rod. The challenge of adopting plate equations of motion is prohibitive compared to the one-dimensional equation for a rod with one, two or three displacement variables. The results obtained from plates in the scientific literature are unfortunately far from convincing, with the objective of solving an inverse problem to find moduli or stiffness coefficients of material[4].

Chapters 5, 6 and 7 present torsional, bending and extensional vibrations. In Chapter 5, a rod with rectangular cross-section is adopted, taking into account the ease of obtaining such a section. Warping of the cross-section is examined for isotropic and anisotropic materials. Saint Venant's dynamic equations of motion are presented as are the higher approximation equations of motion corresponding to more complex section warping.

Bending vibration in Chapter 6 concerns the elementary Bernoulli-Euler's equation of motion. Timoshenko's equation with a higher degree of approximation is preferred when working at a higher frequency. The bending vibration of an off-axis rod is also presented in order to evaluate the compliance a non-diagonal coefficient of anisotropic materials.

Extensional vibrations in a rod are presented in detail in Chapter 7. The longitudinal wave dispersion is surprisingly more difficult to apprehend than the one concerning the two aforementioned vibrations and requires a more elaborate displacement field. For application at higher frequency, the fourth-degree Bishop's equation of motion is not capable of correctly portraying the wave dispersion curve at higher frequencies. Touratier's formulation using internal constraints extends Volterra's work to anisotropic rods.

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4 In Chapter 10, however, progressive waves are used in plates to obtain material stiffness coefficients at ultrasonic frequency range

Chapter 8 is devoted to Le Rolland-Sorin's double pendulum working at very low frequency. This inventive, artful and simple method is practically unknown in English-speaking countries and deserves practicians' attention in the sense it requires so few measuring instruments compared to other test methods. The functioning principle is unusual compared to existing methods used in dynamic tests.

Chapter 9 examines vibrations in rings and hollow cylinders. In many situations we have to deal with a curved rod or straight rod with curved cross-section.

Chapter 10 is devoted to the propagation of ultrasonic waves in thick plates. Ultrasonic progressive dilatational (and/or shear) wave can be chosen in advance as can the wave direction. The second-order equation of motion is simple to handle and, surprisingly, the interpretation of experimental results is much easier to obtain than rod vibrations at lower frequencies. Plate samples with a large thickness compared to wavelength are used to equate the plate with a semi-infinite medium. Progressive waves are used for this purpose.

Chapter 11 concerns evaluation of the viscoelastic complex modulus using characteristic (trigonometric and hyperbolic) functions to express displacement components. Transmissibility function (which relates output displacement to input displacement) is used in the framework of an inverse problem to evaluate complex moduli (or compliance). Methods using some special mathematical algorithms are presented in the framework of research of solutions to the important mathematical inverse problems.

Finally, Chapter 12 complements the preceding chapter, using so-called continuous elements. This method is interesting because it offers us the chance to obtain a response curve in a very large frequency range by numerical computation which takes much less time than the finite element method. In our opinion, the matricial formulation of the problem and integration of elastodynamic equations of motion constitute one of the best ways of tackling the inverse viscoelastic problem.

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