General Introduction

Combinatorics and all the fields deriving from it – the probabilities and graph theories – are no longer peripheral phenomena, at the edge of pure mathematics. We can even consider combinatorics as bringing a breath of fresh air into the universe of theory. Indeed it has its own style of demonstrations, which often require more tricks and common sense than the systematic application of the mainstream mathematical theories. It is also an introduction in a concrete manner of some abstract algebraic tools such as matrices and determinants. The recent development of combinatorics also results from the worldwide emergence of informatics, which offers unlimited possibilities of practice and experimentation, i.e. either to check or anticipate theoretical results, or to solve problems that theory cannot solve. Combinatorics associated with counting and enumerating allows us to encode events in words, made of numbers or letters, which brings the field closer to linguistics. The word dictionaries obtained can in turn be illustrated on computer screen through complex shapes and patterns, such as the new hieroglyphs, which are able to strike the imagination and stimulate the artistic interpretation.

Combinatorics is first studied in science classes at high school, where it is associated with the calculation of probabilities, and recently, with graphs theory. The calculation of the probabilities and its algorithms is also a favorite field used in the entrance examination for major business schools. On the other side of the coin, the number of high-level publications has multiplied in specialized journals, which are only accessible to a few. This book falls between the two camps. It gradually moves from basic introductory sections to the latest theoretical developments in the field, illustrated by numerous examples.[1]

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1. In this respect, we used the work by Graham, Knuth and Patashnik, *Concrete Mathematics* [GRA 90] as a source of inspiration and a model.

This book targets students and researchers, and more broadly knowledgeable amateurs. High school and preparatory school teachers will find here many useful examples and exercises. Depending on the theoretical level of the audience, some readers will prefer to focus on the algorithms, or even the graphic visuals, others will concentrate on the algebraic or combinatorial implementations. The aim of this book is to achieve a global overview of the state of the art in the field.

About the algorithms and programs

One of the specific aspects of this book is to provide a large number of algorithms and programs, all explained in full detail. The programs are designed in abridged versions using C language, and they are easily adaptable to other similar languages such as *Pascal* or *Basic*. Mathematics enthusiasts will need to convert these programs directly into a scientific language such as *Mathematica* if they want to benefit from the graphic functions. In order to learn C programming and SDL graphics, I recommend that readers visit my website, created within the framework of LIASD (Laboratoire d'informatique avancée de Saint-Denis, Paris 8 University): www.ai.univ-paris-8.fr/~audibert/. Under the heading "Book Programs", we give many programs present in this book. They are in full programs, written in C with SDL graphical help, with their codes as well as their executable files.[2]

Structure of the book

The book is divided into three parts:

− Part 1: *Combinatorics*;

− Part 2: *Probability*;

− Part 3: *Graphs*.

2. In order to learn *C* programming and *SDL* graphics, the "happy few" who read French will find a brief introduction about their use, in my web pages. Under the rubric "Education", where IT and mathematics (level 1, L1) and algorithmics (Level 2, L2) courses are listed, in chapter entitled "Graphics SDL: second layer", two functions of basic graphics are listed: *putpixel* and *getpixel* (formerly called *peek* and *poke*), as well as the functions making it possible to draw lines and circles. Readers will also learn the way to draw a segment with arrows, which is highly necessary for drawing graphs. Should the need arise, the chapters dealing with recursivity and linked lists will enable easy comprehension of these more complex concepts. In "Complementary Works", some mathematical games, among others, are explained (such as Marienbad, Instant Insanity, Planarity) with their complete programmings. For algorithmic enthusiasts in general, see the following books for further detail: [AHO 87], [BER 91], [COR 02], [SED 91].

Although these three parts can be read separately, they are connected by counting and enumerating algorithms, following the same reading line, i.e. the concept of generating functions[3]. Despite the large number of subjects studied, this book does not claim to be exhaustive. Interested readers will find more details in [TUC 02], notably on games based on graphs, in [LEN 03] on mathematical linguistics, and in [STA 01] or [LOT 97] for a deeper theoretical vision. Let us not forget to mention [SLO 95], the global reference in the field of integer sequences, or the pioneering book by [COM 70].

Acknowledgements

I want to thanks my colleagues and friends, particularly F. Belhadj, P. Chibaudel, S. El Baz, C. Fer, P. Greussay, C. Lenormand, I. Saleh, H. Wertz, who helped me a lot. I also wish to thank the hundred or so students who wrote their MD or PhD theses under my supervision. They have allowed me to work on a diversity of subjects, but always on an algorithmic basis: ranging from my first student R. Abdoul (on sand avalanches) to my latest I. Mazouzi (about the Chinese theorem), to the other students who explored combinatorial problems and whose works are mentioned in this book: H. Arfa, A. Fathi, N. Grassa, N. Rifaai, Y. Naciri.

3. For a full understanding of generating functions refer to [WIL 94].