

# Contents

<b>Foreword</b> . . . . .	xi
Maurice LEMAIRE	
<b>Preface</b> . . . . .	xv
Christian GOGU	
<b>Part 1. Modeling, Propagation and Quantification of Uncertainties.</b> . . . . .	1
<b>Chapter 1. Uncertainty Modeling</b> . . . . .	3
Christian GOGU	
1.1. Introduction. . . . .	3
1.2. The usefulness of separating epistemic uncertainty from aleatory uncertainty. . . . .	6
1.3. Probability theory . . . . .	10
1.3.1. Theoretical context . . . . .	10
1.3.2. Probabilistic approach for modeling aleatory uncertainties . . . . .	13
1.3.3. Probabilistic approach for modeling epistemic uncertainties . . . . .	16
1.4. Probability box theory (p-boxes) . . . . .	21
1.5. Interval analysis . . . . .	24
1.6. Fuzzy set theory . . . . .	25
1.7. Possibility theory . . . . .	27
1.7.1. Theoretical context . . . . .	27

---

1.7.2. Comparison between probability theory and possibility theory . . . . .	30
1.7.3. Rules for combining possibility distributions . . . . .	34
1.8. Evidence theory . . . . .	35
1.8.1. Theoretical context . . . . .	35
1.8.2. Rules for combining belief mass functions . . . . .	38
1.9. Evaluation of epistemic uncertainty modeling . . . . .	40
1.10. References. . . . .	40
<b>Chapter 2. Microstructure Modeling and Characterization . . . . .</b>	<b>43</b>
François WILLOT	
2.1. Introduction. . . . .	43
2.2. Probabilistic characterization of microstructures. . . . .	45
2.2.1. Random sets . . . . .	45
2.2.2. Covariance . . . . .	47
2.2.3. Granulometry . . . . .	50
2.2.4. Minkowski functionals. . . . .	51
2.2.5. Stereology . . . . .	53
2.2.6. Linear erosion . . . . .	53
2.2.7. Representative volume element. . . . .	54
2.3. Point processes. . . . .	55
2.3.1. Homogeneous Poisson point processes . . . . .	56
2.3.2. Inhomogeneous Poisson point processes . . . . .	58
2.4. Boolean models . . . . .	59
2.4.1. Definition and Choquet capacity . . . . .	59
2.4.2. Properties. . . . .	61
2.4.3. Covariance . . . . .	63
2.4.4. Other characteristics . . . . .	63
2.5. RSA models . . . . .	66
2.6. Random tessellations . . . . .	67
2.6.1. Voronoi tessellation . . . . .	68
2.6.2. Johnson–Mehl tessellation. . . . .	69
2.6.3. Laguerre tessellation . . . . .	69
2.6.4. Random Poisson tessellation . . . . .	70
2.6.5. The dead-leaves model. . . . .	71
2.6.6. Generalized random partition models . . . . .	72
2.7. Gaussian fields . . . . .	73
2.8. Conclusion . . . . .	76
2.9. Acknowledgments. . . . .	77
2.10. References. . . . .	77

<b>Chapter 3. Uncertainty Propagation at the Scale of Aging Civil Engineering Structures . . . . .</b>	<b>83</b>
David BOUHJITI, Julien BAROTH and Frédéric DUFOUR	
3.1. Introduction. . . . .	83
3.2. Problem positioning . . . . .	85
3.2.1. Probabilistic formulation. . . . .	85
3.2.2. Thermo-hydro-mechanical-leakage transfer function . . . . .	86
3.2.3. Resulting probabilistic THM-F problem . . . . .	87
3.3. Random field-based modeling of material properties . . . . .	88
3.3.1. Random fields . . . . .	88
3.3.2. Generation methods for discretized random fields . . . . .	88
3.3.3. Random fields and autocorrelations . . . . .	91
3.3.4. Application: contribution to modeling the cracking of reinforced concrete works by self-correlated r.f. . . . .	92
3.4. Modeling uncertainty propagation using response surface methods . . . . .	98
3.4.1. Probabilistic coupling strategies . . . . .	98
3.4.2. Polynomial chaos method . . . . .	101
3.5. Conclusion . . . . .	108
3.6. References . . . . .	108
<b>Chapter 4. Reduction of Uncertainties in Multidisciplinary Analysis Based on a Polynomial Chaos Sensitivity Study . . . . .</b>	<b>113</b>
Sylvain DUBREUIL, Nathalie BARTOLI, Christian GOGU and Thierry LEFEBVRE	
4.1. Introduction. . . . .	113
4.2. MDA with model uncertainty . . . . .	115
4.2.1. Formalism . . . . .	115
4.2.2. Solving the random MDA . . . . .	119
4.2.3. Approximation of the quantity of interest using sparse polynomial chaos . . . . .	122
4.3. Sensitivity analysis and uncertainty reduction . . . . .	124
4.3.1. Introduction . . . . .	124
4.3.2. Sobol' indices approximated by polynomial chaos . . . . .	126
4.4. Application to an aeroelastic test case . . . . .	128
4.4.1. Presentation . . . . .	128
4.4.2. Construction of disciplinary metamodels . . . . .	131
4.4.3. Sensitivity analysis and uncertainty reduction . . . . .	133
4.5. Conclusion . . . . .	140
4.6. References . . . . .	140

<b>Part 2. Taking Uncertainties into Account: Reliability Analysis and Optimization under Uncertainties</b> . . . . .	143
<b>Chapter 5. Rare-event Probability Estimation</b> . . . . .	145
Jean-Marc BOURINET	
5.1. Introduction. . . . .	145
5.1.1. Mapping to the multivariate standard normal space. . . . .	147
5.1.2. Copulas and correlation . . . . .	149
5.1.3. Isoprobabilistic transformations . . . . .	152
5.2. MPFP-based methods. . . . .	159
5.2.1. First-order reliability method . . . . .	159
5.2.2. Second-order reliability method . . . . .	163
5.3. Simulation methods . . . . .	166
5.3.1. Crude MC simulation . . . . .	167
5.3.2. Subset simulation . . . . .	168
5.3.3. IS and CE methods . . . . .	182
5.4. Sensitivity measures. . . . .	189
5.4.1. Introduction . . . . .	189
5.4.2. FORM . . . . .	191
5.4.3. Crude MC simulation and subset simulation . . . . .	195
5.5. References . . . . .	198
<b>Chapter 6. Adaptive Kriging-based Methods for Failure Probability Evaluation: Focus on AK Methods</b> . . . . .	205
Cécile MATTRAND, Pierre BEAUREPAIRE and Nicolas GAYTON	
6.1. Introduction. . . . .	205
6.2. Presentation of Kriging . . . . .	208
6.2.1. Principle . . . . .	208
6.2.2. Identification of Kriging hyperparameters . . . . .	209
6.2.3. Kriging-based prediction. . . . .	210
6.2.4. Illustration of Kriging-based prediction. . . . .	210
6.3. Employing Kriging to calculate failure probabilities . . . . .	211
6.3.1. The EFF function . . . . .	212
6.3.2. The U function . . . . .	212
6.3.3. The $IMSE_T$ function . . . . .	213
6.3.4. The SUR function. . . . .	213
6.3.5. The H function . . . . .	214
6.3.6. The OBJ function . . . . .	214
6.3.7. The L function. . . . .	214

6.3.8. Discussion . . . . .	214
6.4. The AK-MCS method: presentation and generic principle . . . . .	215
6.4.1. Presentation of the AK-MCS method . . . . .	215
6.4.2. Illustration of the AK-MCS method . . . . .	217
6.4.3. Discussion . . . . .	219
6.5. The AK-IS method for estimating probabilities of rare events. . . . .	219
6.5.1. Presentation of the AK-IS method . . . . .	219
6.5.2. Illustration of the AK-IS method . . . . .	220
6.5.3. Discussion . . . . .	220
6.6. The AK-SYS method for system reliability problems. . . . .	222
6.6.1. Some generalities about system reliability analysis . . . . .	222
6.6.2. Presentation of the AK-SYS method . . . . .	223
6.6.3. Illustration of the AK-SYS method . . . . .	225
6.6.4. Alternatives to the AK-SYS method. . . . .	226
6.6.5. Application to problems indexed by a subset. . . . .	227
6.7. The AK-HDMR1 method for high-dimensional problems . . . . .	229
6.7.1. HDMR functional decomposition . . . . .	230
6.7.2. Presentation of the AK-HDMR1 method . . . . .	231
6.8. Conclusion . . . . .	233
6.9. References . . . . .	234

## **Chapter 7. Global Reliability-oriented Sensitivity Analysis under Distribution Parameter Uncertainty . . . . . 237**

Vincent CHABRIDON, Mathieu BALESDENT, Guillaume PERRIN,  
Jérôme MORIO, Jean-Marc BOURINET and Nicolas GAYTON

7.1. Introduction. . . . .	237
7.2. Theoretical framework and notations. . . . .	242
7.3. Global variance-based reliability-oriented sensitivity indices . . . . .	244
7.3.1. Introducing the Sobol' indices on the indicator function. . . . .	244
7.3.2. Rewriting Sobol' indices on the indicator function using Bayes' Theorem . . . . .	245
7.4. Sobol' indices on the indicator function adapted to the bi-level input uncertainty . . . . .	247
7.4.1. Reliability analysis under distribution parameter uncertainty . . . . .	247
7.4.2. Bi-level input uncertainty: aggregated versus disaggregated types of uncertainty. . . . .	249
7.4.3. Disaggregated random variables . . . . .	250
7.4.4. Extension to the bi-level input uncertainty and pick-freeze estimators . . . . .	251
7.5. Efficient estimation using subset sampling and KDE . . . . .	253

7.5.1. The problem of estimating the optimal distribution at failure . . .	253
7.5.2. Data-driven tensorized KDE . . . . .	257
7.5.3. Methodology based on subset sampling and data-driven tensorized G-KDE . . . . .	258
7.6. Application examples . . . . .	258
7.6.1. Example #1: a polynomial function toy-case . . . . .	261
7.6.2. Example #2: a truss structure . . . . .	264
7.6.3. Example #3: application to a launch vehicle stage fallback zone estimation . . . . .	267
7.6.4. Summary about numerical results and discussion . . . . .	274
7.7. Conclusion . . . . .	274
7.8. Acknowledgments . . . . .	275
7.9. References . . . . .	275
<b>Chapter 8. Stochastic Multiobjective Optimization: A Descent Algorithm . . . . .</b>	<b>279</b>
Quentin MERCIER and Fabrice POIRION	
8.1. Introduction . . . . .	279
8.2. Mathematical refresher . . . . .	281
8.2.1. Stochastic processes . . . . .	281
8.2.2. Convex analysis . . . . .	282
8.3. Multiobjective optimization and common descent vector. . . . .	288
8.3.1. Binary relations . . . . .	288
8.3.2. Multiobjective optimization, Pareto preorder . . . . .	290
8.3.3. Common descent vector . . . . .	296
8.4. Descent algorithm for multiobjective optimization and its extension to the stochastic framework. . . . .	298
8.4.1. Multiple gradient descent algorithm . . . . .	298
8.4.2. Stochastic multiple gradient descent algorithm . . . . .	300
8.5. Illustrations . . . . .	305
8.5.1. Performance of the SMGDA algorithm . . . . .	305
8.5.2. Multiobjective approach to RBDO problems. . . . .	309
8.5.3. Rewriting the probabilistic constraint . . . . .	310
8.6. References . . . . .	316
<b>List of Authors . . . . .</b>	<b>319</b>
<b>Index . . . . .</b>	<b>321</b>