Chapter 1

The Physical Basis of Synthetic Aperture Radar Imagery

1.1. Electromagnetic propagation

The physics behind radar image formation is complex and involves several different topics. Some deal with electronic components devoted to transmission and reception of the wave, but they will not be discussed here. Other aspects, namely wave propagation and the interaction between microwave frequency waves and materials, are more important for our purposes. These two topics are the subject of this chapter. Electromagnetism obviously underlies both these phenomena and we begin with a review of useful results in this area.

1.1.1. *The laws of propagation in homogenous media*

1.1.1.1. *Basic equations*

An electromagnetic wave such as that emitted by radars is characterized at any point in space and at every moment by four vector values: $\vec{\mathbf{E}}$ (electric field), $\vec{\mathbf{D}}$ (electric displacement), $\vec{\mathbf{B}}$ (magnetic induction) and $\vec{\mathbf{H}}$ (magnetic field).

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These quantities verify Maxwell's equations, which in the absence of free charges and current densities are written as [JAC 75]:

$$\mathrm{div}\vec{\mathbf{D}} = 0, \qquad \mathbf{r\vec{o}t}\vec{\mathbf{H}} = \frac{\partial \vec{\mathbf{D}}}{\partial \mathrm{t}},$$
$$\mathrm{div}\vec{\mathbf{B}} = 0, \qquad \mathbf{r\vec{o}t}\vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathrm{t}}.$$

In the linear stationary case, the fields, the electric displacement and the magnetic induction are ruled by the following relations:

$$\vec{\mathbf{D}} = \epsilon\vec{\mathbf{E}}, \quad \text{and} \quad \vec{\mathbf{B}} = \mu\vec{\mathbf{H}}.$$

where ϵ is the permittivity and μ is the permeability. We will consider them as scalar values in this book (they are tensors in the general case of anisotropic dielectrics).

The electric field $\vec{\mathbf{E}}$ and magnetic field $\vec{\mathbf{H}}$ vectors are sufficient to characterize this electromagnetic wave for an unbounded, homogenous, isotropic medium which is free of charges and currents. We will use Maxwell's equations to show that every component of these fields verifies the wave equation:

$$\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0 \quad \text{where} \quad v = \frac{1}{\sqrt{\epsilon\mu}} \qquad [1.1]$$

We thus observe the electromagnetic energy transmission; v is the propagation velocity of the electromagnetic wave.

By denoting ϵ_0 the vacuum permittivity and μ_0 the vacuum permeability, we deduce c, i.e. the speed of light, as being:

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

In the general case and in the absence of any charge or current, the relative permittivity $\epsilon_r = \frac{\epsilon}{\epsilon_0}$ and relative permeability $\mu_r = \frac{\mu}{\mu_0}$ of the propagation medium are normally used, which makes it possible to express the propagation velocity according to *c*:

$$v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

The refractive index *n* for a propagation medium is defined as:

$$n = \frac{c}{v} = \sqrt{\epsilon_r \mu_r}\,.$$

Note that in non-magnetic media we will deal with $\mu_r = 1$, which leads to $n = \sqrt{\epsilon_r}$.

Since the medium is unbounded, $\vec{\mathbf{E}}(\vec{\mathbf{r}})$ and $\vec{\mathbf{H}}(\vec{\mathbf{r}})$ are perpendicular to each other at any $\vec{\mathbf{r}}$, and both are perpendicular to the propagation direction $\vec{\mathbf{s}}(\vec{\mathbf{r}})$ that represents the energy path, which is also called a ray.

If a preferred direction can be specified by convention in the plan ($\vec{\mathbf{E}}$, $\vec{\mathbf{H}}$), we will then be able to characterize $\vec{\mathbf{E}}$ (and therefore $\vec{\mathbf{H}}$) in terms of its polarization, i.e., its orientation with respect to the defined direction.

1.1.1.2. *Propagation equation solution*

In the presence of an isotropic radiation source $g(\vec{\mathbf{r}}_0, t)$ located at $\vec{\mathbf{r}}_0$, the solution of propagation equation [1.1] at any $\vec{\mathbf{r}}$ point in space is written:

$$u(\vec{\mathbf{r}}, t) = \frac{1}{4\pi|\vec{\mathbf{r}} - \vec{\mathbf{r}}_0|}\, g\left(\vec{\mathbf{r}}_0, t - \frac{|\vec{\mathbf{r}} - \vec{\mathbf{r}}_0|}{v}\right) \qquad [1.2]$$

The wave then propagates from the source (homogenous medium) in such a way that the wavefront, i.e., the normal ray surface everywhere in space, is a sphere centered on the source: the propagation between the source and any observer is carried out in a straight line.

In the specific case of satellite systems, the objects impinged by the electromagnetic wave are far enough from the antenna so that the wave can be seen as locally plane around the study zone (Fraunhoffer zone). Moreover, the only waves that are generally taken into account are quasi-monochromatic waves that have a frequency f_c (harmonic case) and are defined by their wavelength $\lambda = \frac{v}{f_c}$ and wave vector $\vec{\mathbf{k}} = \frac{2\pi}{\lambda}\vec{\mathbf{s}}$.

Given these hypotheses, we show that in the presence of a source in $\vec{\mathbf{r}}_0$, u, which is the propagation equation solution in $\vec{\mathbf{r}}$, is written as:

$$u(\vec{\mathbf{r}},t) = \frac{e^{i\vec{\mathbf{k}}(\vec{\mathbf{r}}-\vec{\mathbf{r}}_0)}}{4\pi\left|\vec{\mathbf{r}}-\vec{\mathbf{r}}_0\right|} g(\vec{\mathbf{r}}_0,t) \qquad [1.3]$$

$\vec{\mathbf{r}}$ and $\vec{\mathbf{r}}_0$ fields differ from one another by a phase term and an attenuation term (a term in $\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_0|}$). A surface defined by a set of points sharing the same phase is called a wave surface: $\vec{\mathbf{k}}$ is normal to the wave surface at every point, and the electric and magnetic fields are situated in the plan tangent to the wave surface.

In the general case (equation [1.2]) as well as in the quasi-monochromatic case (equation [1.3]), the term $\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_0|}$ appears describing an attenuation phenomenon arising from energy conservation. By integrating energy over a wave surface or the wave front, energy transmitted by the source should be obtained. This attenuation effect, which is quite strong in airborne radars, may also be significant in satellite imaging radars. With the transmitter located hundreds of kilometers away in orbit and imaged areas extending over dozens of kilometers, the attenuation term may indeed vary by several percentage points and create noticeable effects on the images.

1.1.2. *Propagation equation in heterogenous media*

As the wave no longer propagates through a homogenous medium, electric and magnetic fields no longer obey propagation equation [1.1]. In this case, several major phenomena have to be taken into account, i.e.:

– a change in the propagation, which is no longer a straight line due to a wavefront bend;

– a scattering phenomenon (e.g., backscattering, multiple scattering) that alters the energy transmitted along a ray;

– a potential transfer of energy into heat leading to wave absorption.

As a general rule, a simple expression of the propagation equation will no longer be available. Nevertheless, if the perturbation caused by the heterogenities of the propagation medium is weak enough, we can resort to a traditional method (also in the linear framework) which consists of adding a complementary term to equation [1.1]:

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \Lambda\left(u, \vec{\mathbf{r}}\right) \qquad [1.4]$$

where

$$\Lambda\left(u, \vec{\mathbf{r}}\right) = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

By decomposing the field u into two terms: u_0 which is the incident field and u_p which is the field created by the effects of the perturbation:

$$u = u_0 + u_p \quad u_0 \text{ such that } \quad \nabla^2 u_0 - \frac{1}{c^2}\frac{\partial^2 u_0}{\partial t^2} = 0,$$

we note that, in the end, the problem comes down to solve the following equation:

$$\nabla^2 u_p - \frac{1}{c^2}\frac{\partial^2 u_p}{\partial t^2} = \Lambda\left(u_0 + u_p, \vec{\mathbf{r}}\right)$$

If there is no absorption, we can use the Born first order approximation to solve it by only taking into consideration the incident field within the perturbation term:

$$\nabla^2 u_p - \frac{1}{c^2}\frac{\partial^2 u_p}{\partial t^2} = \Lambda\left(u_0, \vec{\mathbf{r}}\right),$$

This makes it possible to interpret the term $\Lambda\left(u_0, \vec{\mathbf{r}}\right)$ as a source term in the field u_p, thus explaining the wave scattering process.

1.1.2.1. *The permittivity variation case*

In the case of permittivity variations, we can show that the propagation equation verified by field $\vec{\mathbf{E}}\left(\vec{\mathbf{r}}\right)$ is in general written as [LAV 97]:

$$\nabla^2 \vec{\mathbf{E}} - \frac{1}{v^2\left(\vec{\mathbf{r}}\right)}\frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} = -\nabla\left(\vec{\mathbf{E}}.\nabla\left(\mathrm{Log}\epsilon_r\right)\right). \qquad [1.5]$$

In the harmonic case, the second term of this equation can be ignored if the permittivity variations verify the relation:

$$\left|\nabla\left(\mathrm{Log}\epsilon_r\right)\right| << \frac{1}{\lambda}, \qquad [1.6]$$

Taking into account a variation $\Delta\epsilon$ over a displacement Δr, the above relation can also be written as:

$$\frac{\Delta\epsilon}{\epsilon} << \frac{\Delta r}{\lambda}.$$

Under these assumptions, the propagation equation is:

$$\nabla^2 \vec{\mathbf{E}} - \frac{1}{v^2\left(\vec{\mathbf{r}}\right)}\frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} = 0. \qquad [1.7]$$

This occurs as if we had replaced v by $v\left(\vec{\mathbf{r}}\right)$.

Two cases have to be considered:

– when permittivity varies around a stationary mean value, Λ is written:

$$\Lambda(u, \vec{\mathbf{r}}) = \left(\frac{1}{v^2(\vec{\mathbf{r}})} - \frac{1}{c^2} \right) \frac{\partial^2 u}{\partial t^2}$$

making it possible to rewrite the propagation equation within the Born approximation as:

$$\nabla^2 u_p - \frac{1}{c^2} \frac{\partial^2 u_p}{\partial t^2} = \left(\epsilon_r(\vec{\mathbf{r}}) - 1 \right) \frac{1}{c^2} \frac{\partial^2 u_0}{\partial t^2}. \qquad [1.8]$$

Energy scattering is still present, but the rays remain unchanged:

– when permittivity varies slowly (and thus relation [1.6] is greatly verified), we will assume that the notions of wavefront and propagation ray are still valid. In this case, the solution of equation [1.4] is the geometric optical solution, which by applying Fermat's principle makes it possible to establish the curvilinear abscissa s along a ray through the relation:

$$\frac{\mathrm{d}}{\mathrm{d}s} \left(n \frac{\mathrm{d}\vec{\mathbf{r}}}{\mathrm{d}s} \right) = \nabla n$$

Once we are positioned along the ray thus defined, our search for a solution of the type:

$$u = u(\vec{\mathbf{r}}) e^{i\Psi(k_c \vec{\mathbf{r}})}$$

where $k_c = 2\pi \frac{f_c}{c}$ yields non-trivial solutions if Ψ verifies the eikonal equation:

$$\left(\nabla \Psi(k_c \vec{\mathbf{r}}) \right)^2 = \epsilon(\vec{\mathbf{r}}) \mu c^2. \qquad [1.9]$$

1.1.2.2. *The propagation equation in an absorbing medium*

To account for a potential absorption of the incident wave, we can model the source term $\Lambda(u, \vec{\mathbf{r}})$ using integrodifferential operators. In this case, the wave vector may formally have an imaginary component: the wave undergoes a generally significant absorption phenomenon that may even lead to a quasi-total lack of propagation (perfect conductor case).

1.1.3. ***Application to satellite radars***

In order to reach the ground, the electromagnetic radiation emitted by the radar has to travel across the ionosphere, then the neutral atmosphere.

The ionosphere is the region of the atmosphere traditionally extending from 50 to 1,000 km in height, where there are enough free electrons to modify wave propagation. It is made up of three distinct layers, every one of which has a different electronic density ρ, expressed in electrons per cubic meter. Some of the diurnal and nocturnal characteristics of these layers are summarized in Table 1.1.

Ionosphere	Height	Diurnal ρ	Nocturnal ρ
Layer D	from 50 to 90 km	$<10^9$	~ 0
Layer E	from 90 to 150 km	$\sim 10^{11}$	$\sim 10^9$
Layer F	beyond 150 km	$\geq 10^{12}$	$\geq 10^{11}$

Table 1.1. *The different layers of the ionosphere: diurnal and nocturnal rough estimates of the electronic density ρ expressed in electrons per m^3*

In the ionosphere, the index depends on electronic density and is expressed for an *f* frequency in the form of:

$$n(f) = 1 - \frac{f_0^2}{2f^2} \qquad [1.10]$$

where f_0 is the plasma frequency that depends on electronic density and can be approximated by relation $f_0 \approx 9\sqrt{\rho}\,(f_0 \text{ in MHz})$. Given the ionospheric ρ *rough* *estimates*, we see that, in the different layers, this phenomenon has a very weak or even negligible effect on the centimetric waves used by imaging radars.

The neutral atmosphere, defined in terms of thermodynamic concepts, is the gaseous envelope that surrounds the Earth. It consists of several different layers: the troposphere extending from 0 to 15 km in height, where atmospheric heterogenities are located; the stratosphere from 15 to 50 km, where temperature increases with height; and the mesosphere from 50 to 80 km[1], where temperature decreases with increasing height. Within the neutral atmosphere, the n index is essentially a function of pressure, temperature and partial water vapor pressure. Since this index is very close to 1, we will instead use refractive co-index N, which is defined as $N = 10^6 (n-1)$ and expressed by a semi-empirical relation known as the Smith-Weintraub equation [LAV 97]:

$$N(T,P,e) = 77.6\frac{P}{T} - 5.6\frac{e}{T} + 0.375\,10^6\,\frac{e}{T^2} \qquad [1.11]$$

where T is the temperature in Kelvin, P is the atmospheric pressure and e is the partial water pressure[2], both expressed in hPa.

Systematic soundings of the entire surface of the Earth have provided an (at least statistically) accurate knowledge of the index for the stratosphere and mesosphere. In particular, Bean's atlas [BEA 66], which for most of the Earth provides the co-index $N(h)$ according to altitude h in the shape of a 5-parameter model whose coefficients are monthly averages calculated over a period of 5 years. By contrast, major index fluctuations are found in the troposphere, mostly as a result of air moisture and related clouds.

Index variations are low for both the ionosphere and neutral atmosphere: the hypotheses required by the eikonal equation are fully justified and the effects linked to index variation are only perceptible in time-of-flight measurement between the radar and the ground. In the neutral atmosphere, some gaseous components may exhibit resonances within the range of frequencies that is of interest to us, as peripheral electrons of their atoms and molecules are excited. This is the case with water vapor in particular (lines at 22.2 GHz, 183.3 GHz and 325.4 GHz) and oxygen (lines from 50 to 70 GHz and one isolated line at 118.74 GHz). The signal is almost entirely absorbed at these frequencies.

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1 The presence of free electrons in the mesosphere explains some overlapping with the ionosphere.
2 This partial pressure also depends on P and T.

Other phenomena such as hydrometeors may also have an influence on propagation, both on wave delay and wave absorption. Hail, snow and lightning impact considerably on the radar signals but are difficult to model.

1.2. Matter-radiation interaction

The source term in equation [1.4] shows that the propagation of an electromagnetic wave is scattered if the medium is not homogenous. A scattered wave then appears, which is not necessarily isotropic and its radiation pattern depends on the source term. This approach does not easily cover phenomena related to the discontinuities of the propagation medium (e.g., surfaces between two media with different indices) or those related to reflecting targets.

Any phenomenological approach must take into account the radiation wavelength and L, the characteristic length of discontinuities. Even though the general case eludes all analytical considerations – except for the homogenous spherical scatterer treated by the exact Mie model – we can still analytically handle two essentially opposite cases:

– $L \gg \lambda$: this is the case of the flat interface, which can be considered as unbounded so that it possible to use the Snell-Descartes equation;

– $L \ll \lambda$: this is the case of a point target which we will refer to as a Rayleigh target.

However, this approach fails to provide a reasonable account of reality, where in rare cases there may be only one perfectly smooth surface or only one quasi-point target. A pragmatic view of reality will thus prompt us to study in more detail two cases of high practical relevance, namely rough surfaces and point target distributions.

1.2.1. *Theoretical backscattering models*

1.2.1.1. *Unbounded flat interface*

Flat interfaces have been studied since the time of Descartes and Snell. The relations obtained in visible optics (Fermat's principle) are derived from the continuity relations imposed on Maxwell equation solutions. In the case of a flat interface between two media defined by their indexes n and n', if an incident wave impinges this interface at an angle θ with respect to the normal of the interface, we will have a reflected wave at an angle θ and, in the second medium, a wave refracted at an angle θ', so that:

$$n \sin \theta = n' \sin \theta'$$

The only situation in which the Snell-Descartes formalism may be altered is when the second medium is more or less conductive. The wave vector will then include an imaginary component specific to attenuation through the second medium. In the case of a perfect conductor, we will only have an evanescent wave in this second medium, as the energy of the incident wave is entirely conveyed to the reflected wave.

In reality, this is obviously an ideal situation, since interfaces are neither unbounded nor rigorously flat. We may nevertheless consider that an interface can be locally put into its tangent plane: the dimensions on which this approximation is valid correspond to the dimensions of an antenna with a directivity pattern that is directly related to these dimensions. The incident wave will therefore be backscattered with some directivity pattern mainly along the refracted and reflected rays of Snell-Descartes law, as well as according to a radiation pattern for the other directions, in keeping with the Huygens principle and the diffraction theory. This radiation pattern is that of an antenna whose dimensions are those of the approximation area. Despite its simplistic appearance, this analysis gives us the order of magnitude of the backscattered field in other directions than those of the Snell-Descartes angles.

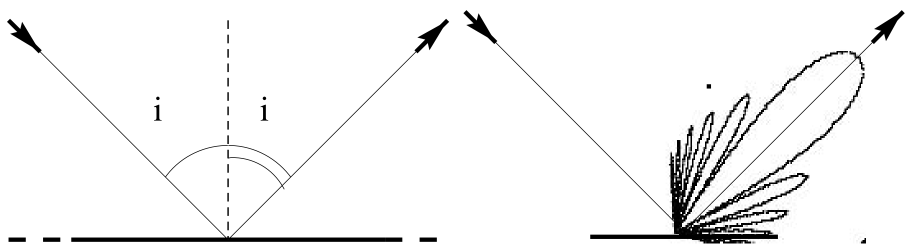

Figure 1.1. *Descartes laws for an unbounded plane (left) with the same original conditions, reflection on a plane sector (right)*

Note that this approach only covers the kinematic aspects. To go into finer details and include the dynamic aspects, wave polarization will need to be considered (as seen in section 1.3) which may require adding a 180° phase rotation in some cases.

1.2.1.2. *Rayleigh scatterer*

This case, based on a target much smaller than wavelength λ, is the opposite of the previous one. By considering a spherical homogenous target with an electric permittivity *e*, we can resort to either exact calculations using the Mie model that makes no assumption as to sphere size, or we can choose approximate calculations (the Rayleigh model in which a sphere much smaller than the wavelength is implied).

The behavior of the Rayleigh model can be deduced from equation [1.8]. Indeed, if the target is homogenous and small enough compared to the wavelength, the source term varies little inside the target. Assuming an incident plane wave $\vec{\mathbf{E}}_i$ it can be written:

$$\left(\epsilon_r - 1\right)\frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{E}}_r}{\partial t^2} \sim \left(\epsilon_r - 1\right)\frac{1}{\lambda^2}\vec{\mathbf{E}}_i,$$

where the proportionality factor involves *V*, i.e. the target volume.

In this way, here we have a secondary source that radiates like a dipole, proportionally to the frequency square, to local permittivity variation inside the target, and to target volume.

1.2.2. *Phenomenological backscattering models*

While the physical models described above provide a better understanding of how electromagnetic waves propagate, they do not make it possible to cover situations found in radar imaging. A pragmatic approach will lead us to consider three more realistic cases that will turn out to be very important for imaging: the rough interface, the case of a generic target and the scattering by a set of (point or not) targets.

1.2.2.1. *Rough interface*

The Snell-Descartes laws assume the interface to be flat. Such an assumption can be called into question in radar imaging as deviations from planarity must not exceed a wavelength fraction (typically $\frac{\lambda}{20}$): for example, a roughcast wall may no longer be considered, for some radar wavelengths, to be a smooth surface.

An interface is said to be rough for an incident ray impinging it at an angle θ, if the mean quadratic deviation of surface irregularities, Δh, verifies the Rayleigh quality criterion:

$$\Delta h > \frac{\lambda}{8\cos(\theta)} \qquad [1.12]$$

meaning a mean quadratic phase shift higher than $\frac{\pi}{2}$.

The higher the roughness, the more the backscattering diagram differs from that of a flat interface. Moreover, it depends on the angle of incidence θ, in particular, the wider the angle of incidence, the more significant the roughness, and the more perturbed the radiation diagram.

The limit case is a surface whose roughness effects completely offset the flat interface appearance. Such a surface will then scatter incident radiation isotropically in a half-plane. The backscattering will in this case be characterized by the albedo, which represents the energetic fraction of the received signal backscattered by this surface.

1.2.2.2. *Scattering by a generic target*

A target that does not satisfy the Rayleigh criterion can still be characterized by using its directivity pattern and by its *Radar Cross-Section* (RCS). In order to define RCS, we will consider that the target behaves at reception like an antenna having an area *a* and as if the entire intercepted power was backscattered isotropically (unit gain antenna); the value of *a* is RCS[3].

The major drawback of this model lies in the fact that RCS is often strongly dependent on the configuration under which the target is illuminated by the incident wave. Even a minor change in this configuration may cause a major change in σ.

1.2.2.3. *Scattering by a set of targets*

Let us consider a set of Rayleigh point targets (they can be seen as isotropic targets). These targets may be distributed on a plane (we then refer to their area density) or in a volume (we then refer to their volume density).

The backscattered wave is the sum of basic waves backscattered by every target. Assuming that target density is not too high, we will be able to omit multi-reflection,

3 This concept deserves a more elaborate definition, especially one that includes polarimetry (see, for example, [LEC 89]).

in which backscattered waves are in turn backscattered by other targets. This often justified assumption verifies the Born approximation hypotheses.

Generally, the emitted radar wave train is far longer than the wavelength: we thus speak of a coherent illumination. In this case, the sum of echoes backscattered by each target will be carried out coherently, i.e. amplitudes are summed up rather than energies[4]. The received signal therefore has a specific appearance induced by speckle, generally well known by opticians. This issue, which is a major one for radar image processing, will be discussed in more depth in Chapter 5.

1.3. Polarization

1.3.1. *Definitions*

When a plane divides a space into two semi-unbounded, isotropic, homogenous media, the incidence plane of an electromagnetic wave characterized by its wave vector $\vec{\mathbf{k}}$ can be defined as the plane containing both $\vec{\mathbf{k}}$ and the normal to the boundary plane dividing the two media.

The polarization of an electromagnetic wave is conventionally defined by the direction of a field $\vec{\mathbf{E}}$: we say that the polarization is perpendicular if the field $\vec{\mathbf{E}}$ is perpendicular to the plane of incidence (TE polarization, $\vec{\mathbf{E}}_\perp$), and that the polarization is parallel if the field $\vec{\mathbf{E}}$ belongs to the plane of incidence (TM polarization, $\vec{\mathbf{E}}_{||}$).

Starting from the Descartes laws and energy conservation, we can calculate the transmission coefficient t and reflection coefficient r for the flat interface. The fields $\vec{\mathbf{E}}_r$ and $\vec{\mathbf{E}}_t$ are related to the incident field $\vec{\mathbf{E}}_i$ by the following relations:

$$\begin{pmatrix} E_{r,\perp} \\ E_{r,||} \end{pmatrix} = \left(\frac{e^{i\vec{\mathbf{k}}\vec{\mathbf{d}}}}{|\vec{\mathbf{d}}|} \right) \begin{pmatrix} r_\perp & 0 \\ 0 & r_{||} \end{pmatrix} \begin{pmatrix} E_{i,\perp} \\ E_{i,||} \end{pmatrix}$$

$$\begin{pmatrix} E_{t,\perp} \\ E_{t,||} \end{pmatrix} = \left(\frac{e^{i\vec{\mathbf{k}}\vec{\mathbf{d}}}}{|\vec{\mathbf{d}}|} \right) \begin{pmatrix} t_\perp & 0 \\ 0 & t_{||} \end{pmatrix} \begin{pmatrix} E_{i,\perp} \\ E_{i,||} \end{pmatrix}$$

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4 We have a very different situation where optical wavelengths are concerned, since on the one hand photons differ in frequency and, on the other hand, have very short coherence lengths. To most receivers, they will appear incoherent, which makes it possible to sum up their intensity contributions resulting in the speckle-free images we are familiar with.

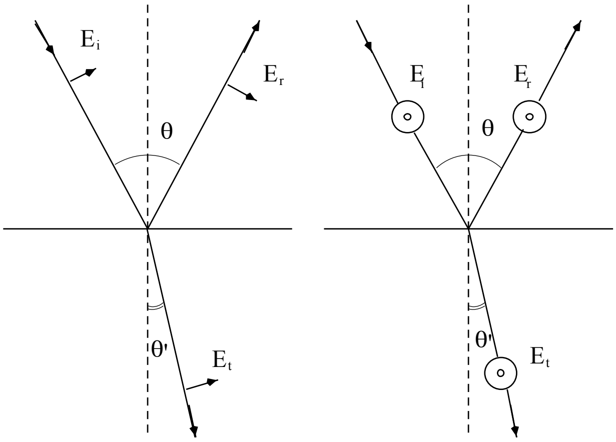

Figure 1.2. *Fresnel laws for parallel polarization, i.e. where $\vec{\mathbf{E}}_i$ is parallel to the incidence plane (left), and for perpendicular polarization, i.e. where $\vec{\mathbf{E}}_i$ is perpendicular to the incidence plane (right)*

where $\vec{\mathbf{d}}$ is the observer's position and [FRA 70]:

$$r_{\|} = -\frac{\tan(\theta-\theta')}{\tan(\theta+\theta')}, \qquad t_{\|} = \frac{2\cos(\theta)\sin(\theta')}{\sin(\theta+\theta')\cos(\theta+\theta')}$$

$$r_{\perp} = -\frac{\sin(\theta-\theta')}{\sin(\theta+\theta')}, \qquad t_{\perp} = \frac{2\cos(\theta)\sin(\theta')}{\sin(\theta+\theta')}.$$

These relations highlight the different behaviors of parallel and perpendicular polarizations. In particular, for $\theta+\theta' = \frac{\pi}{2}$ the parallel polarized wave is no longer reflected; θ is known as the Brewster angle in this case.

In the general backscattering case, the components of the backscattered field $\vec{\mathbf{E}}_r$ are linearly related to the incident field $\vec{\mathbf{E}}_i$ components. This is usually written in matrix form as follows:

$$\begin{pmatrix} E_{r,\perp} \\ E_{r,\|} \end{pmatrix} = \left(\frac{e^{i\vec{\mathbf{k}}\vec{\mathbf{d}}}}{|\vec{\mathbf{d}}|} \right) \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} E_{i,\perp} \\ E_{i,\|} \end{pmatrix} \qquad [1.13]$$

This formulation can be used to describe both the reflection by a plane and the scattering by a target, even though in the latter case parallel and perpendicular polarizations are entirely arbitrary notions.

1.3.2. *Wave polarization*

The polarization of a plane wave describes, versus time, the tip location of an electric field vector $\vec{\mathbf{E}}(t)$ in a plane orthogonal to $\vec{\mathbf{k}}$. Generally, this location is an ellipse (the wave is said *elliptically polarized*), which in some cases may degenerate into a straight line segment (linear polarization) or a circle (circular polarization). An elliptically polarized wave is shown in Figure 1.3 [ULA 90].

For an observer, the ellipse orientation angle ψ is the angle between the horizontal and the major axis of the ellipse describing the polarized wave. It ranges between 0° and 180°. χ is the ellipticity angle, such that, by definition the tangent is the ratio of the ellipse's minor and major axes. It ranges between –45° and +45°, and its sign conventionally determines the direction of polarization: right if $\chi < 0$ or left if $\chi > 0$. Note that opticians refer to polarization as being positive when an observer looking at the wave that propagates towards him sees the ellipse described in the direct sense, i.e., to the left.

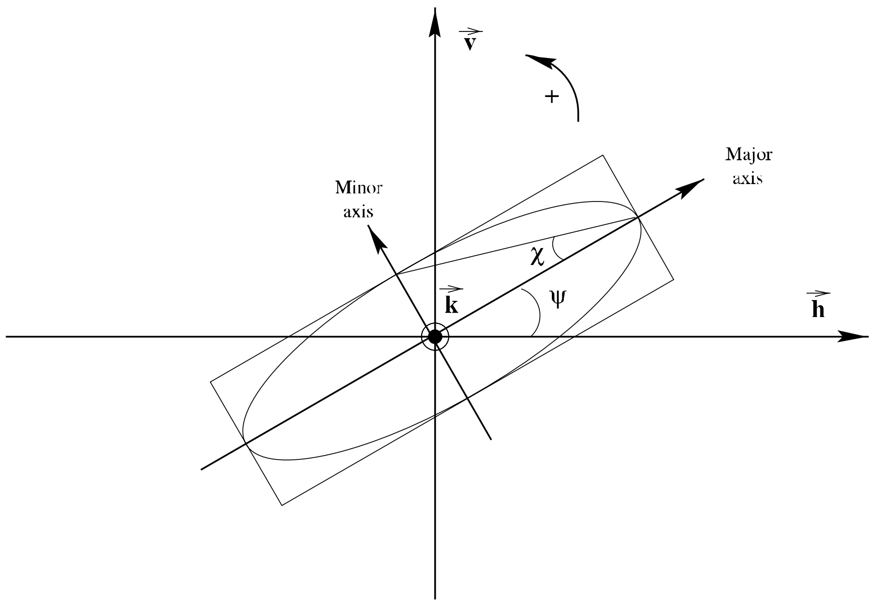

Figure 1.3. *Polarization of a wave: conventions and notations*

The polarization of a wave is then defined using the couple (ψ, χ) deduced from the variations of the E_h and E_v components of field $\vec{\mathbf{E}}(t)$ along the axes h and v. These axes are defined in the plane orthogonal to $\vec{\mathbf{k}}$ and are conventionally related to the observer's reference frame (rather than according to its belonging to an incidence plane related to an interface, as in the previous section):

$$
\begin{aligned}
E_h(r,t) &= |E_h|.\cos\left(\vec{\mathbf{k}}\vec{\mathbf{r}} - \omega.t + \delta_h\right), \\
E_v(r,t) &= |E_v|.\cos\left(\vec{\mathbf{k}}\vec{\mathbf{r}} - \omega.t + \delta_v\right),
\end{aligned}
\qquad [1.14]
$$

where δ_v and δ_h are the original phase shifts. They are linked to (ψ, χ) by parameters $\Delta\delta = \delta_v - \delta_h$ and $\tan(\zeta) = |\boldsymbol{E}_v|/|\boldsymbol{E}_h|$, and relationships [BOR 65]:

$$
\begin{aligned}
\tan(2\psi) &= \tan(2\zeta).\cos(\Delta\delta) \\
\sin(2\chi) &= \sin(2\zeta).\sin(\Delta\delta)
\end{aligned}
\qquad [1.15]
$$

In the case of remote sensing radars, the observer's reference frame is related to the Earth and vector $\vec{\mathbf{h}}$ is horizontal. Particular cases are:

– $\chi = 0°$ corresponds to linear polarizations:

$$
\begin{cases}
\psi = 0°\text{: horizontal polarization} \\
\psi = 90°\text{: vertical polarization}
\end{cases}
$$

– $\chi = \pm 45°$ corresponds to circular polarizations:

$$
\begin{cases}
\chi = -45°\text{: right-hand polarization} \\
\chi = +45°\text{: left-hand polarization}
\end{cases}
$$

The polarization of a wave can also be described by using a real Stokes vector $\vec{\mathbf{F}}$, defined as follows:

$$\vec{\mathbf{F}} = \begin{pmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{pmatrix} = \begin{pmatrix} |E_h|^2 + |E_v|^2 \\ |E_h|^2 - |E_v|^2 \\ 2.\mathrm{Re}\left(E_h.E_v^*\right) \\ 2.\mathrm{Im}\left(E_h.E_v^*\right) \end{pmatrix}$$

From equations [1.15], $\vec{\mathbf{F}}$ can be expressed versus the orientation and ellipticity angles (ψ, χ) as

$$\vec{\mathbf{F}} = g_0 \cdot \begin{pmatrix} 1 \\ \cos(2\chi).\cos(2\psi) \\ \cos(2\chi).\sin(2\psi) \\ \sin(2\chi) \end{pmatrix} \qquad [1.16]$$

The first component $g_0 = |E_h|^2 + |E_v|^2$ is the total power carried by the wave. When the wave is fully polarized, i.e. the parameters $|E_h|$, $|E_v|$, δ_h and δ_v are constant over time, the wave checks the equality: $g_0^2 = g_1^2 + g_2^2 + g_3^2$ (derived from equation [1.16]). This is generally the case for the transmitted wave. Conversely, a backscattered wave is the coherent sum of waves backscattered by elementary targets (that are assumed randomly distributed) in a resolution cell, and it is represented by a random time variable. It verifies the inequality: $g_0^2 \geq g_1^2 + g_2^2 + g_3^2$, where g_i are time averages; the wave is then said to be partially polarized. The polarization degree of a wave, defined as $\sqrt{g_1^2 + g_2^2 + g_3^2} / g_0$, is therefore 1 for a completely polarized wave, less than 1 for a partially polarized wave and 0 for a completely depolarized wave.

1.3.3. *The BSA convention*

When an electromagnetic wave is scattered by a target, the fields are expressed in local coordinate systems related to the transmitting antenna ($\overrightarrow{\mathbf{h_e}}, \overrightarrow{\mathbf{v_e}}, \overrightarrow{\mathbf{k_e}}$) and the receiving antenna ($\overrightarrow{\mathbf{h_r}}, \overrightarrow{\mathbf{v_r}}, \overrightarrow{\mathbf{k_r}}$), while the global system is that of the observed target, as shown in Figure 1.4. In the monostatic case, i.e. when the transmission and reception locations are the same, the variables ($\overrightarrow{\mathbf{h_e}}, \overrightarrow{\mathbf{v_e}}, \overrightarrow{\mathbf{k_e}}$) and ($\overrightarrow{\mathbf{h_r}}, \overrightarrow{\mathbf{v_r}}, \overrightarrow{\mathbf{k_r}}$) coincide according to the backscattering alignment (BSA) convention.

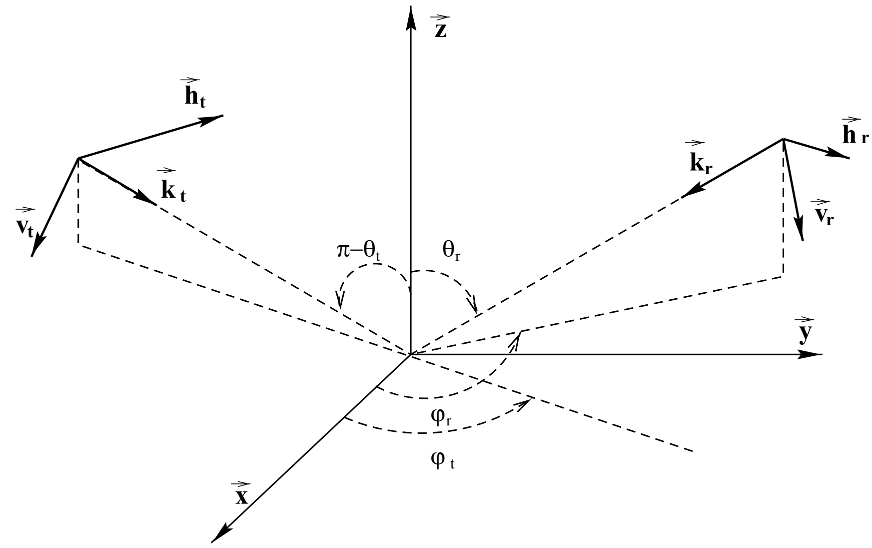

Figure 1.4. *Local coordinate systems and geometry of the BSA convention, describing the incident wave and target scattered wave*

1.3.4. *Complex backscattering matrix S, Mueller matrix*

In the case of backscattering by a target, the reflected wave and the incident wave are related to each other by the scattering matrix **S** (equation [1.13]):

$$\begin{pmatrix} E_{r,h} \\ E_{r,v} \end{pmatrix} = \frac{e^{i\vec{\mathbf{h}}.\vec{\mathbf{d}}}}{|\vec{\mathbf{d}}|} \begin{pmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{pmatrix} . \begin{pmatrix} E_{i,h} \\ E_{i,v} \end{pmatrix} \Leftrightarrow \vec{\mathbf{E}}_r = \frac{e^{i\vec{\mathbf{h}}.\vec{\mathbf{d}}}}{|\vec{\mathbf{d}}|} \mathbf{S}.\vec{\mathbf{E}}_i \qquad [1.17]$$

where $\vec{\mathbf{d}}$ defines the observer's location.

The elements S_{ij} of matrix **S** depend on the target's characteristics, particularly on the geometric (roughness) and dielectric (moisture) features, but also on acquisition characteristics, in particular wave frequency, incidence, etc. In addition, the reciprocity principle [TSA 85] implies that $S_{hv} = S_{vh}$ (rigorously this is true only when the polarized waves H and V are transmitted simultaneously, which is actually not the case in radars alternating V and H transmissions. However, even in the latter case, data are calibrated in order to verify the relationship $S_{hv} = S_{vh}$).

In the following, we represent the complex backscattering matrix either using matrix form **S** or vector form $\vec{\mathbf{S}}$:

$$\vec{\mathbf{S}} = \begin{pmatrix} S_{hh} \\ S_{hv} \\ S_{vh} \\ S_{vv} \end{pmatrix} \qquad [1.18]$$

For calibrated data, $\vec{\mathbf{S}}$ reduces to three components:

$$\vec{\mathbf{S}} = \begin{pmatrix} S_{hh} \\ S_{hv} \\ S_{vv} \end{pmatrix}$$

In numerous applications, our interest focuses on distributed or spread targets and their average properties, rather than point targets. This is the case for studies on farming crops, sea currents and iceberg drifting. For such studies, we would rather not use **S**, but one of the two matrices given below:

– the complex Hermitian covariance matrix **C** (monostatic case):

$$\mathbf{C} = \vec{\mathbf{S}}^{*t}.\vec{\mathbf{S}} = \begin{pmatrix} \left|S_{hh}\right|^2 & S_{hh}.S_{hv}^* & S_{hh}.S_{vh}^* & S_{hh}.S_{vv}^* \\ S_{hv}.S_{hh}^* & \left|S_{hv}\right|^2 & S_{hv}.S_{vh}^* & S_{hv}.S_{vv}^* \\ S_{vh}.S_{hh}^* & S_{vh}.S_{hv}^* & \left|S_{vh}\right|^2 & S_{vh}.S_{vv}^* \\ S_{vv}.S_{hh}^* & S_{vv}.S_{hv}^* & S_{vv}.S_{vh}^* & \left|S_{vv}\right|^2 \end{pmatrix} \qquad [1.19]$$

Assuming reciprocity, **C** reduces to a 3 x 3 matrix:

$$\mathbf{C} = \begin{pmatrix} |S_{hh}|^2 & S_{hh}.S_{hv}^* & S_{hh}.S_{vv}^* \\ S_{hv}.S_{hh}^* & |S_{hv}|^2 & S_{hv}.S_{vv}^* \\ S_{vv}.S_{hh}^* & S_{vv}.S_{hv}^* & |S_{vv}|^2 \end{pmatrix}$$

– the Stokes or Mueller matrix **M**.

The Mueller (or Stokes) matrix has been defined such that polarimetric synthesis might be expressed using either fields $\vec{\mathbf{E}}$ or Stokes vectors $\vec{\mathbf{F}}$:

$$P \equiv \left|\vec{\mathbf{E}}_{\mathbf{r}}.\mathbf{S}.\vec{\mathbf{E}}_{\mathbf{i}}\right|^2 = \vec{\mathbf{F}}_{\mathbf{r}}.\mathbf{M}.\vec{\mathbf{F}}_{\mathbf{i}}. \qquad [1.20]$$

Thus, by analogy with equation [1.17], **M** is defined as the matrix connecting Stokes vectors with incident and reflected waves:

$$\vec{\mathbf{F}}_{\mathbf{r}} = \mathbf{R}.\mathbf{R}^{\mathbf{t}}.\mathbf{M}.\vec{\mathbf{F}}_{\mathbf{i}} \qquad [1.21]$$

where $\vec{\mathbf{F}}_{\mathbf{i}}$ is the transmitted (or incident) Stokes vector, $\vec{\mathbf{F}}_{\mathbf{r}}$ is the received (or scattered) Stokes vector, and:

$$\mathbf{R} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -i & i \end{pmatrix}.$$

M is a real 4×4 square matrix. In the monostatic case, it is symmetric (according to the reciprocity principle) and related to **S** by [ULA 90]:

$$
\begin{aligned}
M_{11} &= \tfrac{1}{4}\left(\left|S_{hh}\right|^2 + \left|S_{vv}\right|^2 + 2.\left|S_{hv}\right|^2\right)\\
M_{12} &= \tfrac{1}{4}\left(\left|S_{hh}\right|^2 - \left|S_{vv}\right|^2\right)\\
M_{13} &= \tfrac{1}{2}\operatorname{Re}\left(S_{hh}.S_{hv}^*\right) + \tfrac{1}{2}\operatorname{Re}\left(S_{hv}.S_{vv}^*\right)\\
M_{14} &= \tfrac{1}{2}\operatorname{Im}\left(S_{hh}.S_{hv}^*\right) + \tfrac{1}{2}\operatorname{Im}\left(S_{hv}.S_{vv}^*\right)\\
M_{22} &= \tfrac{1}{4}\left(\left|S_{hh}\right|^2 + \left|S_{vv}\right|^2 - 2.\left|S_{hv}\right|^2\right)\\
M_{23} &= \tfrac{1}{2}\operatorname{Re}\left(S_{hh}.S_{hv}^*\right) - \tfrac{1}{2}\operatorname{Re}\left(S_{hv}.S_{vv}^*\right)\\
M_{24} &= \tfrac{1}{2}\operatorname{Im}\left(S_{hh}.S_{hv}^*\right) - \tfrac{1}{2}\operatorname{Im}\left(S_{hv}.S_{vv}^*\right)\\
M_{33} &= \tfrac{1}{2}\left|S_{hv}\right|^2 + \tfrac{1}{2}\operatorname{Re}\left(S_{hh}.S_{vv}^*\right)\\
M_{34} &= \tfrac{1}{2}\operatorname{Im}\left(S_{hh}.S_{vv}^*\right)\\
M_{44} &= \tfrac{1}{2}\left|S_{hv}\right|^2 - \tfrac{1}{2}\operatorname{Re}\left(S_{hh}.S_{vv}^*\right)
\end{aligned}
\tag{1.22}
$$

1.3.4.1. *Properties of* **M**

In the case of point targets and a monostatic radar, five relationships exist connecting **M** terms, namely [VAN 87]:

$$
\begin{aligned}
&M_{11} = M_{22} + M_{33} + M_{44}\\
&M_{13}.M_{23} + M_{14}.M_{24} = M_{11}.M_{12} - M_{12}.M_{22}\\
&M_{13}.M_{14} - M_{23}.M_{24} = M_{33}.M_{34} - M_{34}.M_{44}\\
&M_{13}^2 + M_{23}^2 - M_{14}^2 + M_{24}^2 = M_{11}^2 - M_{22}^2\\
&M_{13}^2 - M_{23}^2 - M_{14}^2 + M_{24}^2 = M_{33}^2 - M_{44}^2
\end{aligned}
\tag{1.23}
$$

These relationships are a necessary and sufficient condition for a given Mueller matrix to have a single backscattering matrix associated with it. Therefore, a Mueller matrix only corresponds to an actual “physical” target when [1.23] is verified. Now, the mean Stokes parameters of the waves backscattered by an object varying in either time or space are related to the Stokes parameters of the transmitted wave by an average Mueller matrix E[**M**]. However, as relations [1.23] are generally lost by averaging the **M** matrices, there is no complex backscattered matrix corresponding to E[**M**] and only the first relation out of five in [1.23] is verified.