Contents

Preface ix

Introduction xi

Chapter 1. Elliptic Equations in Canonical Domains with the Dirichlet Condition on the Boundary or its Part 1

1.1. A standard finite-difference scheme for Poisson's equation with mixed boundary conditions 1
1.1.1. Discretization of the BVP 1
1.1.2. Properties of the finite-difference operators 3
1.1.3. Discrete Green's function 8
1.1.4. Accuracy with the boundary effect 10
1.1.5. Conclusion. 17
1.2. A nine-point finite-difference scheme for Poisson's equation with the Dirichlet boundary condition 18
1.2.1. Discretization of the BVP 19
1.2.2. Properties of the finite-difference operators 20
1.2.3. Discrete Green's function 24
1.2.4. Accuracy with the boundary effect 27
1.2.5. Conclusion. 31
1.3. A finite-difference scheme of the higher order of approximation for Poisson's equation with the Dirichlet boundary condition 31
1.3.1. Auxiliary results. 32
1.3.2. Accuracy with the boundary effect 43
1.3.3. Conclusion. 46
1.4. A finite-difference scheme for the equation with mixed derivatives 46
1.4.1. Discretization of the BVP 47
1.4.2. Properties of the finite-difference operators 49
1.4.3. Discrete Green's function 54

1.4.4. Accuracy with the boundary effect 57
1.4.5. Conclusion. 67

Chapter 2. Parabolic Equations in Canonical Domains with the Dirichlet Condition on the Boundary or its Part 69

2.1. A standard finite-difference scheme for the one-dimensional heat equation with mixed boundary conditions 69
2.1.1. Discretization of the problem 69
2.1.2. Accuracy with the boundary effect 71
2.1.3. Accuracy with the initial effect 77
2.1.4. Conclusion. 81
2.2. A standard finite-difference scheme for the two-dimensional heat equation with mixed boundary conditions 82
2.2.1. Discretization of the differential problem and properties of the finite-difference operators 82
2.2.2. Discrete Green's function 85
2.2.3. Accuracy with the boundary effect 86
2.2.4. Conclusion. 101
2.3. A standard finite-difference scheme for the two-dimensional heat equation with the Dirichlet boundary condition 102
2.3.1. Discretization of the differential problem. 102
2.3.2. Accuracy with the boundary effect 103
2.3.3. Accuracy with the initial effect 111
2.3.4. Conclusion. 112

Chapter 3. Differential Equations with Fractional Derivatives 115

3.1. BVP for a differential equation with constant coefficients and a fractional derivative of order ½ 115
3.1.1. A weighted estimate for the exact solution 115
3.1.2. Weighted estimates for approximate solutions 118
3.1.3. Conclusion. 123
3.2. BVP for a differential equation with constant coefficients and a fractional derivative of order $\alpha \in (0,1)$ 124
3.2.1. A scale of weighted estimates for the exact solution. 124
3.2.2. The scale of weighted estimates for approximate solutions 136
3.2.3. A numerical example and conclusion 143
3.3. BVP for a differential equation with variable coefficients and a fractional derivative of order $\alpha \in (0,1)$ 145
3.3.1. Differential properties of the exact solution 145
3.3.2. The accuracy of the mesh scheme 162
3.3.3. Conclusion. 166
3.4. Two-dimensional differential equation with a fractional derivative. 166

3.4.1. A weighted estimate for the exact solution 166
3.4.2. A mesh scheme of the first order of accuracy 172
3.4.3. A mesh scheme of the second order of accuracy 177
3.4.4. Conclusion. 181
3.5. The Goursat problem with fractional derivatives 181
3.5.1. Properties of the exact solution 181
3.5.2. The accuracy of the mesh scheme 198
3.5.3. Conclusion. 212

Chapter 4. The Abstract Cauchy Problem 213

4.1. The approximation of the operator exponential function in a Hilbert space 213
4.2. Inverse theorems for the operator sine and cosine functions 230
4.3. The approximation of the operator exponential function in a Banach space 236
4.4. Conclusion 247

Chapter 5. The Cayley Transform Method for Abstract Differential Equations 249

5.1. Exact and approximate solutions of the BVP in a Hilbert space. 249
5.1.1. Auxiliary results. 250
5.1.2. The exact solution of the BVP 257
5.1.3. The approximate method without saturation of accuracy 267
5.1.4. The approximate method with a super-exponential rate of convergence. . 274
5.1.5. Conclusion. 281
5.2. Exact and approximate solutions of the BVP in a Banach space 282
5.2.1. The BVP for the homogeneous equation 282
5.2.2. The BVP for the inhomogeneous equation 292
5.2.3. Conclusion. 305

References. 307

Index. 315