

Table of Contents

Preface	ix
PART 1. MATHEMATICAL PRELIMINARIES, DEFINITIONS AND PROPERTIES OF FRACTIONAL INTEGRALS AND DERIVATIVES	1
Chapter 1. Mathematical Preliminaries	3
1.1. Notation and definitions	3
1.2. Laplace transform of a function	6
1.3. Spaces of distributions	7
1.4. Fundamental solution	14
1.5. Some special functions	15
Chapter 2. Basic Definitions and Properties of Fractional Integrals and Derivatives	17
2.1. Definitions of fractional integrals and derivatives	17
2.1.1. Riemann–Liouville fractional integrals and derivatives	17
2.1.2. Riemann–Liouville fractional integrals and derivatives on the real half-axis	29
2.1.3. Caputo fractional derivatives	30
2.1.4. Riesz potentials and Riesz derivatives	33
2.1.5. Symmetrized Caputo derivative	36
2.1.6. Other types of fractional derivatives	37
2.2. Some additional properties of fractional derivatives	38
2.2.1. Fermat theorem for fractional derivative	38
2.2.2. Taylor theorem for fractional derivatives	39
2.3. Fractional derivatives in distributional setting	41
2.3.1. Definition of the fractional integral and derivative	41
2.3.2. Dependence of fractional derivative on order	43
2.3.3. Distributed-order fractional derivative	44

PART 2. MECHANICAL SYSTEMS	49
Chapter 3. Restrictions Following from the Thermodynamics for Fractional Derivative Models of a Viscoelastic Body	51
3.1. Method based on the Fourier transform	51
3.1.1. Linear fractional model	52
3.1.2. Distributed-order fractional model	57
3.1.3. Constitutive equations for rod bending	63
3.1.4. Stress relaxation and creep for two special cases of viscoelastic bodies	64
3.1.5. Variable-order fractional derivative: application to stress relaxation problem	70
3.1.6. Linear constitutive equation with fractional derivatives of complex order	71
3.2. Thermodynamical restrictions via the internal variable theory	75
3.2.1. Case I	78
3.2.2. Case II	81
Chapter 4. Vibrations with Fractional Dissipation	83
4.1. Linear vibrations with fractional dissipation	83
4.1.1. Linear vibrations with the single fractional dissipation term	83
4.1.2. Fractional derivative-type creeping motion	86
4.1.3. Linear vibrations with the multiterm fractional dissipation	87
4.1.4. Linear fractional two-compartmental model with fractional derivatives of different order	92
4.2. Bagley–Torvik equation	94
4.2.1. Solution procedure	96
4.2.2. Numerical examples	99
4.3. Nonlinear vibrations with symmetrized fractional dissipation	100
4.3.1. Solvability and dissipativity of [4.58]	103
4.3.2. Stability of the solution	107
4.4. Nonlinear vibrations with distributed-order fractional dissipation	108
4.4.1. Existence of solutions	109
4.4.2. Uniqueness of solutions	117
4.4.3. Nonlinear vibrations with single term of fractional dissipation	121
Chapter 5. Lateral Vibrations and Stability of Viscoelastic Rods	123
5.1. Lateral vibrations and creep of a fractional type viscoelastic rod	126
5.1.1. Rod made of fractional Kelvin–Voigt-type material	128
5.1.2. Rod made of fractional Zener-type material	138
5.1.3. Viscoelastic rod with two different fractional derivatives of strain	149
5.2. Stability of Beck’s column on viscoelastic foundation	162

5.2.1. Solution to systems [5.130]–[5.133]	165
5.2.2. Properties of functions T and V	169
5.3. Compressible elastic rod on a viscoelastic foundation	172
5.3.1. Zeros of D_k	176
5.3.2. Existence of T_k and V_k	178
5.3.3. Asymptotic behavior of T_k	182
5.3.4. Summary of the stability analysis	183
Chapter 6. Fractional Diffusion-Wave Equations	185
6.1. Nonlinear fractional diffusion-wave equation and fractional Burgers/Korteweg-de Vries equation	189
6.1.1. Nonlinear fractional diffusion-wave equation	190
6.1.2. Fractional Burgers/Korteweg–de Vries equation	192
6.1.3. Exact solutions of the nonlinear fractional diffusion-wave equation	194
6.1.4. Numerical solutions to [6.19] and [6.27]	197
6.2. Fractional telegraph equation	201
6.2.1. Dirichlet problem	202
6.2.2. Signaling problem	206
6.2.3. Cauchy problem	208
6.2.4. Numerical results	209
6.3. Distributed-order diffusion-wave equation	213
6.3.1. Existence of a solution to Cauchy problems ([6.81] and [6.82])	215
6.3.2. Solution to the Cauchy problem	230
6.4. Maximum principle for fractional diffusion-wave type equations	249
6.4.1. Maximum principle for fractional telegraph equation	249
6.4.2. Maximum principle for distributed-order diffusion equation	252
Chapter 7. Fractional Heat Conduction Equations	257
7.1. Cattaneo-type space–time fractional heat conduction equation	260
7.1.1. Existence and uniqueness of a solution	262
7.1.2. Explicit form of the solution	268
7.1.3. Numerical examples	275
7.2. Fractional Jeffreys-type heat conduction equation	279
7.2.1. Solution to the Cauchy problem	280
7.2.2. Numerical examples	285
Bibliography	289
Index	311