
Contents

Preface	xi
Chapter 1. Introduction to Non-Local Elasticity	1
1.1. Why the non-local elasticity method for nanostructures?	1
1.2. General modeling of nanostructures	3
1.3. Overview of popular nanostructures	4
1.4. Popular approaches for understanding nanostructures	8
1.5. Experimental methods	9
1.6. Molecular dynamics simulations	9
1.7. Continuum mechanics approach	9
1.8. Failure of classical continuum mechanics	10
1.9. Size effects in properties of small-scale structures	11
1.10. Evolution of size-dependent continuum theories	12
1.11. Concept of non-local elasticity	14
1.12. Mathematical formulation of non-local elasticity	15
1.12.1. Integral form	15
1.12.2. Non-local modulus	17
1.12.3. Differential form equation of non-local elasticity	17
1.13. Non-local parameter	18
1.14. Non-local elasticity theory versus molecular dynamics	19
Chapter 2. Non-local Elastic Rod Theory	21
2.1. Background	21
2.2. Governing equation of motion of the nanorod	24
2.3. Results and discussions	29

Chapter 3. Non-local Elastic Beam Theories	33
3.1. Background	33
3.2. Non-local nanobeam model	36
3.2.1. Non-local Euler–Bernoulli beam theory	36
3.2.2. Non-local Timoshenko beam theory	43
3.2.3. Non-local Reddy beam theory	51
3.3. Torsional vibration of nanobeam	60
3.4. Comparison of the non-local beam theories	64
Chapter 4. Non-local Elastic Plate Theories	69
4.1. Non-local plate for graphene sheets	69
4.2. Non-local plate constitutive relations.	69
4.3. Free vibration of single-layer graphene sheets	72
4.3.1. Transverse-free vibration	73
4.3.2. Graphene sheets embedded in an elastic medium	75
4.4. Axially stressed nanoplate non-local theory	78
4.5. In-plane vibration	79
4.6. Buckling of graphene sheets	80
4.6.1. Uniaxial buckling	81
4.6.2. Graphene sheets embedded in an elastic medium	82
4.7. Summary	84
Chapter 5. One-Dimensional Double-Nanostructure-Systems	87
5.1. Background	87
5.2. Revisiting non-local rod theory	90
5.2.1. Equations of motion of double-nanorod-system	91
5.2.2. Solution methodology	94
5.2.3. Clamped-clamped boundary condition	95
5.2.4. Clamped-free (cantilever) boundary condition	96
5.2.5. Longitudinal vibration of auxiliary (secondary) nanorod	98
5.3. Axial vibration of double-rod system	99
5.3.1. Effect of the non-local parameter in the clamped-type DNRS	100
5.3.2. Coupling spring stiffness in DNRS	102
5.3.3. Higher modes of vibration in DNRS	102
5.3.4. Effect of non-local parameter, spring stiffness and higher modes in cantilever-type-DNRS.	103
5.4. Summary	104
5.5. Transverse vibration of double-nanobeam-systems	104
5.5.1. Background	105
5.5.2. Non-local double-nanobeam-system	107
5.6. Vibration of non-local double-nanobeam-system	110

5.7. Boundary conditions in non-local double-nanobeam-system	111
5.8. Exact solutions of the frequency equations	113
5.9. Discussions	116
5.9.1. Effect of small scale on vibrating NDNBS	117
5.9.2. Effect of the stiffness of the coupling springs on NDNBS	120
5.9.3. Analysis of higher modes of NDNBS	120
5.10. Summary	121
5.11. Axial instability of double-nanobeam-systems	122
5.11.1. Background	123
5.11.2. Buckling equations of non-local double-nanobeam-systems	124
5.12. Non-local boundary conditions of NDNBS	126
5.13. Buckling states of double-nanobeam-system	128
5.13.1. Out-of-phase buckling load: ($w_1 - w_2 \neq 0$)	128
5.13.2. In-phase buckling state: ($w_1 - w_2 = 0$)	129
5.13.3. One nanobeam is fixed: ($w_2 = 0$)	130
5.14. Coupled carbon nanotube systems	130
5.15. Results and discussions on the scale-dependent buckling phenomenon	131
5.15. Summary	136
Chapter 6. Double-Nanoplate-Systems	137
6.1. Double-nanoplate-system	137
6.2. Vibration of double-nanoplate-system	139
6.3. Equations of motion for non-local double-nanoplate-system	139
6.4. Boundary conditions in non-local double-nanoplate-system	142
6.5. Exact solutions of the frequency equations	144
6.5.1. Both nanoplates of NDNPS are vibrating out-of-phase: ($w_1 - w_2 \neq 0$)	144
6.5.2. Both nanoplates of NDNPS are vibrating in-phase: ($w_1 - w_2 = 0$)	146
6.5.3. One nanoplate of NDNPS is stationary: ($w_2(x, y, t) = 0$)	147
6.5.4. Discussions	148
6.5.5. Non-local double-nanobeam-system versus non-local double-nanoplate-system	156
6.5.6. Summary	157
6.6. Buckling behavior of double-nanoplate-systems	158
6.6.1. Background	159

6.6.2. Uniaxially compressed double-nanoplate-system	160
6.6.3. Buckling states of double-nanoplate-system	163
6.7. Results and discussion	167
6.7.1. Coupled double-graphene-sheet-system	167
6.7.2. Effect of small scale on NDNPS undergoing compression	168
6.7.3. Effect of stiffness of coupling springs in NDNPS	170
6.7.4. Effect of aspect ratio on NDNPS	173
6.8. Summary	177
Chapter 7. Multiple Nanostructure Systems	179
7.1. Longitudinal vibration of a multi-nanorod system	180
7.1.1. The governing equations of motion	182
7.1.2. Exact solution	185
7.1.3. Asymptotic analysis	191
7.1.4. Numerical examples and discussions	192
7.2. Transversal vibration and stability of a multiple- nanobeam system	197
7.2.1. The governing equations of motion	199
7.2.2. Exact solution	202
7.2.3. Asymptotic analysis	209
7.2.4. Numerical examples and discussions	210
7.3. Transversal vibration and buckling of the multi- nanoplate system	215
7.3.1. The governing equations of motion	217
7.3.2. Exact solutions	221
7.3.3. Asymptotic analysis	227
7.3.4. Numerical results and discussions	227
7.4. Summary	232
Chapter 8. Finite Element Method for Dynamics of Nonlocal Systems	235
8.1. Introduction	236
8.2. Finite element modeling of non-local dynamic systems	239
8.2.1. Axial vibration of nanorods	239
8.2.2. Bending vibration of nanobeams	241
8.2.3. Transverse vibration of nanoplates	243
8.3. Modal analysis of non-local dynamical systems	247
8.3.1. Conditions for classical normal modes	248
8.3.2. Non-local normal modes	250

8.3.3. Approximate non-local normal modes	251
8.4. Dynamics of damped non-local systems	254
8.5. Numerical examples.	256
8.5.1. Axial vibration of a single-walled carbon nanotube	256
8.5.2. Bending vibration of a double-walled carbon nanotube	261
8.5.3. Transverse vibration of a single-layer graphene sheet	265
8.6. Summary	269
Chapter 9. Dynamic Finite Element Analysis of Nonlocal Rods: Axial Vibration	271
9.1. Introduction	272
9.2. Axial vibration of damped non-local rods	275
9.2.1. Equation of motion	275
9.2.2. Analysis of damped natural frequencies	277
9.2.3. Asymptotic analysis of natural frequencies.	279
9.3. Dynamic finite element matrix	281
9.3.1. Classical finite element of non-local rods.	281
9.3.2. Dynamic finite element for damped non-local rod	282
9.4. Numerical results and discussions	285
9.5. Summary	291
Chapter 10. Non-local Nanosensor Based on Vibrating Graphene Sheets	293
10.1. Introduction	294
10.2. Free vibration of graphene sheets	295
10.2.1. Vibration of SLGS without attached mass	297
10.3. Natural vibration of SLGS with biofragment	299
10.3.1. Attached masses are at the cantilever tip	301
10.3.2. Attached masses arranged in a line along the width	301
10.3.3. Attached masses arranged in a line along the length	302
10.3.4. Attached masses arranged with arbitrary angle.	302
10.4. Sensor equations and sensitivity analysis	303
10.5. Analysis of numerical results	305
10.6. Summary	311

Chapter 11. Introduction to Molecular Dynamics for Small-scale Structures	313
11.1. Background	313
11.2. Overview of the molecular dynamics simulation method	314
11.3. Acknowledgement	325
Bibliography	327
Index	353