Foreword

My colleague Maurice Lemaire has invited me to write a few words as a foreword to this treatise on structural reliability. The importance of the subject is obvious to everybody. Undertaking a scientific study of it requires a superior mastery of two disciplines, namely mechanics and probabilities. Maurice Lemaire was inspired by a remark made by the authors of the report that the President of the Republic Giscard d’Estaing had requested in 1980 from the Académie des Sciences, titled ‘*Les sciences mécaniques et l’avenir industriel de la France*’ (‘Mechanical sciences and the industrial future of France’). The report in fact observed that it was quite rare to find in French laboratories a team in which skills in these two disciplines mixed. This observation caught Lemaire’s attention and motivated him to orient his research and that of his students in order to remedy the situation. This book testifies to the undeniable success of the decision made by Maurice Lemaire. I thank him for this opportunity to convey my heartfelt congratulations, and express my satisfaction in seeing that young researchers have found in this report an inspiration that has proved to be so fruitful.

The preface and the introduction present the content of this book as is customary, and also indicate its place in the history of mechanics. The reader can also discover how the book has been developed by its author, who has made good use of his lectures enriched by his personal reflections, tested his developments with his students and discussed his progress with colleagues abroad in international meetings and thus earned a well-merited reputation. That is why I believe that this book is a treatise, which I think will be a reference in this discipline for a long time to come.

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Chapter 1

Introduction

1.1 An old history

It would be fascinating to take some time to go back in history in order to understand how man gradually conquered enough 'certainties' to accept rationally the risk of his uncertainties. We will find that great personalities have reflected on this question and contributed to the gradual acquisition of more comprehensive heuristic and axiomatic information, which has made it possible to design more and more ambitious structures and systems.

It is certainly Hammurabi's code that first established rules governing the acceptance of risk in construction.[1] Around 1755 B.C., this Babylonian sovereign put together a set of prescriptions, dictated by the gods, constituting the first legal code ever known. It remained in force in Mesopotamia for a thousand years. The code related to the construction of houses, and the mason's responsibility was strongly binding.[2] Let us judge for ourselves:

> *Article 229:* If a mason has constructed a house for someone but has not strengthened his construction, and if the house that he has constructed collapses and kills the house owner, that mason shall be put to death.
>
> *Article 230:* If it is the child of the house owner that has been killed, one of the mason's children shall be put to death.

It is interesting to note that the insistence on safety was then based on the transfer onto the builder of a risk that related to his own security: linking the notion of risk to the outcome of the feared event remains quite a contemporary mindset.

In fact, risk is defined by the existence of a feared event that has a probability (or a possibility) of occurrence, and by the gravity of the consequences of this event.

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[1] *Les Grands Procès*, under the guidance of N. Laneyrie-Dagen, Larousse, Paris, 1995.

[2] *Le Code d'Hammourabi*, A. Finet, Editions du Cerf, Paris, 1983.

The following equation is often given:

$$\boxed{\text{Risk} = \text{Probability} \times \text{Gravity.}}$$

In order to diminish the probability of an event feared by the user, the penalty should be increased for the person who takes the responsibility for the construction. This is a direct application of the Farmer graph illustrated in Figure 1.1, according to which the mason will try to reduce the probability of the occurrence of a feared event if its consequences are disastrous for him.[3]

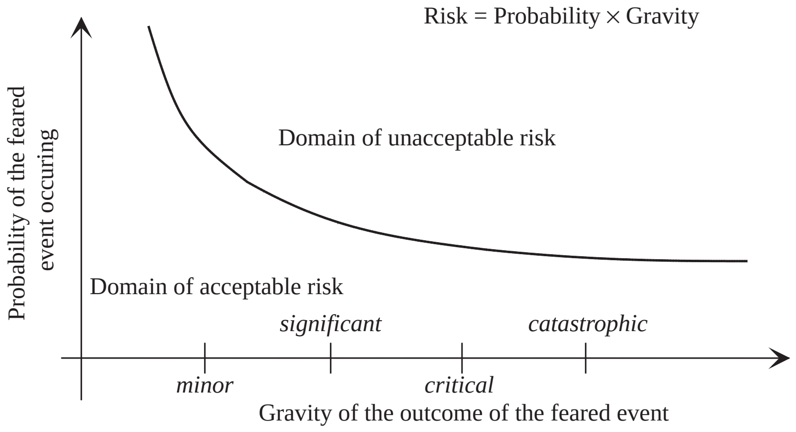

Figure 1.1 *Farmer graph (1967): probability-gravity.*

Hammurabi imposed responsibility for results and left the choice of the means of achieving them open. He anticipated what the European Union would much later call *directives*. Today this practice seems barbaric, whereas it essentially aimed at limiting the effects of an endless vendetta between the concerned parties due to the application of the law of 'an eye for an eye'.

Humanity's scientific quest therefore consisted of accumulating experience and constructing projected models that give today, not for one man but collectively to engineers, the possibility of taking on the risks of civil and mechanical constructions on behalf of society with real success, in a context in which great catastrophes are a reminder that humility is always necessary.

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[3] *Concepts et méthodes probabilistes de base de la sécurité*, A. Desroches, Editions Lavoisier, Paris, 1995.

1.2 A modern attitude

If the knowledge of geometry and static mechanics advanced rapidly in ancient times, the mastery of the uncertain in the construction of cathedrals in the Middle Ages proceeded by trial and error and led to well-known failures.[4] Leonardo da Vinci (1452–1519) was one of the first to look for a relationship between load effect and resistance in the case of beams. A little later, Galileo was particularly interested in the optimization of the resistance of a cantilever beam, thereby initiating the first modeling.[5]

We know the great developments in modeling the behavior of materials and constructions that followed. This conquest could lead us to believe that one day the knowledge of laws, models and solutions will attain such perfection that engineers will be able to trust them completely. However, in parallel and sometimes simultaneously, scientists explained that we should live with chance.

Should we return to a philosophical debate by wondering whether chance exists, or whether what we call chance is only the fruit of our ignorance and our inability to take into account all the initial conditions of a process? Using the language of fluid mechanics, the Lagrange approach to chance is impossible because it requires monitoring a trajectory, whereas the Euler approach is content to observe the variability at the time and the place where the observer is placed. This debate is certainly futile for us today, and it is no doubt more pragmatic to think that between the present mastery of knowledge and physical reality, there will always be a gap that cannot be modeled with certainty. Moreover, we all know very well that very small disturbances of an initial state can lead to huge potential differences in the consequences, as seen in the phenomena of instability, for example in meteorology or in structural mechanics.

On the other hand, perfectly determined outcomes can be predicted by methods based on the modeling of uncertainty. This is the case for geostatistics, for example, where the content of an ore is considered a unique outcome of a random process.

In view of this inability to master all the data – which he possesses in theory, however, as they are in front of him and he has only to know how to read them – man gambles, and most often he wins.

Blaise Pascal (1623–1662) invites us to reflect on this theme:

> when we work for tomorrow, and do so on an uncertainty, we act reasonably; for we ought to work for an uncertainty according to the doctrine of chance which has been demonstrated.

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[4] Trials on the construction of very high vaults ended at Beauvais with a partial collapse in 1284.

[5] *Discorsi e Dimostrazioni Matematiche, intorno à due nuoue scienze, attenenti alla Mecanica & i Movimenti Locali*, Galiléo Galiléi Lincéo, Leida 1638.

The first part of this proposition is simple. We work for tomorrow using studies that contribute to strengthening scientific progress, to increasing our knowledge of phenomena and to creating models in order to anticipate; we work for the uncertain because all the data will never be at our disposal for sure. Pascal tells us that we do it with reason, and he justifies it by the hope of success. It is therefore important to know what success brings us, and what makes us fear failure. Replying to Chevalier de Méré, Pascal demonstrated how to share – in other words, to give everyone his due – in a game which is subject to risk-taking and hopes of success. The doctrine of chance therefore enables each person to justify his commitment depending on his hopes.

This *pensée* by Pascal regarding a commercial stake can also be applied relevantly to a technical context. In his quest for knowledge, in his desire to want to achieve the most daring of constructions or machines, man takes risks, hoping for progress, for the benefit of humankind. Thus, Pascal made a contribution to decision-making in uncertainty, by showing that chance, whatever its origin, has a geometry: Pascal thereby laid the foundations for the calculation of probabilities.

If everyone accepts the uncertainty of data, the uncertainty of models has only recently been admitted.

One of the most powerful drivers in the advancement of science was the observation of the solar system, as much by what it induced in terms of the capacity for precise measurements as in the capacity for modeling. Until the end of the 19th century, the majority of astronomer–mathematicians were searching for mathematical laws to describe the movement of the planets, particularly in terms of the well-known problem of three bodies in gravitational interaction (sun, earth, moon). According to them, it would have been enough to add to the main components of movement successive corrections brought about by weak perturbations, in order to obtain exact predictions of the ephemerids. The works of Henri Poincaré, on the occasion of the award given in 1889 by the King of Sweden and Norway, Oscar II, led to a conclusion accepted with difficulty: *unpredictability* had its place in deterministic systems.[6] Poincaré wrote in 1903, in his book *Science et méthode*:

> A very small cause, which escapes us, produces a considerable effect that we cannot avoid seeing, and then we say that this effect is due to chance. If we knew the laws of nature exactly and the situation of this very universe at its initial instant, we would be able to predict exactly the situation of the same universe at a later time. However, even though the laws of nature would no longer have any secrets for us, we could know the initial situation only *approximately.* If this allows us to anticipate the latter situation with the *same*

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[6] *Le Chaos dans le système solaire*, Ivars Peterson, Pour la Science, Paris, 1995.

> *approximation*, it is all that we need; we say that the phenomenon was anticipated, that it is governed by laws. But it is not always so; sometimes small differences in the initial conditions generate large ones in the final phenomena: a small error in the former will cause an enormous error in the latter. Prediction becomes impossible and we have a random phenomenon.

This does not challenge the principles of Newtonian mechanics in any way. For this, we had to wait for the theory of relativity, once again in relation to astronomy: an elucidation of Mercury’s trajectory, then quantum mechanics uniting a certain macroscopic determinism with the necessary taking into account of the random of uncertainty in the infinitely small.

Furthermore, the 20th century has also removed certain illusions following Kurt Gödel’s incompleteness theorem (1931), which introduced the *undecidability* of certain propositions. The theorem results in the impossibility of solving certain well-formulated problems algorithmically.

Unpredictability and *undecidability* therefore invite us to assume considerable humility in our hope of analyzing risks – humility that the engineer should have permanently in his mind each time he is confronted with a decision about reliable and economic design.

In risk analysis, reliable design belongs to the engineer, and the gravity of the consequences belongs to the economist and citizen. Serious accidents and lower tolerance toward damages on the part of society, though often paradoxical, have recently given rise to a political deliberation leading to the evolution of an essentially deterministic French approach via the integration of probabilistic concepts into risk analysis. The French law of 30 July 2003 makes the following provision:

> the measures envisaged in plans for the prevention of technological risks . . . are implemented progressively depending particularly on the probability, the gravity and the kinetics of potential accidents as well as the relationship between the cost of the measures envisaged and the expected benefit in terms of safety.[7]

However, O. Gudmestad reminds us that constraints owing to budgets or unrealistic schedules are never taken into account in risk analysis, and suggests the restoration of the role of court jester in a project team, for he is the only person capable of telling the truth![8]

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[7] Law of 30 July 2003 regarding the prevention of technological and natural risks and repair of damages.

[8] ASRaNet Conference, Barcelona, 5 July 2004, ‘Implementation of human factors analysis in QRA & SRA’.

1.3 Reliability: a definition

We still need to be sure that mastery of the uncertain is sufficient for risks to be well evaluated and remain acceptable; otherwise, it would be like playing the sorcerer's apprentice. This is the objective of theories of *reliability*.

A technical definition of this word is given in the text retained by AFNOR:

> the ability of a system to accomplish a required function in given conditions, during a given period ... the term is also used as an attribute describing a probability of success or a percentage of success.[9]

This text demonstrates the importance of a closely associated qualitative definition (ability) and quantitative definition (probability).

It should be noted that such a definition immediately settles a possible debate by associating the analysis of the uncertain with probabilistic modeling. It favors an approach using random variables and stochastic processes, which is not the only choice: the methods and tools of fuzzy logic, convex sets and robustness may also play their role.

Reliability may be considered as an element of a larger whole, constituting the 'ities' RAMS: which can be understood as *reliability* (the subject of this book), as *availability* (the ability of a device or a commodity to accomplish a required purpose at a given moment), as *maintainability* (the ability of a device or a commodity to be maintained or re-established with the intention of accomplishing a required purpose), and finally as *safety* (concerning risks of physical, material and environmental damage related to the system or commodity considered).

1.4 Which risk is acceptable?

As we have just seen, the theory of reliability uses the estimation of probability as a measure. Whether this probability is acceptable or not is a very complex question, and it is clear that a decision involves taking into consideration the quality of the available information, the estimated level of the risk, the consequences of a failure, such as material and human damage, and the duration of exposure to the risk, given that certain risks are imposed on everyone by living conditions and that others are freely accepted. One reasoned argument, two research studies and a few expert opinions will enable us to define the values of the probabilities that we will be searching for.

The reasoning is by Laplace. If an event has been observed n times consecutively, the probability that it will take place $n+1$ times is $n/(n+1)$, and the probability that it will not take place $n+1$ times is therefore $1/(n+1)$.

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[9] AFNOR, NF X50–120, 1988.

Activity	Rate
Plague epidemic in London in 1665	15,000
Training for rock climbing on a rockface	4,000
Firefighter in London during the air raids in 1940	1,000
Travel by helicopter	500
Travel by motorbike or moped	300
Travel by bicycle	60
An average police officer in North Ireland	70
Using tobacco	40
Walking along the road	20
Travel by plane	15
Travel by car	15
Travel by train	5
Accident during construction	1
Travel by bus	1
Accidents at home	1
Acceptable limit when facing an unexpected exceptional event	1
Effects of natural radon	0.1
Terrorist attack in a street in London	0.1
Collapse of a building	0.002

Source: from J. P. Menzies.

Table 1.1 *Risk rate by activity, per hour for* 10^8 *people exposed.*

According to the estimated duration of the universe (around 15 billion years), the probability that the sun will not rise tomorrow is of the order 10^{-12}. It is pointless to be interested in such weak probabilities.[10]

The first study is that of J. P. Menzies, who carried out an *objective* evaluation of death risks in the United Kingdom by type of activity.[11] Table 1.1 represents some values of the fatal accident rate per hour of activity for 10^8 persons exposed. It is possible to draw two conclusions from it. The first is the extremely low risk due to the collapse of buildings in comparison with many other risks of daily life, although the duration of exposure is very long. The second is the definition of an acceptable limit when facing an unexpected exceptional event, that is, the level of risk accepted more or less consciously by everyone, which is $\tau = 10^{-8}/h$.

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[10] Quoted by A. Leroy and J. P. Signoret, *Le Risque technologique*, PUF, Paris, 1992.

[11] Quoted by J. A. Calgaro, *Fiabilité des matériaux et des structures*, Hermes, Paris, 1998.

Activity	Rate τ $(10^8/\text{h})$	Exposure n_e (h/yr)	Rate τ_a $(10^4/\text{yr})$	Ratio
Mountain climbing (international)	2,700	100	27	
Trawl fishing (deep sea, 1958–1972)	59	2,900	17	
Flying (crew)	120	1,000	12	$\ll 1$
Coal mining	21	1,600	3.3	
Automobile travel	56	400	2.2	20
Construction	7.7	2,200	1.7	450
Flying (passengers)	120	100	1.2	$\ll 1$
Home accidents	2.1	5,500	1.1	
Factory work	2	2,000	0.4	
Building fires	0.15	5,500	0.08	5
Structural failure	0.002	5,500	0.001	6

Source: H. O. Madsen *et al.*, *Methods of Structural Safety* (Prentice Hall, 1986).

Table 1.2 *Risk rate by activity.*

Menzies’ study can be compared with Table 1.2, given by H. O. Madsen *et al.*, who also concluded that there is a very low risk due to structural failure, with the same value.[12] This table also introduces hours of exposure per year n_e, the annual rate τ_a and the ratio between the number of injured and the number of dead.

These figures represent different levels of risk well, but do not directly introduce a subjective appreciation of risks. For the population, risk is subjectively different if it is freely accepted or if it is imposed (nobody has to fly a plane or to climb a mountain) and if it allows – or not – a high chance of survival (is the plane any safer than the car?).

The level of risk acceptable to the population is not a totally objective value. It depends on social relations: the level of nuclear risk apparently tolerated in Germany, France and even in Russia is not the same. It also depends on constraints, because no-one can escape the risk associated with consuming food without the certainty of dying of hunger.

Structural reliability seems a trivial issue in this context, but that would be to forget that certain catastrophes are linked not only to an installation, but also to its environment. This is the case, for example, with industrial risks inducing pollution that it is not possible to contain, such as oil slicks and radioactive

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[12] *Methods of Structural Safety*, H. O. Madsen, S. Krenk, N. C. Lind, Prentice Hall, New York, 1986 (source: CIRIA, 1977).

Average number of people placed in danger	Economic consequences		
	Not serious	Serious	Very serious
Small (<0.1)	10^{-3}	10^{-4}	10^{-5}
Average	10^{-4}	10^{-5}	10^{-6}
Large (> 10)	10^{-5}	10^{-6}	10^{-7}

Table 1.3 *Order of magnitude of target probabilities in construction.*

pollution. It would also be forgetting the localization of strong natural risks such as earthquakes and floods.

A probability of structural failure causing a risk of 10^{-7} deaths per year therefore appears a sufficiently strong value to prompt particular attention from designers. However, it is not the only criterion, and the level of reliability also depends on an economic optimization. A lower level is perfectly acceptable if the consequent risks remain well within strict geographical and temporal limits. Table 1.3, often quoted in documents on civil construction, suggests target values according to various situations in construction, yet without clearly specifying the reference period.[13] The table shows the order of magnitude of target probabilities in construction. Hence, very small quantities in relation to one have to be estimated – something that poses specific problems in terms of data and calculations.

Figure 1.2 adopts the principle of the Farmer graph by demarcating zones for which the design is acceptable, and others for which the design is unacceptable.[14] Obviously, designers seek to place themselves in the most favorable economic situation, that is, the closest possible to the danger zone. The figure shows that installations in the North Sea were designed for a much higher risk than those in the rest of the world; the representative point of the Piper Alpha platform, destined for tragedy, is indeed in the zone of unacceptable design.

Figure 1.3 summarizes probabilities of failure estimated by experts in different industrial areas involving all the greatest risks, but with very different lifetimes.[15]

Any decision-making supposes the acceptance of a more or less well-assessed risk. Reliability methods help by contributing to better identification of the component of the risk associated with the occurrence of a feared event.

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[13] Suggested by the Norwegian NKB regulations in 1978.

[14] Suggested by J. P. Signoret, Total company, in his speech during the 'Risks and Reliability of Industrial Systems' course at the SPI Doctoral School of Clermont-Ferrand, 11 June 2001.

[15] Source: EDF.

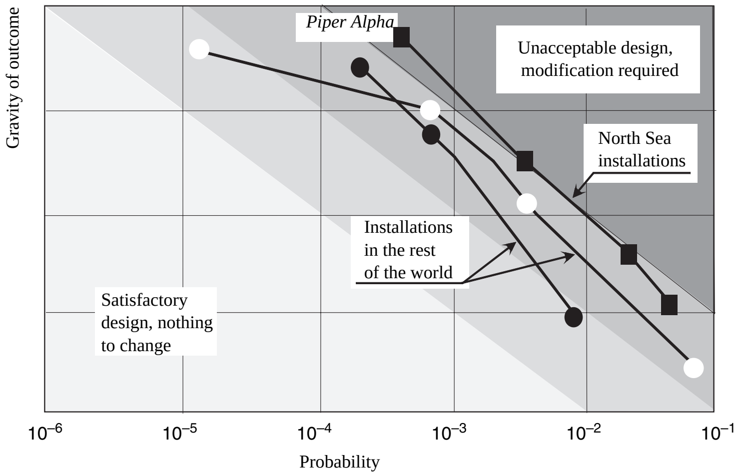

Figure 1.2 *Diagram of gravity of outcome versus probability: positioning of various petroleum research installations (from J. P. Signoret).*

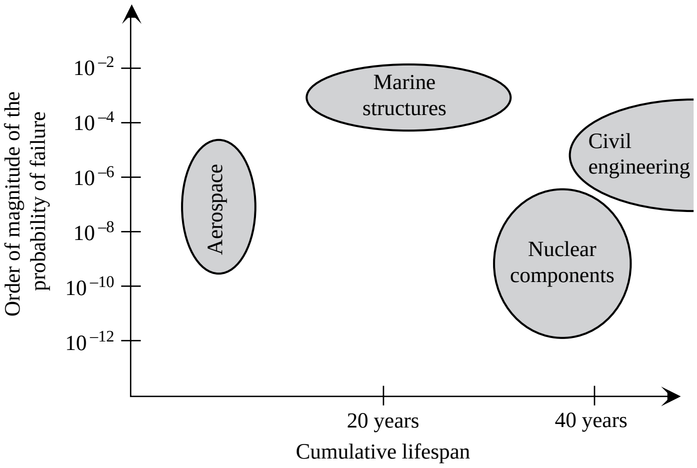

Figure 1.3 *Level of probabilities estimated in different industrial branches.*

1.5 Today

Enriched by a mechanical and probabilistic culture, progressively accepting the uncertain, the 20th century first of all gave rise to pioneers who attempted progressively to rationalize the concepts of reliability in structural mechanics.

In Germany, M. Mayer was certainly one of the first to suggest, in 1926, the use of average values and variances of variables in design.[16] However, it was in 1947 that A. M. Freudenthal initiated, in an article, the present scientific debate:

> place the concept of safety of structures in the realm of physical reality, where there is no absolute and where knowledge is not perfect.[17]

At practically the same time, A. R. Rjanitzyne introduced, in the context of metallic construction, a *characteristic of safety* γ.[18] This is equal to the inverse of the coefficient of variation of a performance function, and it became the first definition of a reliability index. The Soviet school was distinguished by V. V. Bolotin in particular.[19] In Europe, we should mention the research studies of J. Ferry-Borges and M. Castanheta at the Laboratório Nacional de Engenharia Civil in Lisbon, whose summary was outlined in 1971.[20]

In France, we can note, for example, the anticipatory study of R. Lévi published in the *Annales des Ponts et Chaussées*[21] in 1949 and in the *Revue Générale des Chemins de Fer*[22] in 1951.

For over half a century now, ongoing scientific advancement has enabled researchers and engineers to construct design methods that take into account predictable limits and uncertainties. It is of course those fields where these advances were the greatest which have mobilized efforts – for civil engineering first of all, and geotechnics in particular, for natural loads and then for mechanical structures and constructions. Thus the application of probabilities and statistics to engineering problems is becoming more and more necessary for design and maintenance.

Any probabilistic approach involves the explicit acknowledgment of a risk that appears not to exist in a deterministic approach because it is not identified. Thus, it is naturally subject to rejection, and the application must overcome cultural and also educational inertia.

Risk analysis first of all results from expert and axiomatic knowledge. As illustrated in Figure 1.4, these two forms of knowledge grow exponentially with the development of humanity and, in every age, man depends on one or the

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[16] *Die Sicherheit der Bauwerke*, M. Mayer, Springer Verlag, Berlin, 1926.

[17] ‘The Safety of Structures’, A. M. Freudenthal, *ASCE transactions* 112, 1947.

[18] *Strovoenmorizdat*, 1949. French translation, *Calcul à la rupture et plasticité des constructions*, Eyrolles, Paris, 1959.

[19] *Statistical Methods in Structural Mechanics*, translated from Russian, Holden Day, San Francisco, 1969.

[20] *Structural Safety*, 1971.

[21] ‘Calculs probabilistes de la sécurité des constructions’, R. Lévi, *Annales des Ponts et Chaussées* 26, July–August 1949.

[22] ‘Conceptions modernes relatives à la sécurité des constructions’, M. Prot and R. Lévi, *Revue Générale des Chemins de Fer*, June 1951.

other. However, the modern age also constructs theories and tools of reliability. By contributing to probability theory they bring, according to A. M. Hasofer, 'an additional precision'.[23]

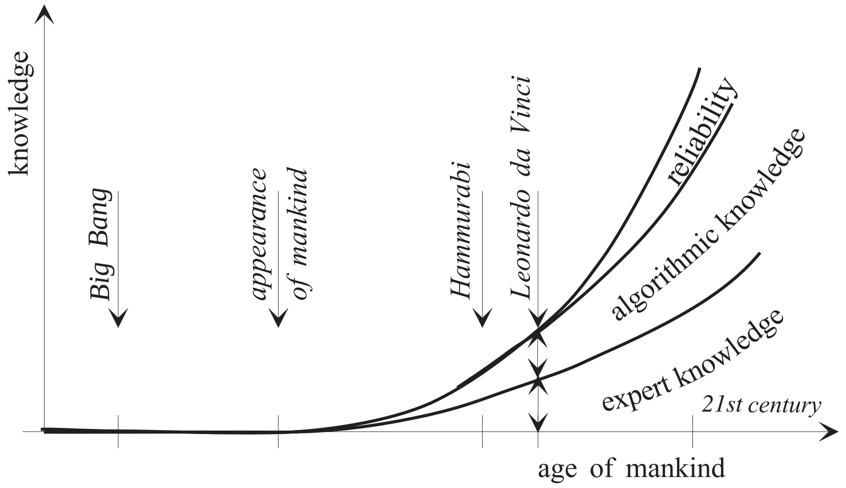

Figure 1.4 *The evolution of knowledge in history.*

If probabilistic approaches to the RAMS of systems, particularly electronic systems, and the management of breakdowns have been developing for several years now, it is only very recently that they have penetrated civil and mechanical engineering for risk analysis linked to dimensional choice. They also constitute a link between modeling in mechanics and the codes and rules of design, and represent an essential element in the transfer of knowledge between research and regulation.

The combination of mechanical modeling to simulate (in the physical sense) mechanical behavior and reliability modeling constitutes a **mechanical-reliability coupling** indispensable for an approach to design that includes risk analysis in an economic context.

1.6 A little glossary

All disciplines use a vocabulary whose current meaning should be clarified. Some words will be repeated frequently and hence deserve to be explained,

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[23] Quotation taken from a lecture delivered at the University of Clermont in the framework of doctoral training 'Matériaux, Structures, Fiabilité', 29 January 1992.

either from classic definitions found in books on mechanics or probability theory or based on a more general meaning.[24]

In mechanics

Action (load) – a force, or field of forces, translating the effect of the environment on a medium or a system that consequently undergoes transformations. Example: gravity.

Load effect (internal strength) – the result of an action on any medium, on a structure. Example: stress. The term is often wrongly used in the place of ‘action’. The effect of the loading induces internal stresses in a structure. Due to the essential role of stresses, a typical variable is noted by the letter S, but S includes all load effect variables such as displacements, strains, etc. More generally, S is a demand variable.

Resistance – the capacity of materials to resist actions, which is also called ‘strength of materials’. The chosen letter is R. More generally, R is a resource variable. A typical problem is the elementary case R–S which compares the balance between the resource and the demand, the resistance and the stress.

Structure – an orderly assembly of materials conforming to a geometry. A structure has relations and, placed in a field of action, it undergoes load effects. Example: a beam. The entire set of actions constitutes the **loading** of a structure.

Structural component – a geometric and material element belonging to a structure. Example: a rod in a framework.

On reliability approaches

Hazard – from Arabic *az-zahr*, a game of dice; the word is variously linked, to good luck in its Arabic origin, to the uncertain in French and to danger in English.

Chance – attributed to any phenomenon of which the root cause is unknown. This ignorance leads to the impossibility of determined prediction. Is chance a confession of our ignorance or does it have an existence of its own? ‘Chance is a vehicle used by God when He wants to travel incognito’ is a quotation sometimes attributed to A. Einstein, but it seems to go back to the first centuries of our era.

Random – from the old French verb ‘randir’. A ‘randy’ foal is a young horse that is gamboling in any direction, without having a precise goal. Example: a random number is any number selected in a set, such as 1 or 2 or 3 or 4 or 5 or 6, when playing dice.

Stochastic – from Greek *στοχαστής*, soothsayer. The word characterizes phenomena related to chance. ‘Random’ and ‘stochastic’ are equivalents, and the use of one or the other is more from habit than a semantic

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[24] See, for example, *Le Trésor, dictionnaire des sciences*, under the direction of Michel Serres and Nayla Farouki, Flammarion, Paris, 1997.

difference. Example: 'random processes' and 'stochastic processes' are often used interchangeably.

Test – a protocol aimed at obtaining a particular result among several possible results: the set of possible results from a test constitutes the universe Ω (or event space). A result is a selection $\omega_i \in \Omega$. Example: throwing a die (not loaded) with six sides and obtaining one of the following results $\{1, 2, 3, 4, 5, 6\}$.

Outcome – the result of a test, it is therefore a selection $\omega_i \in \Omega$. Example: rolling a 6 when throwing a die. The word 'outcome' interprets a physical and concrete concept.

Event – a part of Ω that can be defined by a logical proposition (statement) corresponding or not to an outcome. The event is realized if the logical proposition is true. The word 'event' expresses a mathematical and abstract concept. An event may not correspond to any physical outcome. Example: rolling 2 and 3 in a single throw of the die.

Protocol – all the formal conditions of an experiment.

Random experiment – a test in which the repetition of the same protocol leads to different results.

Probability – frequency definition: associated with the frequency of an event, that is, with the number of favorable situations out of the total number of situations. Axiomatic definition of probability P as a mathematical measure (A. N. Kolmogorov, 1933).

Uncertain – characterizes all the possible outcomes in a given situation. Example: the weather is uncertain; it could rain in the afternoon.

Unexpected – characterizes an event that has not been predicted. Example: the destruction of the Tacoma Narrows bridge was due to an unexpected phenomenon, the coupling of vibrations of bending and of torsion.[25]

Imprecise – characterizes a result evaluated in a very summary manner; also applicable to reasoning. Example: the imprecision of a measurement.

Error – the result of a test is an error when the protocol has not been respected.

Imprecise leads to error – when there is an imprecision it induces a difference between the result potentially anticipated and the result obtained. The difference is therefore called an error. Example: measurement error.

Reliability component – a complete description of a protocol including a structural component and its mode of mechanical operation with respect to a test. Example: a rod in elastic traction tested in relation to its yield limits.

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[25] The Tacoma Narrows bridge was situated in the State of Washington in the United States. It collapsed in 1940 under very moderate wind conditions.

> **Safety and security** – The meaning of the words ‘safety’ and ‘security’ tends to vary, depending on the communities which use them. In the reliability context, this book is addressed to engineers for whom safety is related to the performance of a mechanical system, and security is related to the risk which dysfunction could occasion for people and for the environment.

Let us note that the word ‘simulation’ takes on a different meaning depending on whether it is used in a physical or a mathematical context. If it relates to representing a physical phenomenon by a mathematical model, it is the word ‘modeling’ that is accepted. The ‘resolution’ of a model is therefore the process that moves from modeling to numerical operation. In this context, the word ‘simulation’ is reserved for the repetition of the solution with random data.

1.7 The structure of this book

This book is located within the framework of mechanical-reliability coupling, that is, the coupling between mechanical and probability methods, with the objective of a reliable design of structures. It has been written in such a way as to introduce the concepts in as simple a manner as possible and it remains limited to an approach independent of time; we will, however, note that a number of phenomena repeated in time may be modeled using the statistics of extremes. It is dependent on probabilistic methods using reliability indexes. This book provides study material aimed at defining an essential tool for students and engineers confronted with the use of probabilistic methods in mechanics. The structure is as follows.

1. **Introduction:** this chapter.
2. **Preliminary Approach to Reliability in Mechanics:** first of all, this chapter is about situating the hypotheses within the framework of the book, and then opening up certain fields of application.
3. **Elementary $R - S$ Case:** this chapter introduces the basic concepts (failure scenario, random design variables) in the elementary case of two variables, resistance R and internal strength or stress S. It presents elementary methods of resolution – direct or by simulation – and introduces the notion of the reliability index.
4. **Isoprobabilistic Transformation:** the calculation of the index uses a transformation between physical and standardized random variables. This chapter studies three transformation principles (Rosenblatt, Nataf and Hermite).
5. **Reliability Index:** this concerns the problem of non-linear optimization for which the general methods (of gradient, by quadratic programming) are adapted to the particular form of research on the reliability index.

6. **Products of Reliability Analysis:** apart from the index and the probability of failure, reliability analysis also produces important factors associated with sensitivity.
7. **Probability of Failure:** this chapter deals with approximations of the calculation of probability of failure, P_f, from the reliability index.
8. **Simulation Methods:** they offer the possibility of researching the probability of failure, either directly or in combination with reliability index methods.
9. **Reliability of Systems:** in cases where failure may result from several components, this chapter studies their combination.
10. **'Safety' Coefficients:** from the knowledge of failure parameters, this chapter shows how to define partial coefficients, and introduces methods for reliable design.
11. **Mechanical-Reliability Coupling:** the finite elements method is the essential tool for modeling in mechanics. This chapter shows how to couple mechanical and reliability models.
12. **Stochastic Finite Elements:** the introduction of randomness into the variational models of the finite elements method or at the discretized level leads to new families of finite elements, called stochastic finite elements.
13. **A Few Applications:** demonstration of the potential of reliability methods.
14. **Conclusion:** certain remarks in conclusion, and to pursue the subject further.

Being study material, this book is complemented first of all by comprehension exercises, and also by the presentation of results obtained for certain engineering problems.

This is intended to be a methodological preliminary. For implementation, it should be embedded within various types of knowledge:

- knowledge of statistical methods and random modeling using random variables,
- knowledge of specific scientific fields (integrity of structures, fatigue, cracking and rupture, stability and instability of shells, and so on) which call for reliability approaches,
- knowledge of technical fields (civil engineering, mechanical construction, naval construction, aeronautics, inspection, maintenance, repair, and so on) which introduce random analysis in their design rules.

All these scientific and technical questions are the subject of numerous publications that the reader can refer to later.

Remarks: *it is quite obvious that the calculation of probabilities of failure is concerned with orders of magnitude, and that the engineer cannot assert a large number of significant digits in a result, taking into account what he knows about the data. We will depart from this rule because, on the methodological level, the comparison of methods and axioms should be carried out with the greatest possible precision.*

Chapter 14

Conclusion

This brief chapter, by way of conclusion, summarizes all the results described in this book. It then gives some indications as to the potential fields of application of reliability methods, and it concludes with perspectives for the future, since the field of mechanical reliability is constantly progressing.

14.1 Reliability methods in mechanics

The 19th century saw the first approaches to design by the evaluation of a resistance value and that of a load effect value, and the introduction of a 'safety' coefficient between these two values. It was the century of characteristic values with a more or less defined content. The 20th century introduced variability through the coefficient of variation. It was the century of means and standard deviations. Better still, it introduced the modeling of random variables. Each level of approach has its own methods of calculation and justification. This book helps us study mechanical design, taking into account the following assumptions.

1. There is a performance function dependent on random variables and whose probability of negative outcomes we must calculate; there is a combination of performance functions (system reliability).
2. The random variables are identified using a stochastic model. In principle, we must know the joint probability density of a random variable vector, which is often inaccessible. In practice, only knowledge of the marginal distributions of each variable and their correlations is at our disposal.

In accordance with these two assumptions, time-independent mechanical-reliability methods enable us calculate a probability of failure, a probability conditioned on the quality of the information available: the quality of the mechanical representation of the performance function and the quality of the stochastic model. They also enable us to calculate the importance factors (sensitivities and elasticities) which are relevant in optimizing the design of a mechanical part. Ignoring them would be tantamount to rejecting an additional precision that is available to us.

Of course, these two assumptions limit the field of application and we will discuss this further in Section 14.3 in the perspectives for progress.

Three summary sheets, presented in the following pages, summarize the objective and the method. The first (Figure 14.1) recalls the methodology:

The goal: FORM /SORM approximation methods are efficient for the calculation of weak probabilities, less than $10^{-2} \sim 10^{-3}$. They can be used alone or in combination with conditional Monte Carlo methods.

Methodology:
1 – Selection of design variables and modeling of *random variables* X_i,
2 – Construction of the performance function $G(X_i)$ of random variables X_i,
such that $G(X_i) > 0$ defines success
and $G(X_i) \leqslant 0$ defines failure.
3 – Definition of the *probability of failure*: $\boxed{P_f = \text{Prob}(G(X_i) \leq 0)}$
4 – Evaluation using FORM/SORM methods:
– of the *reliability index* β,
– of the *probability of failure* P_f,
– of the *reliability sensitivity factors.*

Variables X_i**:** they are modeled with random variables by choosing their probability density function:
$\boxed{X_i \rightarrow f_{X_1,X_2,\ldots,X_n}(x_1,x_2,\ldots,x_n)}$
or, at least:
the marginal distribution of each variable, $\boxed{X_i \rightarrow f_{X_i}(x_i)}$
and the correlation between each couple. $\boxed{X_i, X_j \rightarrow \rho_{ij}(X_i, X_j)}$

Performance function: $G(X_i)$ expresses the mechanical link between variables, such that a positive value indicates success:
Elementary case: $X_1 = R$, R is a *resistance* or a resource,
$X_2 = S$, S is an *internal strength* (stress) or a need,
and then $G(R,S) = R - S$.
General case: $G(X_i)$ is any function resulting from a physical, mechanical, economic, etc ..., model.
If $G(X_i)$ is *explicit* and *derivable*, FORM/SORM algorithms can be easily applied,
If $G(X_i)$ is *implicit*, various approaches enable us to implement FORM /SORM methods, *like numerical design of experiment*.

Obtained results: the *reliability index* β, which forms an intermediate calculation but also a reliability measure; the *probability of failure* P_f and the *importance factors* (sensitivity or elasticity) of type: $\dfrac{\partial P_f}{\partial X_i}, \ldots, \dfrac{\partial \beta}{\partial X_i}$

Figure 14.1 *FORM/SORM methods – assumptions and expected results.*

- choosing a stochastic model of design variables; in general, marginal densities and correlations,
- choosing the mechanical model and the performance function $G(X_i)$ and its numerical implementation, validated in the entire variation domain of the design variables.

Depending on the nature of the performance function, simple explicit or more complex models, often based on finite element discretization, will result in either an easy or a much more complicated solution. The results expected are the probability of failure and the importance factors.

The second sheet (Figure 14.2) illustrates the calculation of the reliability index through the following two operations:

- application of the isoprobabilistic transformation to move from physical space to the space of standardized, centered and independent Gaussian variables,
- solution of the optimization problem to locate the most probable failure point (or design point).

The third sheet (Figure 14.3) summarizes the first-order approximation and the higher-order approximations:

- asymptotic Breitung approximation,
- 'exact' SORM calculations by integration along a quadric,
- exact RGMR integration.

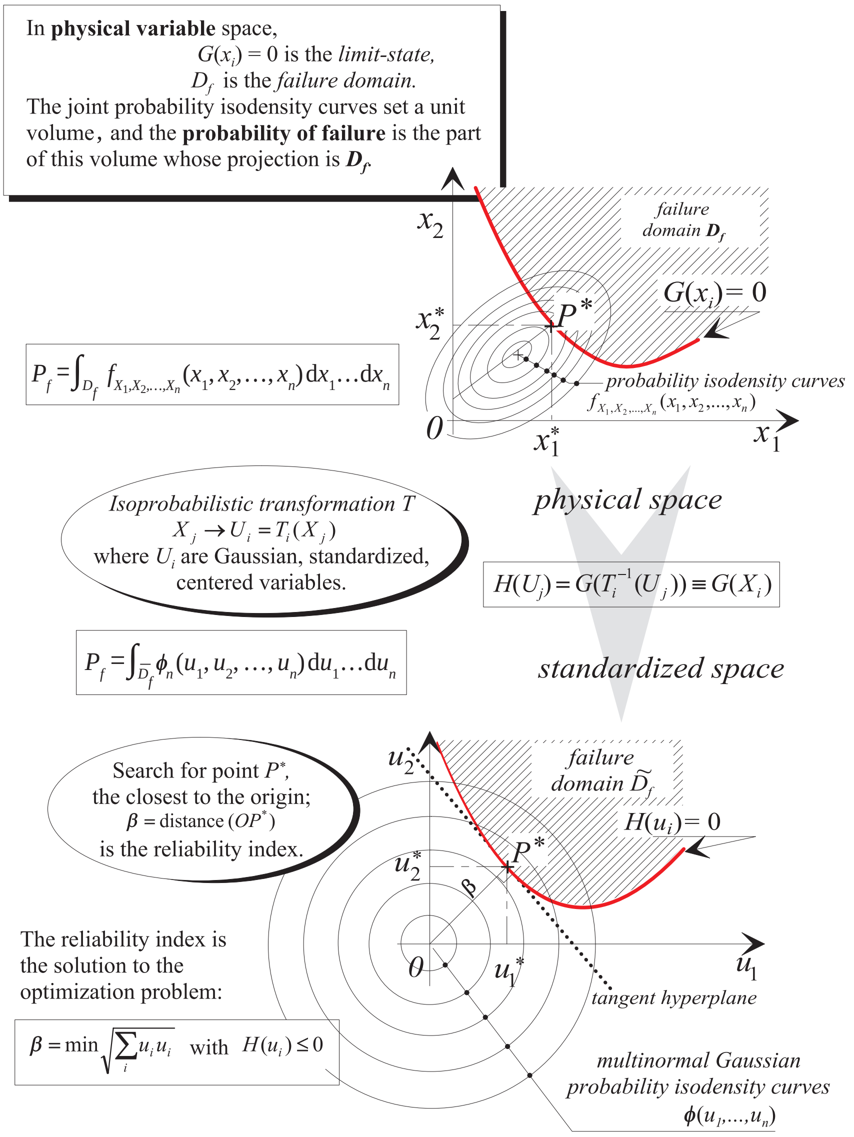
Lastly, it underlines the fact that the results must be meticulously validated by conditional simulations, and that the duality between approximation methods and simulation methods must always be borne in mind in order to achieve optimum efficiency.

Among the results of the analysis, importance factors link the probability of failure to the partial coefficients of the design rules. It comes to the same thing either to choose partial coefficients or to choose target reliability as an index associated with a stochastic model. The door is then open for the calibration of design rules.

14.2 Application of the methods

14.2.1 Design and maintenance

Let us recall once again the observation made by Hasofer: reliability methods '*offer an additional precision*' to the designer who can use them both in the initial design phase of a structure or mechanical part, and to optimize inspection, maintenance and repair operations. It is this latter aspect that seems the most likely to stimulate motivation, given the economic stakes. In fact,



In **physical variable** space,
$G(x_i) = 0$ is the *limit-state,*
D_f is the *failure domain.*
The joint probability isodensity curves set a unit volume, and the **probability of failure** is the part of this volume whose projection is $\boldsymbol{D_f}$.

$P_f = \int_{D_f} f_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n)\,\mathrm{d}x_1 \ldots \mathrm{d}x_n$

Isoprobabilistic transformation T
$X_j \rightarrow U_i = T_i(X_j)$
where U_i are Gaussian, standardized, centered variables.

$H(U_j) = G(T_i^{-1}(U_j)) \equiv G(X_i)$

$P_f = \int_{\tilde{D}_f} \phi_n(u_1, u_2, \ldots, u_n)\,\mathrm{d}u_1 \ldots \mathrm{d}u_n$

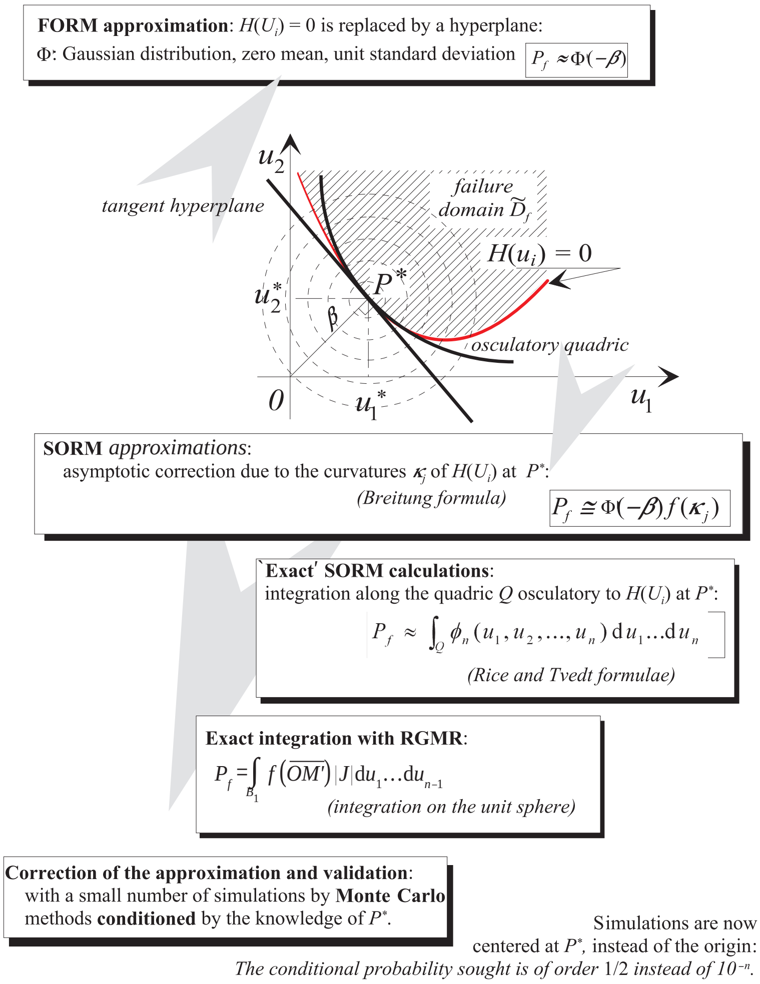
Search for point P^*,
the closest to the origin;
$\beta = \text{distance}\,(OP^*)$
is the reliability index.

The reliability index is the solution to the optimization problem:

$\beta = \min \sqrt{\sum_i u_i u_i}$ with $H(u_i) \leq 0$

Figure 14.2 *FORM/SORM methods – calculation of the reliability index.*

deterministic design methods are very often capable of providing satisfactory designs in the case of prototypes (unique constructions) at a time when it is not possible to imagine all the defects and all the deviations between physical reality



FORM approximation: $H(U_i) = 0$ is replaced by a hyperplane:
Φ: Gaussian distribution, zero mean, unit standard deviation $P_f \approx \Phi(-\beta)$

SORM *approximations*:
asymptotic correction due to the curvatures κ_j of $H(U_i)$ at P^*:
(Breitung formula) $P_f \cong \Phi(-\beta) f(\kappa_j)$

`Exact′ SORM calculations:
integration along the quadric Q osculatory to $H(U_i)$ at P^*:

$$P_f \approx \int_Q \phi_n(u_1, u_2, \ldots, u_n)\, \mathrm{d}u_1 \ldots \mathrm{d}u_n$$

(Rice and Tvedt formulae)

Exact integration with RGMR:

$$P_f = \int_{B_1} f\left(\overline{OM'}\right) |J| \mathrm{d}u_1 \ldots \mathrm{d}u_{n-1}$$

(integration on the unit sphere)

Correction of the approximation and validation:
with a small number of simulations by **Monte Carlo** methods **conditioned** by the knowledge of P^*.

Simulations are now
centered at P^*, instead of the origin:
The conditional probability sought is of order 1/2 *instead of* 10^{-n}.

Figure 14.3 *FORM/SORM methods – approximation and validation of the probability of failure.*

and the model. On the other hand, when a structure is in service, the question is to know whether it is still capable of performing its mission, in view of the actual deviations, this time not imagined but observed. The variation of the reliability index and the evolution of importance factors are then precious indications. In the case of mass production, the initial reliability analysis contributes to the definition of tolerances.

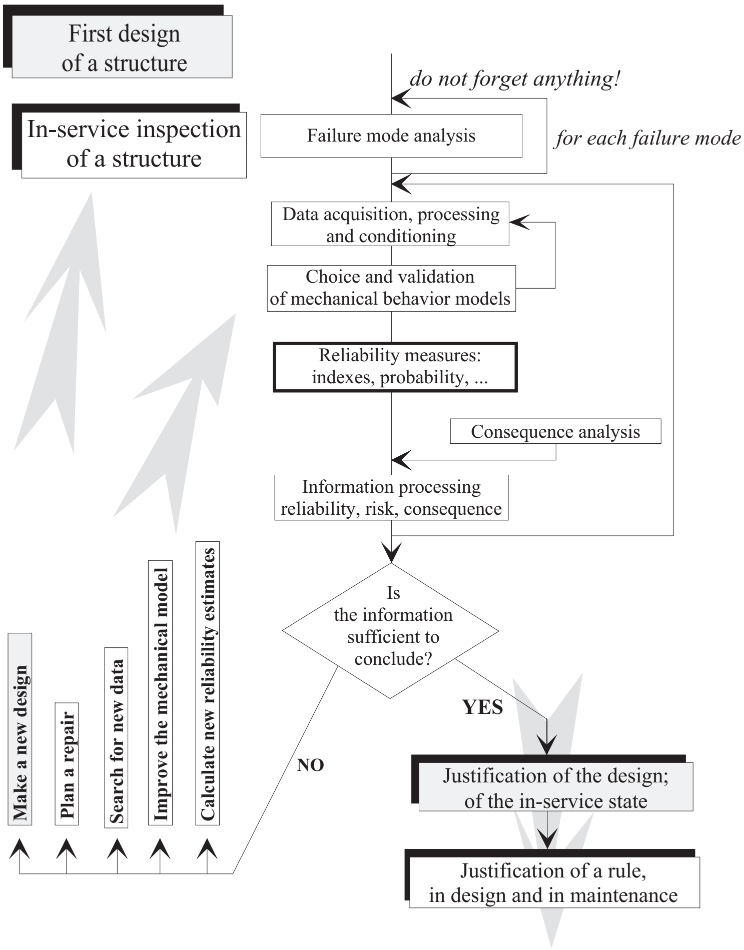


Figure 14.4 *Sequence of actions for a reliable design.*

Figure 14.4 illustrates the entire procedure. Reliability methods appear as an element in a set of actions to be undertaken for a reliable and optimal design, during both initial analysis and follow-up.

14.2.2 Software tools

It is clear that such an approach will require considerable software resources to analyze the failure modes and their criticality, to acquire and process data, and, in the light of this book, to apply reliability methods. As mechanical modeling is essential, it is necessary to have tools capable of conversing with the tools to which the designer is accustomed. It is for this purpose that the RYFES tool was developed at the initiative of A. Mohamed: *Reliability with Your Finite Element Software.* Depending on the needs and resources available, a first solution is to develop a tool to couple the mechanical model and the reliability model. If the needs are limited, it can be advantageous to write the reliability procedures directly; if they are considerable, we can use commercially-distributed software, among which we can mention the following:

- STRUREL (STRUctural RELiability) is the fruit of research by Professor Rackwitz at the Technical University of Munich. It comprises modules for component reliability and system reliability as well as time-invariant and time-variant approaches. Contact: http://www.strurel.de/.
- PROBAN (PROBabilistic ANalysis) was developed by Det Norske Veritas, evolving from the first versions oriented toward the offshore oil industry to a general code available today. Contact: http://www.dnv.at/services/software/products/safeti/safetiqra/proban.asp.
- PHIMECA software is the general purpose software whose development is based on the research carried out by the author of this book. Its specificity lies in its openness and the possibilities of dialog with any mechanical or physical behavior model. Contact:http://www.phimeca.com/.
- OpenTURNS is an Open source initiative to Treat Uncertainties, Risks 'N Statistics in a structured industrial approach. Since the beginning of 2005, a partnership of three companies – Electricité de France (EDF), European Aeronautic Defense and Space (EADS) and an engineering consultancy company (PHIMECA) – has been working on building together a tool designed to perform reliability analyses. Contact: http://trac.openturns.org/.

The reader can also refer to Structural Safety (28), 2006. This reference presents an overview of general purpose software in structural reliability.

14.3 Perspectives

We hope that this book has convinced the reader of the advantages of probabilistic methods. Furthermore, we hope it has enabled the reader to discover the bases of these methods so that he/she can himself/herself apply them and validate the results obtained with a clear knowledge of the assumptions. These assumptions will certainly leave him/her dissatisfied. To discover the probabilistic approach is also to discover its limits.

To go further, the reader can consult the international literature on the subject as well as review books; see, for example, [DM96].

In a scientific and industrial context, the author would point out the following four avenues for progress.

14.3.1 Stochastic finite elements

As during the early years of the development of finite element codes for the number of degrees of freedom, the question today concerns solution capacity, considering the number of random variables. We have indicated that the efficiency of FORM/SORM methods can decrease with an increase in the number of variables, which then call for other approaches or a greater use of conditional simulations. Moreover, the volume of calculation increases very rapidly.

Gordon Moore’s law, which predicts that the power of computers will double every two years – up to what limit? – will come to our aid.

From a scientific point of view, it is the stochastic finite element method that should result in the creation of codes. The challenge today is to find the correct spectral decomposition capable of capturing the maximum variability with a minimum number of modes. The creation of software tools will promote their application in increasingly complex real cases. As for deterministic finite element methods, it will be more by an accumulation of experiences than by algorithmic justifications – indispensable but insufficient – that we can gradually develop a skill.

14.3.2 The time factor

The time factor has been discussed very little in this book. Two approaches are associated with it.

First, stochastic dynamics deals with the response of a dynamic system excited by a random signal and having, as the case may be, parameters of stiffness, mass and damping that are also random. This is a field in its own right described in depth in the works of Krée and Soize [KS83] in France and in Chapter V of [Soi01].

In addition, all the time-dependent phenomena urge us to replace modeling with a random variable by modeling with a stochastic process. A time-invariant approach results in the probability calculation:

$$P_{f,i}(t) = \text{Prob}\ (G(t, \mathbf{x}(t), \omega) \le 0) \quad \text{at fixed } t$$

whereas a time-variant approach poses the question of the calculation of cumulative probability:

$$P_{f,c}(t) = \text{Prob}\ (\exists \tau \in [t_0, t] \text{ such that } G(\tau, \mathbf{x}(\tau), \omega) \le 0)$$

where t_0 is the date of commissioning of the mechanical system.

The response to this question can be addressed using several methods based on the concept of outcrossing rate according to the Rice formula [Ric44a, Ric44b]. This involves estimating the number of trajectories of the process entering the failure domain in the interval $[t_0, t]$:

- The Monte Carlo method can be applied under broad assumptions, but at the cost of very great demand on computing resources.
- For differentiable or jump processes [BFR91, SFR91], there are explicit equations for the outcrossing rate.
- Hagen and Tvedt [HT91b, HT91a] have suggested considering outcrossing as a system formed by the intersection of a positive outcome on date t and a negative outcome on date $t + \mathrm{d}t$. Such an approach has been developed further in [ARSL04].

Under certain assumptions, often verified in the degradation of materials, point-in-time probability is equal to cumulative probability. This is the case if all the trajectories are decreasing.

14.3.3 Data acquisition and processing

The reliability methods described in this book are based on an exhaustive knowledge of the stochastic modeling of the variables. Of course, the information available is always poor and we only have estimates for the parameters of the random variable distributions. They introduce an additional uncertainty that can be offset at the cost of an increase in the number of random variables. It is then possible to construct a reliability index as a function of the size of the samples available. This information indicates whether it is advisable or not to increase the statistical richness of the sample, at the price of additional tests. [PHLM00, Pen00] describe this approach.

The quality of the data model is essential. It will never be satisfactory, particularly because of the importance of distribution tails. However, it always provides better information than the choice of a deterministic characteristic value. The absence of standards for the modeling of variables is currently an obstacle to the dissemination of probabilistic methods. We can imagine that one day a code for the representation of variables will be adopted, as there is today a code for the partial coefficient approach.

14.3.4 Mechanical-reliability optimization

The design of mechanical structures and systems progressed by leaps and bounds in the second half of the 20th century. This is primarily thanks to the considerable increase in computing power. Thus, the following milestones were crossed:

- linear calculation of structures during the 1970s,
- geometric and material non-linear calculation during the 1980s,

- coupling between the calculation of structures and optimization on the one hand and design of the geometry on the other hand during the 1990s,
- coupling between reliability methods and mechanical models, also in the 1990s.

Tomorrow's tools will see the development of optimization methods. Firstly, classic optimization in a deterministic context, but also reliability-based design optimization (RBDO), as the criterion of data variability must enter into the optimization scheme for the latter to guarantee a minimum level of reliability. In fact, neglecting it can lead to solutions that are perhaps theoretically optimal, but in reality hardly robust in the face of the deviations between the target values and the physical outcomes.

Such an approach has been developed by the author of this book and his team, in the laboratory [KML02b, KOML04]. As the search for the design point is itself an optimization problem, RBDO is tantamount to building two interleaved optimization loops.

By denoting:

1. the optimization variables $\{x\}$; these are the deterministic variables to be controlled in order to optimize the design; they represent the control parameters of the mechanical system (i.e. dimensions, materials, loads, etc.) and of the probabilistic model (i.e. means and standard deviations of the random variables),
2. the random variables $\{y\}$, which represent the uncertainties; each of these variables is identified by its distribution type and the associated parameters; these variables can be geometric dimensions, material characteristics or external loadings,
3. $f(\{x\})$, the objective function of the design variables to be optimized,
4. $G(\{x\},\{y\})$, the performance function,
5. $g_k(\{x\},\{y\}) \geq 0$, the set of k constraints of the optimization.

Then the two optimization procedures are:

1. The deterministic optimization of the system, by integrating the reliability constraint:

$$\min_{\{x\}}: f(\{x\})$$
$$\text{under the constraints: } g_k(\{x\},\{y\}) \geq 0 \quad \text{and} \quad \beta(\{x\},\{y\}) \geq \bar{\beta}$$

   where $\bar{\beta}$ is the target reliability index and $\beta(\{x\},\{y\})$ is that of the structure.

2. The search for the design point by an optimization procedure:

$$\min_{\{y\}}: d^2(\{x\},\{y\}) \quad \text{under the constraint} \quad G(\{x\},\{y\}) \geq 0$$

   where $d(\{x\},\{y\}) = \|T(\{x\},\{y\})\|$ is the distance, in standardized space, between the origin and the design point.

14.4 Reliability analysis and sensitivity analysis

Reliability analysis consists of searching for a particular point of the distribution function of the performance function. By introducing a threshold and by varying the reliability level, it becomes possible to construct the density point by point. Sensitivity analysis consists of developing the performance function from its first statistical moments. By increasing the number of moments, their density can be approximated, and thus a reliability analysis can be performed. If there are in fact two ways to address the same problem, the expected results are indeed different.

The first analysis prefers an accurate study at a given point, located generally in a distribution tail, whereas the second favors the knowledge of the central tendencies. The choice therefore depends on the questions posed.

- *Static mechanical-reliability coupling is a tool already at the disposal of engineers for an optimal and reliable design. It will be complemented in the coming years with the arrival of tools integrating spatial variability, the time dimension and optimization.*
- *These tools will be used to refine prediction methods by the development of specific tools, according to the type of behavior of materials and structures: fracture mechanics, fatigue, instability, etc.*
- *They will one day join with failure analysis tools to integrate reliability more completely into operating safety.*

To be continued . . .