

## Table of Contents

<b>Foreword</b> . . . . .	xi
<b>Acknowledgements</b> . . . . .	xiii
<b>Chapter 1. Introduction</b> . . . . .	1
1.1. Overview of the book . . . . .	2
1.1.1. Constraint programming . . . . .	3
1.1.2. Ant colony optimization . . . . .	4
1.1.3. Constraint programming with ant colony optimization . . . . .	4
<b>Chapter 2. Computational Complexity</b> . . . . .	7
2.1. Complexity of an algorithm . . . . .	8
2.2. Complexity of a problem . . . . .	10
2.2.1. The $\mathcal{P}$ class . . . . .	10
2.2.2. The $\mathcal{NP}$ class . . . . .	11
2.2.3. $\mathcal{NP}$ -complete problems . . . . .	12
2.2.4. $\mathcal{NP}$ -hard problems . . . . .	14
2.2.5. Undecidable problems . . . . .	14
2.2.6. Complexity of optimization problems . . . . .	15
2.3. Where the most difficult instances can be found . . . . .	15
2.3.1. Phase transition . . . . .	16
2.3.2. Search landscape . . . . .	19
2.4. Solving $\mathcal{NP}$ -hard problems in practice . . . . .	21
2.4.1. Exploitation of particular cases . . . . .	23
2.4.2. Approximation algorithms . . . . .	23
2.4.3. Heuristics and metaheuristics . . . . .	24
2.4.4. Structuring and filtering the search space . . . . .	24

<b>PART I. CONSTRAINT PROGRAMMING . . . . .</b>	<b>27</b>
<b>Introduction to Part I . . . . .</b>	<b>29</b>
<b>Chapter 3. Constraint Satisfaction Problems . . . . .</b>	<b>31</b>
3.1. What is a constraint? . . . . .	31
3.1.1. Definition of a constraint . . . . .	32
3.1.2. Arity of a constraint and global constraints . . . . .	33
3.2. What is a constraint satisfaction problem? . . . . .	33
3.2.1. Complexity of CSPs . . . . .	34
3.3. Optimization problems related to CSPs . . . . .	35
3.3.1. Maximizing constraint satisfaction . . . . .	35
3.3.2. Constrained optimization . . . . .	36
3.4. The $n$ -queens problem . . . . .	37
3.4.1. Description of the problem . . . . .	37
3.4.2. First CSP model . . . . .	38
3.4.3. Second CSP model . . . . .	39
3.4.4. Third CSP model . . . . .	40
3.4.5. Influence of the model on the solution process . . . . .	41
3.5. The stable marriage problem . . . . .	43
3.5.1. Description of the problem . . . . .	43
3.5.2. CSP model . . . . .	45
3.6. Randomly generated binary CSPs . . . . .	46
3.7. The car sequencing problem . . . . .	47
3.7.1. Description of the problem . . . . .	47
3.7.2. CSP model . . . . .	49
3.8. Discussion . . . . .	50
<b>Chapter 4. Exact Approaches . . . . .</b>	<b>53</b>
4.1. Construction of a search tree . . . . .	53
4.2. Constraint propagation . . . . .	57
4.2.1. Forward checking . . . . .	57
4.2.2. Maintaining arc consistency . . . . .	59
4.3. Ordering heuristics . . . . .	60
4.3.1. Heuristics for choosing variables . . . . .	61
4.3.2. Heuristics for choosing values . . . . .	62
4.3.3. Randomized restart . . . . .	63
4.4. From satisfaction to optimization problems . . . . .	63
4.5. Discussion . . . . .	65

<b>Chapter 5. Perturbative Heuristic Approaches . . . . .</b>	69
5.1. Genetic algorithms . . . . .	70
5.1.1. Basic principles . . . . .	70
5.1.2. Using GAs to solve CSPs . . . . .	73
5.2. Local search . . . . .	73
5.2.1. Basic principles . . . . .	73
5.2.2. Metaheuristics based on LS . . . . .	75
5.2.3. Using LS to solve CSPs . . . . .	77
5.3. Particle swarm optimization . . . . .	78
5.3.1. Basic principles . . . . .	78
5.3.2. Using PSO to solve CSPs . . . . .	80
5.4. Discussion . . . . .	80
<b>Chapter 6. Constructive Heuristic Approaches . . . . .</b>	85
6.1. Greedy randomized approaches . . . . .	86
6.1.1. Basic principles . . . . .	86
6.1.2. Using greedy randomized algorithms to solve CSPs . . . . .	88
6.2. Estimation of distribution algorithms . . . . .	88
6.2.1. Basic principles . . . . .	88
6.2.2. Using EDAs to solve CSPs . . . . .	90
6.3. Ant colony optimization . . . . .	90
6.4. Discussion . . . . .	91
<b>Chapter 7. Constraint Programming Languages . . . . .</b>	93
7.1. Constraint logic programming . . . . .	94
7.2. Constraint programming libraries . . . . .	96
7.3. Constraint-based local search . . . . .	96
7.4. Discussion . . . . .	99
<b>PART II. ANT COLONY OPTIMIZATION . . . . .</b>	101
<b>Introduction to Part II . . . . .</b>	103
<b>Chapter 8. From Swarm Intelligence to Ant Colony Optimization . . . . .</b>	105
8.1. Complex systems and swarm intelligence . . . . .	106
8.2. Searching for shortest paths by ant colonies . . . . .	108
8.3. Ant system and the traveling salesman problem . . . . .	111
8.3.1. Pheromone structure . . . . .	112
8.3.2. Construction of a Hamiltonian cycle by an ant . . . . .	114

8.3.3. Pheromone updating step . . . . .	115
8.3.4. Artificial versus real ants . . . . .	115
8.4. Generic ACO framework . . . . .	116
8.4.1. Pheromone structure and construction graph . . . . .	116
8.4.2. Construction of combinations by ants . . . . .	118
8.4.3. Improving combinations with local search . . . . .	120
8.4.4. Pheromone updating step . . . . .	121
8.4.5. Parameters of an ACO algorithm . . . . .	122
<b>Chapter 9. Intensification versus Diversification</b> . . . . .	125
9.1. ACO mechanisms for intensifying the search . . . . .	125
9.2. ACO mechanisms for diversifying the search . . . . .	127
9.3. Balancing intensification and diversification . . . . .	128
9.4. Measures of diversification/intensification . . . . .	135
9.4.1. The $\lambda$ -branching factor . . . . .	136
9.4.2. Resampling ratio . . . . .	136
9.4.3. Similarity ratio . . . . .	137
<b>Chapter 10. Beyond Static Combinatorial Problems</b> . . . . .	141
10.1. Multi-objective problems . . . . .	141
10.1.1. Definition of multi-objective problems . . . . .	141
10.1.2. Solving multi-objective problems with ACO . . . . .	143
10.2. Dynamic optimization problems . . . . .	145
10.2.1. Definition of dynamic optimization problems . . . . .	145
10.2.2. Solving dynamic optimization problems with ACO . . . . .	146
10.3. Optimization problems over continuous domains . . . . .	147
10.3.1. Definition of continuous optimization problems . . . . .	147
10.3.2. Solving continuous optimization problems with ACO . . . . .	148
<b>Chapter 11. Implementation Issues</b> . . . . .	151
11.1. Data structures . . . . .	151
11.1.1. Data structures associated with pheromone factors . . . . .	151
11.1.2. Data structures associated with heuristic factors . . . . .	153
11.1.3. Data structures associated with ants . . . . .	154
11.2. Selection of a component with respect to probabilities . . . . .	154
11.3. Implementation of a local search procedure . . . . .	157
11.4. Computation of diversification/intensification measures . . . . .	157
11.4.1. Resampling ratio . . . . .	158
11.4.2. Similarity ratio . . . . .	158

<b>PART III. CP WITH ACO . . . . .</b>	161
<b>Introduction to Part III . . . . .</b>	163
<b>Chapter 12. Sequencing Cars with ACO . . . . .</b>	165
12.1. Notation . . . . .	165
12.2. A first pheromone structure for identifying good car sequences . . . . .	166
12.2.1. Pheromone structure . . . . .	167
12.2.2. Construction of a sequence by an ant . . . . .	168
12.2.3. Pheromone laying step . . . . .	170
12.3. A second pheromone structure for identifying critical cars . . . . .	171
12.3.1. Pheromone structure . . . . .	171
12.3.2. Construction of a sequence by an ant . . . . .	172
12.3.3. Pheromone updating step . . . . .	172
12.4. Combining the two pheromone structures . . . . .	173
12.4.1. First pheromone structure . . . . .	173
12.4.2. Second pheromone structure . . . . .	173
12.4.3. Construction of a sequence by an ant . . . . .	173
12.5. Comparison of the different ACO algorithms . . . . .	174
12.5.1. Considered algorithms . . . . .	174
12.5.2. Test suite . . . . .	175
12.5.3. Parameter settings . . . . .	175
12.5.4. Experimental results . . . . .	177
12.6. Comparison of ACO with state-of-the-art approaches . . . . .	178
12.6.1. Considered approaches . . . . .	178
12.6.2. IDWalk . . . . .	179
12.6.3. VFLS . . . . .	179
12.6.4. Experimental set-up . . . . .	180
12.6.5. Experimental results . . . . .	180
12.7. Discussion . . . . .	182
<b>Chapter 13. Subset Selection with ACO . . . . .</b>	185
13.1. Subset selection problems . . . . .	186
13.1.1. Maximum clique . . . . .	186
13.1.2. Multidimensional knapsack . . . . .	187
13.1.3. Maximum Boolean satisfiability . . . . .	187
13.1.4. Maximum constraint satisfaction . . . . .	187
13.1.5. Minimum vertex cover . . . . .	188
13.1.6. Maximum common subgraph . . . . .	188

13.1.7. Edge-weighted $k$ -cardinality tree . . . . .	189
13.2. Description of Ant-SSP . . . . .	189
13.2.1. Construction of a combination by an ant . . . . .	190
13.2.2. Pheromone laying step . . . . .	192
13.3. Instantiations of Ant-SSP with respect to two pheromone strategies . . . . .	192
13.3.1. The vertex pheromone strategy . . . . .	193
13.3.2. The clique pheromone strategy . . . . .	193
13.3.3. Comparison of the two strategies . . . . .	194
13.4. Instantiation of Ant-SSP to solve CSPs . . . . .	196
13.4.1. Heuristic factor . . . . .	196
13.4.2. Local search . . . . .	197
13.5. Experimental results . . . . .	197
13.5.1. Randomly generated binary instances . . . . .	197
13.5.2. Results on instances of the 2006 solver competition .	199
13.6. Discussion . . . . .	202
<b>Chapter 14. Integration of ACO in a CP Language</b> . . . . .	205
14.1. Framework for integrating ACO within a CP library . . . . .	206
14.1.1. Pheromone strategy . . . . .	207
14.1.2. Construction of assignments . . . . .	208
14.1.3. Pheromone updating step . . . . .	210
14.2. Illustration of ACO-CP on the car sequencing problem . . . . .	210
14.2.1. CSP model . . . . .	210
14.2.2. Variable ordering heuristic . . . . .	211
14.2.3. Pheromone strategies . . . . .	211
14.2.4. Heuristic factor . . . . .	212
14.2.5. Experimental results . . . . .	213
14.3. Discussion . . . . .	214
<b>Chapter 15. Conclusion</b> . . . . .	215
15.1. Towards constraint-based ACO search . . . . .	215
15.2. Towards a reactive ACO search . . . . .	216
<b>Bibliography</b> . . . . .	219
<b>Index</b> . . . . .	231