# Contents

**Chapter 1. Introduction** . . . . . 1

1.1. General introduction . . . . . 1
1.2. History and three mainstays . . . . . 2
1.3. Scientific context . . . . . 2
1.3.1. Dioids . . . . . 3
1.3.2. Petri nets . . . . . 4
1.3.3. Time and algebraic models . . . . . 5
1.4. Organization of the book . . . . . 7

**Chapter 2. Consistency** . . . . . 9

2.1. Introduction . . . . . 9
2.1.1. Models . . . . . 9
2.1.2. Physical point of view . . . . . 11
2.1.3. Objectives . . . . . 12
2.2. Preliminaries . . . . . 14
2.3. Models and principle of the approach . . . . . 17
2.3.1. P-time event graphs . . . . . 17
2.3.2. Dater form . . . . . 21
2.3.3. Principle of the approach (example 2) . . . 23

2.4. Analysis in the “static” case . . . . . 25
2.5. “Dynamic” model . . . . . 28
2.6. Extremal acceptable trajectories by series of matrices . . . . . 31
2.6.1. Lowest state trajectory . . . . . 32
2.6.2. Greatest state trajectory . . . . . 35
2.7. Consistency . . . . . 36
2.7.1. Example 3 . . . . . 41
2.7.2. Maximal horizon of temporal consistency . . . . . 44
2.7.3. Date of the first token deaths . . . . . 47
2.7.4. Computational complexity . . . . . 48
2.8. Conclusion . . . . . 50

**Chapter 3. Cycle Time** . . . . . 53

3.1. Objectives . . . . . 53
3.2. Problem without optimization . . . . . 55
3.2.1. Objective . . . . . 55
3.2.2. Matrix expression of a P-time event graph . . . . . 56
3.2.3. Matrix expression of P-time event graphs with interdependent residence durations . . . . . 57
3.2.4. General form $\boldsymbol{Ax} \leq \boldsymbol{b}$ . . . . . 59
3.2.5. Example . . . . . 60
3.2.6. Existence of a 1-periodic behavior . . . . . 61
3.2.7. Example continued . . . . . 65
3.3. Optimization . . . . . 67
3.3.1. Approach 1 . . . . . 67
3.3.2. Example continued . . . . . 69
3.3.3. Approach 2 . . . . . 70
3.4. Conclusion . . . . . 75
3.5. Appendix . . . . . 76

**Chapter 4. Control with Specifications** . . . . . 79

4.1. Introduction . . . . . 79
4.2. Time interval systems . . . . . 80
4.2.1. (min, max, +) algebraic models . . . . . 81
4.2.2. Timed event graphs . . . . . 82
4.2.3. P-time event graphs . . . . . 83
4.2.4. Time stream event graphs . . . . . 84
4.3. Control synthesis . . . . . 88
4.3.1. Problem . . . . . 88
4.3.2. Pedagogical example: education system . . . . . 89
4.3.3. Algebraic models . . . . . 91
4.4. Fixed-point approach . . . . . 92
4.4.1. Fixed-point formulation . . . . . 92
4.4.2. Existence . . . . . 95
4.4.3. Structure . . . . . 103
4.5. Algorithm . . . . . 107
4.6. Example . . . . . 111
4.6.1. Models . . . . . 111
4.6.2. Fixed-point formulation . . . . . 113
4.6.3. Existence . . . . . 114
4.6.4. Optimal control with specifications . . . . . 116
4.6.5. Initial conditions . . . . . 117
4.7. Conclusion . . . . . 118

**Chapter 5. Online Aspect of Predictive Control** . . . . . 119

5.1. Introduction . . . . . 119
5.1.1. Problem . . . . . 119
5.1.2. Specific characteristics . . . . . 120
5.2. Control without desired output (problem 1) . . . 122
5.2.1. Objective . . . . . 122
5.2.2. Example 1 . . . . . 123
5.2.3. Trajectory description . . . . . 124
5.2.4. Relaxed system . . . . . 125

5.3. Control with desired output (problem 2) . . . . . 127
5.3.1. Objective . . . . . 127
5.3.2. Fixed-point form . . . . . 128
5.3.3. Relaxed system . . . . . 129
5.4. Control on a sliding horizon (problem 3): online and offline aspects . . . . . 130
5.4.1. CPU time of the online control . . . . . 131
5.5. Kleene star of the block tri-diagonal matrix and formal expressions of the sub-matrices . . . 132
5.6. Conclusion . . . . . 138

**Bibliography** . . . . . 141

**List of Symbols** . . . . . 149

**Index** . . . . . 153