
Contents

Preface	ix
1 Fundamentals of Tensor Theory	1
1.1 Tensor algebra	1
1.1.1 Contravariant and covariant components of a vector	1
1.1.2 Dual basis	2
1.1.3 Different representations of a vector	3
1.1.4 Results related to the orientation of the 3D space	4
1.1.5 Tensor	5
1.1.6 Metric tensor	8
1.1.7 Tensor product	9
1.1.8 Tensor basis - Different representations of a tensor	10
1.1.9 Contraction - Contracted Product	12
1.1.10 Results specific to 2nd-order tensors	15
1.1.11 Results specific to 4th-order tensors	18
1.2 Tensor analysis	18
1.2.1 Curvilinear coordinates	19
1.2.2 Natural basis - Natural frame	19
1.2.3 Derivatives of the natural basis vectors - Christoffel symbols	20
1.2.4 Covariant derivative	20
1.2.5 Expressions for differential operators in curvilinear coordinates	22
2 Initial Position of a Plate	23
2.1 Initial position of the mid-surface of the plate	23
2.1.1 Definitions	23
2.1.2 First fundamental form of S_0 - Primal and dual bases	25
2.2 Initial position of the plate	29
2.2.1 Definitions	29
2.2.2 Natural covariant basis and dual basis	30
2.2.3 Gauss formula	33
2.3 Covariant derivative on a surface	34
2.4 Divergence theorem	35
3 Cosserat Plate Theory	37
3.1 Current position of the plate mid-surface	38
3.2 Current position of the plate - Displacement field	39
3.3 Displacement gradient	46

3.4 Strain tensor	46
3.5 Velocity field	49
3.6 Principle of Virtual Power (PVP)	50
3.7 Virtual velocity field	50
3.8 Virtual velocity gradient	50
3.9 Virtual power of inertia forces	51
3.10 Virtual power of internal forces	52
3.11 Virtual power of external forces	54
3.11.1 Virtual power of body forces	55
3.11.2 Virtual power of forces over the upper and lower faces	55
3.11.3 Virtual power of forces along the edge	57
3.11.4 To recapitulate	59
3.12 Equations of motion and boundary conditions	60
3.13 Static problems	61
3.14 Another method to obtain the equations	61
3.15 Overview of the equations and unknowns	65
4 Reissner-Mindlin Plate Theory	67
4.1 Current position of the plate mid-surface	67
4.2 Current position of the plate - Displacement field	68
4.3 Gradient of displacement	73
4.4 Strain tensor	73
4.5 Velocity field	74
4.6 Virtual velocity field	77
4.7 Virtual power of inertia forces	78
4.8 Virtual power of internal forces	78
4.9 Virtual power of external forces	78
4.10 Equations of motion and boundary conditions	79
4.11 Note on couples	80
4.12 Static problems	81
4.13 Overview of equations and unknowns	81
5 Kirchhoff-Love Plate Theory	83
5.1 Current position of the plate mid-surface	83
5.1.1 First fundamental form of S - Primal basis and dual basis	84
5.1.2 Second fundamental form of S - Curvature tensor of S	85
5.1.3 Third fundamental form of S	89
5.2 Current position of the plate - Displacement field	89
5.3 Strain tensor	94
5.4 Velocity field	96
5.5 Virtual velocity field	96
5.6 Virtual powers of inertia forces	97
5.7 Virtual power of internal forces	98
5.8 Virtual power of external forces	105
5.9 Equations of motion and boundary conditions	111
5.10 Static problems	114
5.11 Overview of equations and unknowns	115
5.12 Example: Kirchhoff-Love plate in cylindrical bending	115
5.12.1 Statement of the problem	115
5.12.2 Equations of the problem	117
5.12.3 Comparison with Bernoulli's beam in finite bending	119

5.12.4 Case of an inextensible plate	120
6 Constitutive Law of Plates	129
6.1 Hyperelastic 3D constitutive law	130
6.1.1 Saint Venant-Kirchhoff material	131
6.1.2 Neo-Hookean material	132
6.2 Strains in terms of the Z-coordinate	133
6.3 Stress resultants for Cosserat plates	134
6.4 Zero normal stress hypothesis $\sigma^{33} = 0$	137
6.5 Plane stress state	138
6.6 Reduced constitutive law	141
6.6.1 Saint Venant-Kirchhoff material	141
6.6.2 Neo-Hookean material	144
6.7 Stress resultants for Reissner-Mindlin plates	146
6.8 Stress resultants for Kirchhoff-Love plates	147
6.9 Review of the hypotheses used	152
6.9.1 Hypotheses to establish the equations by means of the PVP	152
6.9.2 Hypotheses to establish the reduced constitutive law	153
6.9.3 Hypotheses to establish the plate constitutive laws	154
6.9.4 Additional remarks on some usual hypotheses	154
7 Linearized Kirchhoff-Love Plate Theory	157
7.1 Statement of the problem	157
7.2 Linearization principle	160
7.3 Linearization of the vectors of the current natural basis	164
7.3.1 Expression for vector \mathbf{a}_α	164
7.3.2 Linearized current normal vector \mathbf{a}_3	165
7.3.3 Linearized vector \mathbf{a}^α	166
7.4 Linearized current curvatures	167
7.5 Linearized current Christoffel symbols	167
7.6 Linearized strain tensor	167
7.7 Linearized integrated constitutive laws	168
7.8 Linearized governing equations and boundary conditions – Vibrations of a pre-stressed plate	170
7.9 Overview of the equations and unknowns	176
7.10 Displacement equations	176
7.10.1 Common boundary conditions in terms of W	179
7.10.2 Problem solving	180
7.11 Equilibrium of a pre-stressed plate	181
7.12 Plate buckling problem	183
7.13 Example: Buckling of a simply-supported rectangular plate	184
7.14 Example: Buckling of a circular plate	188
7.14.1 Clamped circular plate	188
7.14.2 Simply supported circular plate	191
Appendix Some Mechanical Relations in 3D Curvilinear Coordinates	195
Bibliography	199
Index	201